ALTERNATING CURRENTS FOR TECHNICAL STUDENTS

BY

CALVIN C. BISHOP, E.E.

Head of the Department of Drafting and Design
Technical High School, Buffalo, N. Y.

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PREFACE

The purpose of the book is to explain graphically and with simple mathematics the fundamental principles of alternating-current theory, circuits, and apparatus. The book is intended for technical and vocational students, engineering students and others who are familiar with direct-current theory but find themselves called upon to become familiar with alternating-current theory and apparatus.

In selecting material from the vast field of alternating-current theory and practice, the author has been guided by his association with young engineers, technical assistants and students for the past twenty years. He has included those topics of theory that are fundamental, and apparatus that is standard and in common use in which the application of principles readily appears. He believes that a mastery of the subjects chosen will form the necessary background for an understanding of other apparatus, that a young man working in the electrical field will encounter in his daily work.

Topics closely related to direct currents are covered as briefly as possible, others at more length. Details of construction and operation are made clear by sketches, scale drawings, and pictures of apparatus representing different manufacturers. Enough practical problems are included to test the reader's progress but not enough to discourage him.

The author wishes to express his appreciation to Professor W. P. Graham for examination of the manuscript and for helpful suggestions, and to the various companies who have so generously furnished technical data and photographs of their apparatus.

CALVIN C. BISHOP

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ALTERNATING CURRENTS FOR TECHNICAL STUDENTS

CHAPTER I

ALTERNATING CURRENTS

Present Ideas of Current Flow. The present idea of curre flow is that extremely minute particles called electrons, which them selves are negative, move within and among the molecules of substance. Current flow is, then, electron flow

In brief, the theory is as follows: matter is made up of sma particles known as molecules which are in size almost within th range of a microscope These molecules are in turn made up c particles much smaller, known as atoms. The atom is not a soli particle, but consists of a central part or nucleus around whic move other small particles known as electrons. These electror move in shells or orbits. A common illustration is that of th solar system in which we live The sun may be thought of as th nucleus and the planets as the electrons To make the analog complete, the sun must be reduced to a diameter about that of th earth, and the whole solar system made microscopic in size.

The nucleus holds a positive charge, and the electrons negativ charges. The electrons are prevented from falling out of thei orbits and going to the nucleus by the kinetic energy they possess Since the distance between the nucleus and the electrons that en circle it is very great in proportion to the size of the electrons, i follows that electrons can pass readily among the various group of atoms that form the molecules. While electrons are thought o as being held in their orbits by the nucleus, forming complete atomi systems, some free electrons that become detached from regula systems are supposed to exist. Conductors have many free elec trons, insulators few.

As stated in the opening paragraph current flow is electron flow. While it is accepted that electrons move about at the same speed as light (300,000,000 meters or 186,000 miles per second), it is not to be understood that electrons are forced from one end of a transmission line to the other at each impulse of the generator. The impulse is transmitted from electron to electron or atom to atom.

A most excellent illustration of current flow is given by Mills who states that a conductor may be thought of as a large basket ball court in which there are many players, each having a definite section assigned to him in which he may play Many balls are put in play and thrown in a haphazard manner from player to player Each player is kept busy throwing away the balls that come to him. If suddenly a large number of balls is thrown into one end of the court, and an equal number of balls is withdrawn at the other end, the number of balls in the court does not change nor is it necessary for a given ball to go the whole length of the court, yet "current flows" through the court or "conductor."

Positive and Negative Charges — Electromotive Force. Following out the theory outlined briefly, the nucleus is positive and the electrons negative Under certain normal conditions, each nucleus may have the necessary number of electrons so that a condition of balance exists Suppose now, that balance is disturbed, either by rubbing two substances together and tearing apart electron systems, or disturbing them by other mechanical means as in the case of an electric generator. Then a force will exist between the electron systems that can only be satisfied by the systems being pulled together, or a conducting path being provided through which the electrons can flow. Such a flow would restore balance. This force is spoken of as *electric potential* or *electromotive force*.

Direction of Flow. For the present, only the method of conducting electricity most common in engineering, namely by metal conductors, will be considered. In metals, the atoms have a large mass and are not readily movable. That is, the substance is "solid." It follows, then, that the positive charges which are on the nuclei cannot move about freely in the conductor. Conduction of an impulse set up at one end of a conductor can take place, then,

only by the movement of electrons. Negative charges are attracted towards positive charges, that is, electrons flow from the negative end of a wire through the wire towards the positive end. Unfortunately, the electron theory which is now well established by recent discoveries in radioactive materials, was not developed when the markings of + and − were made. The old idea of current flow is the reverse of the new

At present we still maintain the old notation, namely that current flows from the + side of a battery through the external circuit to the negative side. This notation readily enables us to analyze most direct and alternating current circuits, but is confusing in the case of circuits containing apparatus such as vacuum tubes used in X-Ray, radio, and other work. In circuits containing such apparatus, we must keep in mind electron flow, and consider it always the reverse of the present notation of current flow.

Electrons and Lines of Force. The facts of magnetism can be explained by the electron theory, but as the common "lines of

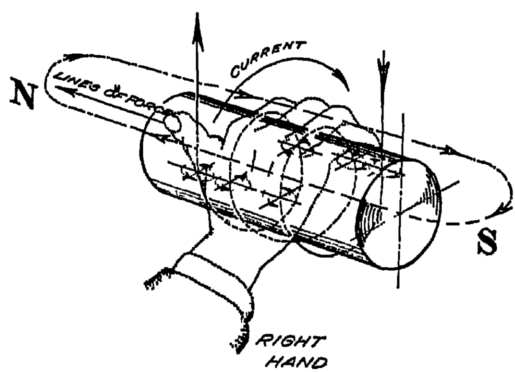


Fig 1. — Fundamental Relations of Current and Lines of Magnetic Force.

force"[1] scheme is so simple to apply, it will be used in this book. A *north pole* is understood to be the end from which lines of magnetic force proceed from the magnet, and the *south pole* the

[1] More strictly "Lines of magnetic induction."

end at which they return, having made a circuit through the space outside the magnet.

Lines of force "encircle" a conductor, pointing clockwise if we stand facing the end of the conductor at which current flows away from us. An electro-magnet is simply a coil of wire which multiplies the number of these encircling lines of force. If the magnet has an iron core, the core forms a better path than air for the lines and they are "bunched" and we have a "strong magnet."

The fundamental relations of current and lines of force are shown by Fig. 1.

Alternating Electromotive Force and Current. An alternating electromotive force is one that periodically changes its direction from plus to minus according to a definite law. An alternating current is the current that would flow in a closed circuit if an alternating electromotive force were impressed across the terminals of the circuit.

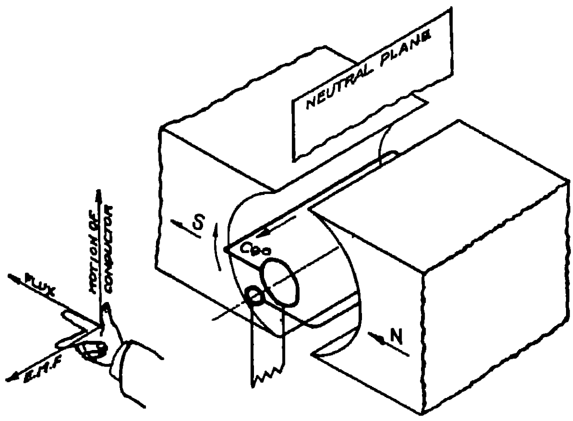


Fig 2 — Simple Two Pole Alternator

Generation of an Alternating Electromotive Force. Consider two poles as in Fig. 2 with a coil arranged to turn in the space between the poles Considering conductor C_{90} it is clear from the sketch that, as the coil turns, E. M. F will be generated in one direction while C_{90} is passing pole S and in the opposite direction

while it is passing pole N. Further, while it is passing the center line of the poles it will be cutting squarely across the lines of force from the pole, and while it is passing across a line at right angles to this center line (through the neutral plane) it will be moving parallel to the lines of force and therefore not cutting them at all. These facts may be shown by applying the three-finger rule to Fig. 2 and Fig. 3. Figure 3 is an end view of Fig. 2 with the lines of

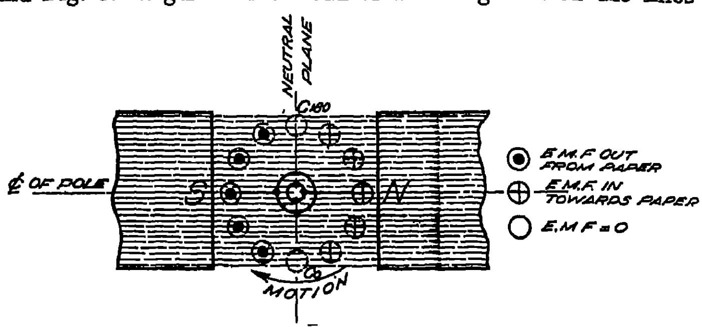


Fig 3. — End View of Alternator Showing Evenly Distributed Flux

force uniformly distributed across the poles. When the field is uniformly distributed and the coil turns at a uniform rate of speed, it has been found by experiment and mathematically that the value of the E M. F. generated is proportional to the sine of the angle through which the coil has turned from the neutral plane. The sine of 0° is 0 and the sine of 90° is 1, so if we take the zero position of the coil in the neutral plane as C_0, Fig. 3, then after the coil has turned 90° or to C_{90}, the E. M F. will have risen from zero to maximum After the coil has turned from C_{90} to a position 90° farther or to C_{180} the E. M F. will have fallen to zero. When it has turned to C_{270} it will have risen to a maximum in the other direction and finally, when it has reached C_0 again, the E. M. F. will have dropped to zero.

A curve showing the change in E M. F as the coil turns may be plotted by taking degrees along a horizontal line (abscissa), and plotting to scale, above each degree chosen, the value of the sine of that angle Figure 4 shows such a curve, and seven positions of the coil with the corresponding directions and values of E. M. F.

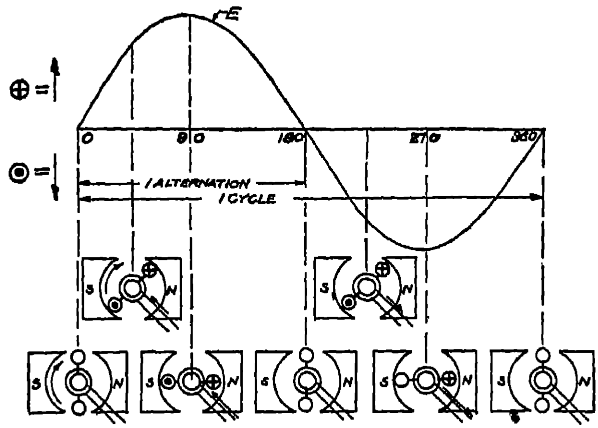


Fig 4.—E. M. F Wave with Generating Coil in Various Positions

Alternation and Cycle. The E. M. F. in rising from zero to a maximum and returning to zero again is said to make an alternation. When the E. M F. has made two alternations it is said to have completed a cycle or period. The number of cycles in a second is called the frequency of the circuit. A study of Fig. 4 will show that the E. M. F. will have made a cycle when a conductor has passed a pair of poles In the case of a two-pole machine, this is the same as the coil making one revolution. In a four-pole machine the coil will have to make only one half a revolution to pass a pair of poles and complete a cycle; in a six-pole machine only one third of a revolution, etc. From the above it follows that:

When p = number of pairs of poles

f = frequency in cycles per second

v = revolutions per minute

That $f = \frac{pv}{60}$, (1) $p = \frac{60f}{v}$, (2) $v = \frac{60f}{p}$ (3)

Wave of Alternating Current. Suppose that the E. M. F. generated by the simple alternator of Fig. 2 were impressed upon a circuit containing only resistance. As the coil turned from the neutral plane to a position 90° from the neutral plane, the E. M. F. would rise from zero to maximum. The actual amount of current

that would flow in the circuit at each instant as the coil turned, would equal the E. M F at the particular instant, divided by the resistance As the coil passed the center of the pole the E. M. F. would begin to fall and the current would fall likewise, its value always being the instantaneous E. M. F. divided by the resistance. Similarly after the E. M. F. had passed the zero point, the E M. F. and current would rise and fall again but in the opposite direction. A curve showing these changes in current is shown by

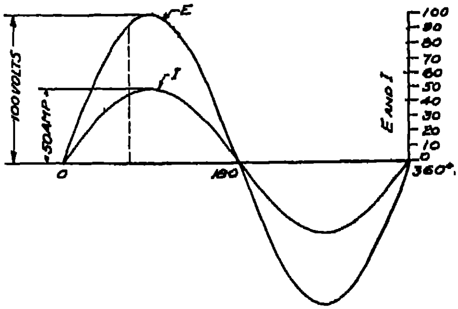


Fig 5. — E M F and Current Waves. (R = 2 Ohms)

Fig 5. The drawing shows the current wave in a circuit containing only resistance of a value of 2 ohms. The E. M. F. has a maximum value of 100 volts. A study of the drawing will show that the value of any ordinate of the current curve is equal to the ordinate of the E. M F wave at the same instant, divided by the resistance

Meaning of "In Phase." Referring to Fig. 5 it will be seen that both the E M F. wave and the current wave pass through zero at the same time, and that they have their maximum values at the same time and in the same direction. When two waves have their zero values at the same time and their maximum values at the same time and in the same direction, they are said to be in phase.

E. M. F.'s may be in phase with each other; currents may be in phase with each other, and E. M. F.'s may be in phase with currents.

Lag and Lead. When one wave starts at a later time than another, the second wave lags behind the first. Conversely the first wave leads the second. In plotting waves of E. M. F. or current, time is reckoned from the left towards the right, hence a

wave drawn in the same direction as another wave, but with its zero and maximum values to the right of the first wave, lags the first wave. In Fig. 6, I lags E by 30°, or E leads I by 30°.

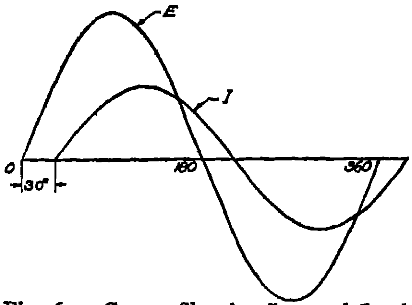


Fig 6. — Curves Showing Lag and Lead.

Effective Value of Current. In measuring alternating currents the ordinary meter indicates what is known as the effective value of current. An alternating current of such strength that it will heat a conductor just as much as one ampere of direct current is called one "effective" ampere of alternating current. To clearly understand what is meant by "same heating value," plot a wave of alternating current as in Fig. 7. Take ordinates at any convenient points as

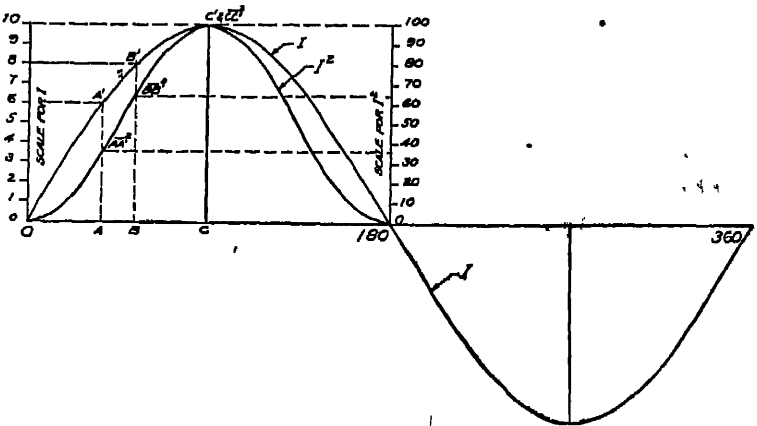


Fig. 7 — Current Wave and Curve Showing Values of Current Squared.

A, B and C. Measure them. Suppose AA′ = 6, BB′ = 8, CC′ = 10. If, at the instants chosen, these currents were allowed to flow through a resistance, they would heat the resistance proportional to $\overline{AA'}^2 = 36$, $\overline{BB'}^2 = 64$, $\overline{CC'}^2 = 100$. So, if we should

plot a point 36 over A, 64 over B, 100 over C, etc., we would have points on a curve proportional to the heating that these instantaneous currents would give. In order to draw this curve on Fig. 7, we must select a different scale for it to have the curve come the same height. If we use a scale for I^2 one tenth as large as for I, then $\overline{CC'}^2 = 100$ will be the same height as CC′ and $\overline{BB'}^2 = 64$, $\overline{AA'}^2 = 36$ will be plotted as shown at $\overline{BB'}^2$ and $\overline{AA'}^2$. If ordinates are taken throughout the entire cycle and squared and plotted, a curve like Fig. 8 will result. Both lobes of the curve of Fig. 8 are above the horizontal line, because, after 180 degrees are reached, the current values are minus, and a minus quantity squared gives a plus result.

Since the curve of Fig. 8 is plotted from values of current squared

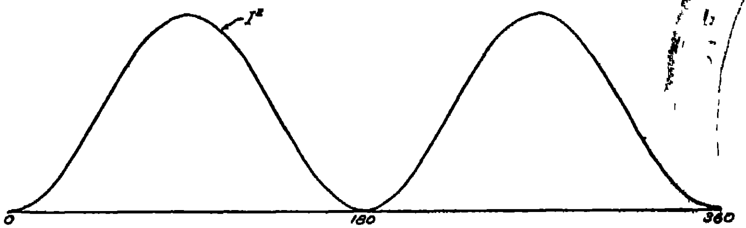


Fig 8 — Complete Curve of Current Squared Values.

which are proportional to the heating, if we take the average ordinate of the curve, we shall have the "average square" or "mean square." The square root of this quantity is called Root of Mean Square or Effective Value. A current that is of such a maximum value that it gives a "root of mean square" value of 1 will heat a conductor just as much as one ampere of direct current.

Methods of Finding Effective Value. The average ordinate (mean square) may be found from a curve like Fig 8 by measuring the area included between the curve and the horizontal line by a planimeter and dividing this area by the base line 0–180. The square root of the value thus found will be the effective value.

The effective value may be found fairly accurately without a planimeter as follows: Divide the base line of one lobe of the curve into any convenient number of parts. Erect a full line at

each point as AA′ in Fig. 9. Half way between these full lines erect dash lines as BB′. On a strip of paper, or with a scale, total up the length of all these dash lines and divide the quantity that you obtain by the number of dash lines. The quotient will be the average height of the curve. The square root of this quotient will be the effective value.

In Fig. 9 the sum of the current squared ordinates of curve I^2 (dash lines) totals 22.8 divisions or units. From curve I, one division equals 2000 current squared units. There are 9 dash lines, each of which represents closely the average height of the section under the curve of which the dash line forms a center line, so that the "average square" equals $\frac{22.8 \times 2000}{9} = 5067$ units. Hence the "square root of average square" equals $\sqrt{5067}$, which is approximately 71. That is, the effective value is approximately 71 per cent of the maximum value.

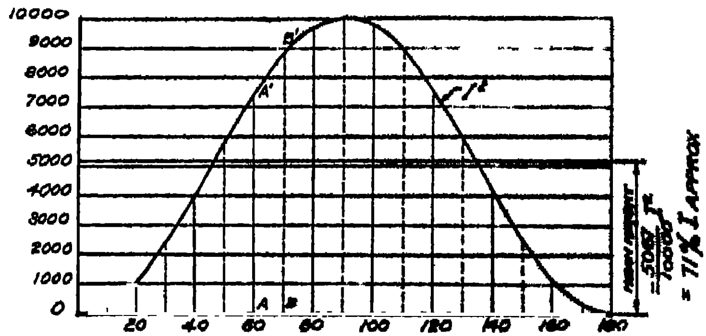


Fig. 9. — Method of Finding Effective Value.

By a more accurate method of calculation involving higher mathematics, the effective value for a sine wave has been found to be 70.7 per cent of the maximum value. In computations, use this more accurate value, or $I_{ef} = I_{max} \times .707$

Average Value of Current. It is desirable in certain alternating current work to know the average value of the current during a cycle. When the shape of the current wave is known, the average value may be found by measuring by means of a planimeter, the

area included between the wave and base line, and dividing this area by the length of the base line. Since both half-waves are alike, this method may be shortened by using only one half-wave.

A method similar to that used in obtaining the effective value is shown by Fig. 10. The wave of Fig 10 is curve I of Fig 7 redrawn. Divide the base into equal parts and erect full lines as AA′ at each point of division. Erect dash lines as BB′ half way between the full lines. The length of each dash line represents very nearly the average height of the section of which it is a center line. The larger the number of sections, the more nearly these dash lines approach the true average height of the sections. The sum of the dash lines, divided by the number of them used, is the average height of the wave.

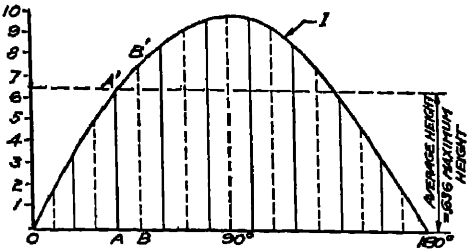


Fig 10. — Method of Finding Average Value of Current.

Effective and Average Voltages. If we consider that the sine wave of current used in explaining the effective and average value of current was produced by a voltage acting across a constant resistance, the voltage wave would have the same shape as the current wave, since at every instant E = RI.

The effective and average voltages are in the same proportion to the maximum value of voltage, that the effective and average values of current are to the maximum value of current.

Thus for a sine wave of voltage

$$E_{eff} = E_{max} \times .707 \tag{4}$$

$$E_{av} = E_{max} \times .636 \tag{5}$$

Harmonics. The true sine wave has been considered thus far in the discussion of alternating electromotive force and current. In actual practice we have to deal with waves that are distorted from the true sine wave, due to certain conditions that may exist in the apparatus or circuits. Such distorted waves [illegible]

are found to be made up of a main or fundamental wave of the frequency of the circuit, and other waves of higher frequency which are superimposed on the fundamental wave. The effect of these higher frequency waves is to give the fundamental wave a rippled, peaked, or flat topped effect, its exact shape depending on the particular frequency and amplitude of the waves that are superimposed on the fundamental wave.

The superimposed waves are called harmonics and a wave distorted by such higher frequency waves is said to have harmonics.

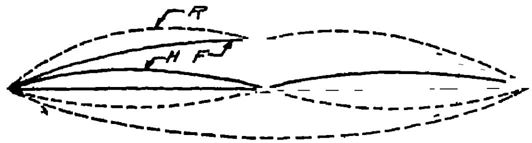


Fig 11 — Effect of Harmonics on Fundamental Wave

An analogy occurs in the case of musical instruments having vibrating strings, and may be easily illustrated If a string be tightly stretched between two points and set in vibration it will vibrate as a whole and give out a certain tone. If now a rider such as a piece of wire be held at the center of the string and one half be made to vibrate, the other half of the string will vibrate also. The frequency will be twice that of the string vibrating without the rider. If now the rider be released and allowed to vibrate with the string, the string will vibrate as a whole, and also in the two sections. The tone that it will give out will be different with the rider than without the rider, due to the effect of the higher frequency vibrations. These higher frequency vibrations are called overtones or harmonics. The effect may be pictured by the diagram of Fig. 11.

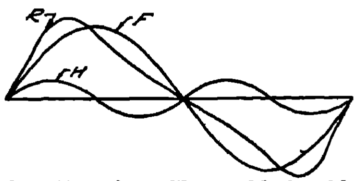


Fig 12 — Sine Wave with Double Frequency Harmonics.

F represents the string vibrating as a whole and H the double frequency vibrations due to the rider If we plot curve R using as indicated the sum of ordinates of F and H we will get a curve that illustrates the effect of the harmonics.

Carrying the analogy to the case of a wave of alternating E. M. F. we might have a condition like Fig 12. Here we have a fundamental wave F and a harmonic wave of double frequency H. The sum of these two waves gives a wave R whose two lobes are ***unlike in shape.***

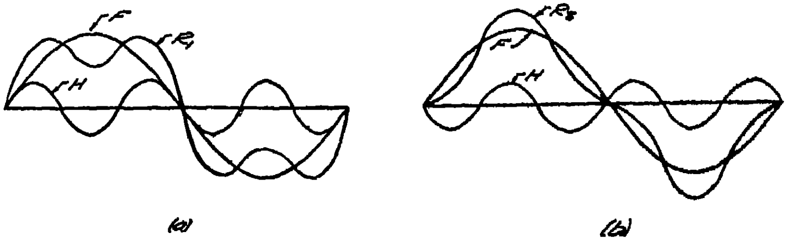


Fig 13. — Sine Waves with Triple Frequency Harmonics.

If we plot a sine wave F as in Fig. 13 and superimpose on it a wave H of triple frequency we shall get a wave either of the shape of R_1 or R_2 depending on the phase relation of the harmonic wave to the fundamental In Fig. 13 (a) and (b) the two lobes of the resultant waves are of the same shape. As we are accustomed

Fig. 14 — Wave Containing 5th and 7th Harmonics
(L. F. Curtis Trans. A.I.E.E. 1919)

to waves with lobes of the same shape in actual practice, we conclude that the odd harmonics appear only in such waves and the even harmonics do not exist or cancel each other.

Fig 14 shows a photograph of an actual wave taken by means of an oscillograph (Chap. XII). This wave contains fifth and seventh harmonics.

Power in an Alternating Current Circuit. Figure 15 shows an E. M. F of 100 volts and a current of 60 amperes in phase with the E. M F. At every instant throughout the cycle the power developed in the circuit is EI. For instance, when E = 50, I = 30 and P = EI = 50 × 30 = 1500 watts. The power developed through the whole cycle is the total of all the power developed by each instantaneous E. M. F. multiplied by the cor-

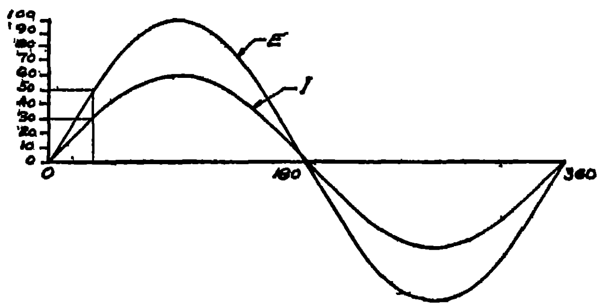


Fig. 15. — Curves Showing E. M. F. of 100 Volts (Max) and 60 Amperes (Max) in Phase

responding instantaneous current. Hence, if ordinates are erected to the current and E. M. F. curves at any convenient points along the horizontal line, and at each point the ordinate to the E. M. F. curve be multiplied by the ordinate to the current curve, the product of the two ordinates will be the power at the point chosen. If all these instantaneous powers be added together and averaged, the result will be the power developed in the circuit. To illustrate, draw curve E, Fig. 16, with a maximum value of 100 and curve I with a maximum value of 60 in phase with curve E. Divide each lobe of the E. M. F. and current curve into 18 parts by erecting full vertical lines, such as AE, AI, etc. Scale the vertical lines and multiply each value of E obtained, by the corresponding value of I. For instance, at A, AE = 76.6, AI = 46, AE × AI = 3524, the power at that point. Plot the points that you obtain by multiply-

ing all the E's and I's. You will obtain curve P. Draw dash lines BP, etc., half way between the full lines and add all the dash lines together on a strip of paper, or total with a scale. Divide the sum that you obtain by the number of dash lines and the quotient will be the average height of the curve P or the average power for the cycle.

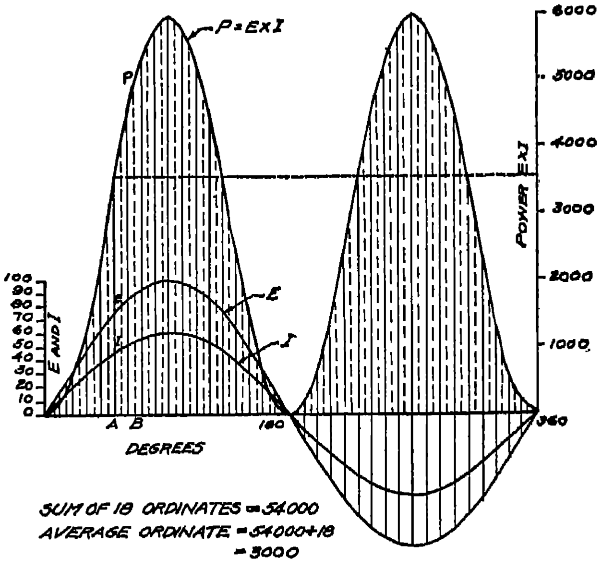


Fig 16. — Power Curve for Circuit with 100 Volts (Max) and 60 Amperes (Max.) in Phase.

In Fig. 16 the sum of the dash lines was 54000 and since the number of dash lines was 18, the average was $\frac{54000}{18}$ or the power was 3000 watts. In the case of Fig. 16 where the E. M. F. and current are in phase it will be noted that the power obtained by the method explained above is the same as the product of $E_{ef} \times I_{ef}$, viz. $100 \times .707 \times 60 \times .707 = 3000$. That is, when the E. M. F. and current are in phase, the power in an alternating current circuit is equal to the product of the effective volts and effective current.

When the E. M. F. and current are not in phase, the power in an alternating current circuit is less than the product of the effective E. M. F. and current. This is shown by Fig. 17 which shows an E. M. F. of 100 volts (max.) and a current of 60 amperes (max.) the

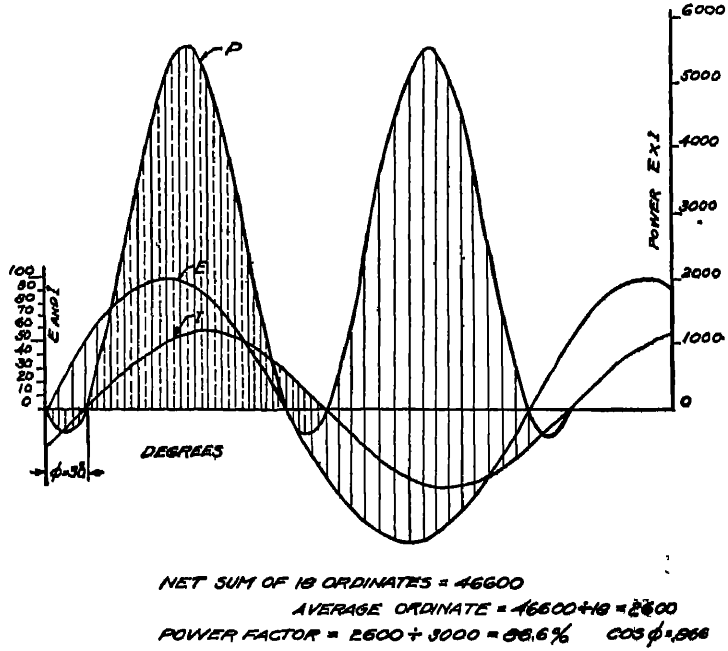


Fig. 17. — Power Curve for Circuit with 100 Volts (Max.) and 60 Amperes (Max) Lagging 30 °

same as Fig. 16 except that the current lags behind the E. M. F. by 30°. Using the method of computation explained for Fig. 16, the average power for the conditions of Fig. 17 is found to be 2600 watts. That is, when a current of 60 amperes (max.) lags an E. M. F. of 100 volts (max.) by 30°, the power in the circuit is only $\frac{26}{30}$ or 86.6 per cent of what it would be if the E. M. F. and current were in phase.

Power Factor. In an alternating current circuit a wattmeter reads the true power. Using Fig. 17 as an illustration, it would read the power as computed by taking the average of all the instantaneous values of power throughout the cycle, that is, 2600 watts. If a voltmeter and an ammeter were placed in the circuit, the voltmeter would read $100 \times .707 = 70.7$ volts and the ammeter would read $60 \times 707 = 42.4$ amperes. The product of the volts and amperes is called the "volt-amperes" or "apparent power," the wattmeter reading is called the "true power." The ratio of the true power to the apparent power expressed as a per cent is called the "power factor."

That is
$$\text{Power Factor} = \frac{\text{Wattmeter reading}}{\text{Volts} \times \text{amperes}} = \frac{P}{EI} \qquad (6)$$

where
$$P = \text{power in watts}$$
$$E = \text{effective volts}$$
$$I = \text{effective current}$$

$$\text{For Fig. 17, P F.} = \frac{2600}{3000} = .866 = 86\ 6\%$$

The power factor will be 100 per cent only when the current and E. M. F. are in phase. It will be less than 100 per cent if the current leads or lags by any angle to 90°. The more it lags or leads, the less the power factor. The power factor can never be more than 100 per cent

It has been found that in all cases with sine E. M F. and current the power factor is equal to the cosine of the angle of lag or lead Using Fig. 17 as an illustration again, the cosine of 30° is .866 which is the same as the value obtained from computations from the curves.

The above discussion of power factor applies to sing
circuits only, that is to circuits fed by a simple alternator
line wires like the one in Fig. 2.

Commercial Importance of High Power Factor. A study of the equation $P = EI \cos \phi = EI \times P.F.$ shows that for a given amount of power in kilowatts and with a constant voltage we have to increase the current as we lower the power factor. A v

important reason for selecting equipment and laying out circuits to insure a high power factor is at once apparent. We need smaller apparatus and smaller conductors.

While it is true that a machine like a fan or pump, needs for its operation, the same amount of power regardless of the power factor, it is more economical to supply this power at a high power factor than a low one as the following example will show.

Suppose that at 100 per cent power factor that 90 amperes are required at 1000 volts the power will be $90 \times 1000 = 90{,}000$ watts = 90 k w. If we use apparatus that operates on 90 per cent power factor, the current will be $\frac{90}{.90} = 100$ amperes. The 100 amperes consist of the 90 ampere component that does the useful work as before, but there is now a magnetizing or reactive current that may be considered as flowing back and forth in the line and apparatus only for the purpose of supplying the extra magnetic field needed with the lower power factor.

Graphically, the energy component may be represented by the base of a right angle triangle, the magnetizing current by the vertical side and the total current by the hypothenuse.

This magnetizing current causes an I^2R loss in the conductors just the same as the energy component that does the useful work. The magnetizing current causes trouble in generators by demagnetizing the fields and requiring extra excitation, and it makes trouble in transformers by causing poor voltage regulation. The apparatus fed from the transformers will not operate at its best on lowered voltage and so a further boosting up of generator voltage is required.

From the above it will appear that in order to get the 90 kilowatts that we need in the example given, we have to increase the power we put into the generator when we lower the power factor. We can go further with the illustration and imagine the generator just operating at its maximum efficiency when carrying 90 amperes and that when it is overloaded by the 100 amperes its efficiency will drop off. In this case we would have to put in still more power.

If we were buying the power from a power company, it would either have to install larger equipment for our needs, or give us service at reduced voltage when we operated at low power factor.

Methods of compensating for low power factor and thereby keeping down the size of generators, transformers, and line wires are discussed under synchronous motors and static condensers.

PROBLEMS

1. Find direction of E. M. F.

Find direction of motion.

Find proper polarity for poles.

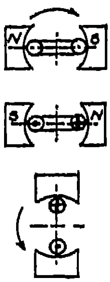


Fig. 18. — Application of Three-Finger Rule.

2. Plot a sine wave of E. M. F whose maximum value is 100. Use for ordinates $\frac{1}{4}'' = 10$ volts and for abscissas $\frac{1}{4}'' = 10°$.

3. What is the frequency of a machine with four poles that runs at 1800 r p.m ?

4. What is the effect of doubling the speed of an alternator on (a) the frequency, (b) the voltage?

5. How many poles must a machine have to give 25 cycles at 1500 r.p.m.?

6. How fast must a 6-pole machine run to give a frequency of 60 cycles?

7. On the same sheet with Problem 2, plot a curve of current whose maximum value is 60 amperes. The current is to lag behind the E. M. F. by 20°.

8. Replot the curve of Problem 2. On the same sheet, plot to a scale $\frac{1}{10}$ as large, a curve of I^2. Find the effective value of current. Use a planimeter or the method of ordinates

9. An E M. F. of a sine form has a maximum value of 150 volts. What is its effective value?

10. What is the maximum value of an E. M. F. of sine form in a circuit where the voltmeter reads 2300 volts?

11 Plot a sine wave of current of maximum value 30, and on the same sheet a second sine wave of current whose maximum value is 20 but which lags the first wave by 30°. Plot a third wave which you obtain by adding together the ordinates of the first and second waves Find the effective value of the wave that you obtain

12 A wattmeter reads 150 and at the same time the ammeter reads 20 and the voltmeter reads 100 What is the power factor of the circuit?

13 In a circuit the power factor is 80 per cent. How much is the current out of phase?

14 How many watts are delivered to a load if the amperes are 30 and the voltage 1000, if the power factor meter indicates 60 per cent?

CHAPTER II

ALTERNATORS

Constructional Features of Alternators. Alternating-current generators, according to their constructional features, may be divided into two general classes: first, those which have stationary-field magnets and revolving armatures, and second, those which have revolving-field magnets and stationary armatures. Alternators of the first class are called revolving-armature type machines and those of the second class, revolving-field type machines.

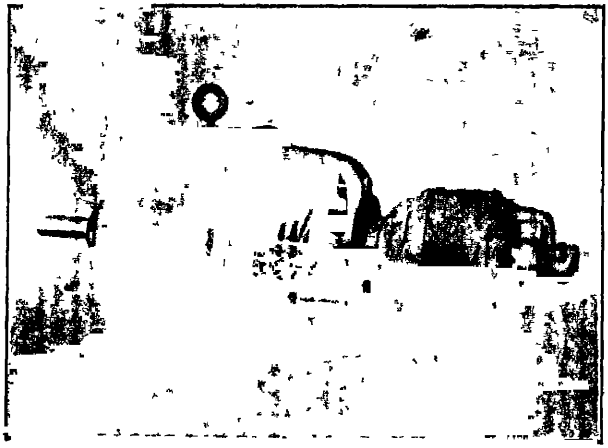


Fig. 19. — Revolving-Armature Type Alternator with Exciter. (Westinghouse Electric & Mfg. Co.)

Figure 19 shows a small belt-driven revolving-armature alternator. In general appearance it resembles a direct-current generator. It has, however, slip rings in place of a commutator. Brushes ride on these slip rings and conduct the alternating cur-

rent to the line. The field current, which is direct current, is supplied from a separate machine called an exciter. Revolving-armature machines are usually of small size and run at a fairly high speed. They are suitable for low and medium voltages.

Fig 20 — Revolving-Field Type Alternator (General Electric Company)

Figure 20 shows a revolving-field type alternator suitable for direct connection to a water wheel or engine This type of machine has slip rings also, but they carry the direct current to the fields. The windings of the armature are imbedded in slots in the iron punchings which form the armature core. These punchings are firmly clamped in a housing which forms the frame of the machine. Alternating current is taken from the armature through cables directly connected to the armature coils. There are no moving parts carrying the alternating current, so that these machines can be insulated for high voltages and can be built for large currents. Large, high-voltage machines are of the revolving-field type.

Modification of Direct-Current Armature to Obtain A. C. In the study of the generation of an electromotive force, as in Chapter I, it is clearly brought out that the electromotive force generated by a single coil of wire, as in Fig. 2, is alternating in character. In direct-current studies it is further shown that this electromotive force can be rectified by a commutator which consists of segments connected to the winding terminals instead of slip rings. From these facts, it follows that if an ordinary D. C. drum winding be tapped and slip rings connected to the taps, alternating current can be taken from the slip rings. Further, if the commutator be kept on, the machine will supply direct current to the brushes resting on the commutator and alternating current to those resting on the slip rings. This is shown diagrammatically

by Fig. 21. From these facts it is seen that it would be possible to wind the armatures of alternators with closed windings similar to direct-current machines but leave off the commutator and bring out taps to slip rings. Commercial machines, however, are wound with open windings, which are a development of the type shown by Fig. 2 instead of the closed type shown by Fig. 21.

Single-Phase and Polyphase Currents. A machine which has a winding of the type of Fig. 2 or Fig. 21 and but two slip rings generates what is known as single-phase current. There are but two line wires, the current flowing out on one wire and back on the other during one half-cycle, then reversing its direction for the other half-cycle. Such current is suitable for lamps, heating elements, certain types of motors and other apparatus. Reference to the elementary alternator of Fig. 2 again will show that much of the available space on the armature is unused. If a second winding exactly like the first be placed in the space and 90° from the first, this second winding will generate an E. M. F. exactly like the first but its wave will be displaced from the first by 90°. A circuit formed by two such windings and their line wires is called a two-phase circuit. If three windings are placed on the armature 120° apart, a three-phase winding is made. A two-phase winding as described would require four wires and a three-phase winding six wires. By connecting the windings as shown later a two-phase machine may have but three wires and a three-phase winding but three wires also. Two-phase and three-phase currents are known as polyphase currents and are necessary in order to operate certain types of motors and apparatus.

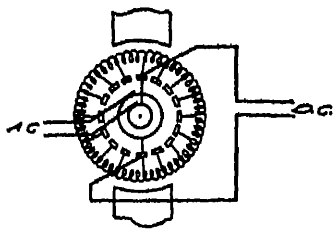


Fig. 21 — Diagram Showing how A. C. can be Obtained from a D. C. Armature.

Armature Windings for Alternators. In the pages that follow, the revolving-armature type of machine will be described first, then the revolving-field type machine.

Referring to the elementary alternator of Fig. 2 and Fig. 3,

the simple-loop winding can be better shown for the purpose of study if spread out flat That is, consider the winding as if drawn on a piece of paper which is wrapped around the armature,

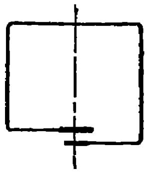


Fig. 22 —Loop Winding Spread Out Flat.

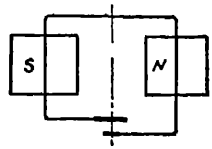


Fig 23 —Loop Winding with Poles Added.

and then the paper unwrapped. Figure 2 would appear as in Fig 22. If the poles are added to the drawing, the machine would appear as in Fig. 23. It is desirable that the discussion be general, so two more poles will be added, and by changing the winding accordingly, Fig. 24 will represent the simplest form of alternator with four poles.

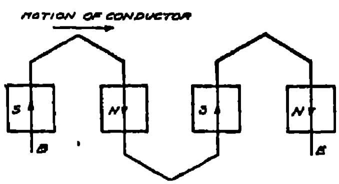


Fig. 24.—Simple Single-Phase Four-Pole Alternator Winding.

In this simple form of alternator shown by Fig. 24 there is one conductor per pole. If the conductors move from left to right as shown by the arrow, by applying the three-finger rule, E. M. F 's will be acting upward in the conductors which are under the S

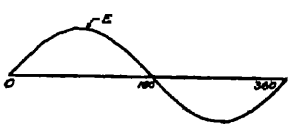


Fig 25.—E. M F Wave of Single-Phase Alternator

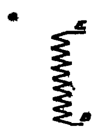


Fig 26 —Single-Phase Winding Simplified.

poles and downward in the conductors which are under the N poles The machine will generate a wave like Fig. 25. The winding itself may be represented by Fig. 26. Such a machine, that is, one with a single winding carried out to two line wires is

called a single-phase machine. In the winding shown, it is assumed that each pole generates 25 volts so that the voltage across B and E is 100 volts. This value of voltage will be used for the phase voltage in all the windings discussed.

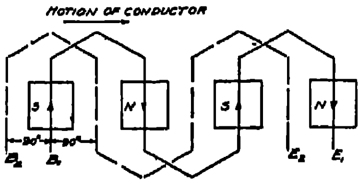


Fig 27. — Two-Phase Alternator Winding

If a second winding, exactly like the first, be placed in the space between the poles, 90 electrical degrees from the first winding, as shown by the winding drawn by a long dash line in Fig 27, a two-phase machine is made. The second winding will generate a wave exactly like the first but 90 degrees from it, Fig. 28.

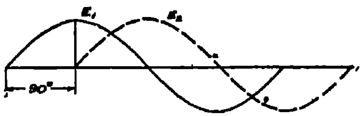


Fig 28 — E M F Waves of a Two-Phase Alternator.

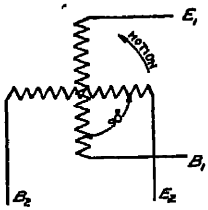


Fig 29. — Two-Phase Winding Simplified

If three windings are used and spaced 120° apart, a three-phase 6-lead machine is made. The windings are shown properly in place by Fig 30. Figure 31 shows them diagrammatically.

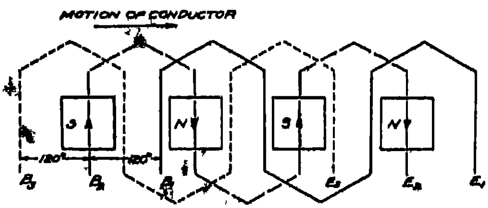


Fig 30. — Three-Phase Alternator Winding

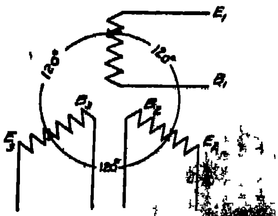


Fig 31. — Three-Phase Winding Simplified

Commercial machines have the following complications added to the simple winding of Figs. 24 to 30.

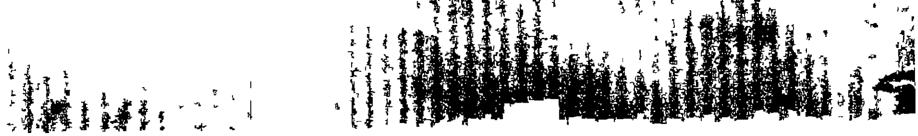
1. There may be more than one conductor per pole per phase.
2. There may be more than one slot per pole per phase.
3. Windings may be connected so that all the leads shown in the simple diagrams are not brought out.

1 *Machine with More than One Conductor per Pole per Phase.*

Fig 32.—Formed Coil. (General Electric Co.)

Machines are usually wound with formed coils, one of which is shown by Fig. 32. The coil may consist of a large number of turns of wire which are first wound on a wood or metal form and then taped. The span of the coil is such that, when in the slot, one side comes under the center of one pole and the other side comes near the center of the pole of opposite polarity. Coils which span from center to center of poles are said to have full pitch, those which do not span quite all of this

Fig. 33.—Formed Coils in Place in Slots (General Electric Co.)



distance are said to have fractional pitch. Only coils with full pitch will be considered in this discussion.

Figure 33 shows formed coils as they appear from the end when in place in the slots.

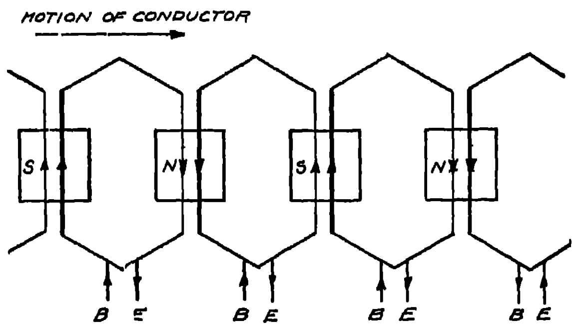


Fig. 34. — Formed Coils in Place on Armature.

For convenience in describing the sketches that follow, assume that the side of the coil that goes to the top of the slot is the left side as viewed when the armature is spread out, and show this by

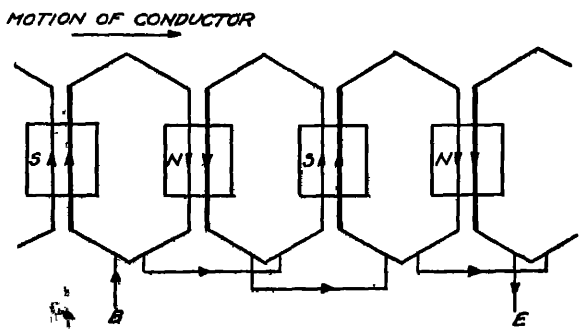


Fig. 35. — Formed Coils Connected for Highest Voltage.

a heavy line. Show the side of the coil that goes to the bottom of the slot by a light line. Figure 34 shows the machine of Fig. 24 as it would appear if wound with formed coils.

After the coils are in place on the armature, they must be connected. If they are to be put in series so as to get the greatest

voltage, connect so that the E M. F. of one coil will add itself to the E. M. F. of the next coil, etc. Figure 34 would be connected like Fig. 35 to get the highest possible voltage, that is, adjacent poles in series, or like Fig. 36 to get the least voltage, that is, alternate poles in parallel.

2. *Machines with More than One Slot per Pole per Phase.* When the winding is distributed so that there is more than one

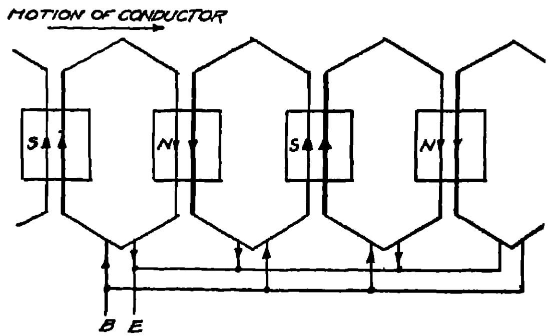


Fig 36 —Formed Coils Connected for Lowest Voltage.

slot per pole per phase, the coils may be put in the slots as in Fig 37. The coils lying in adjacent slots are first connected together as shown by the small loops, then the "B's" and the

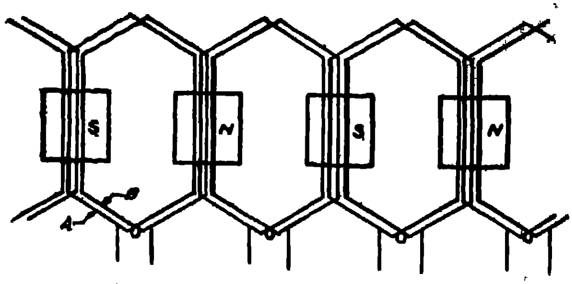


Fig 37 —Use of Two Coils per Pole per Phase.

"E's" are connected together to form poles, using the method of Fig. 35 or Fig 36 or a combination of these methods. In connecting up coils placed as in Fig. 37, one should remember not to

use a scheme for connecting that will put coils that are in adjacent slots, such as coils A and B, in parallel, because these coils are in slightly different fields and circulating current will result.

3. *Connection of Phases.* The following will be described:

(*a*) Single-phase
(*b*) Two-phase four-wire
(*c*) Two-phase three-wire
(*d*) Three-phase star (Y)
(*e*) Three-phase delta (mesh)

(*a*) Single-Phase. Since there are but two line wires in a single-phase machine, the two terminals lettered B and E in Fig. 24 form the line wires. If the machine is a revolving-armature type, these terminals are connnected to slip rings: if the machine is a revolving-field type, the line wires are connected directly to these two ends of the winding as they are brought out from the armature.

(*b*) Two-Phase Four-Wire Two wires are carried out from each phase. The effect is just the same as if the machine had two separate armatures, each delivering single-phase current and the armatures were keyed to the shaft 90 electrical degrees apart.

(*c*) Two-Phase Three-Wire. The windings shown by Fig. 29 may be connected so that one line wire may be eliminated, and work satisfactorily as in Fig. 38. It will be seen by inspection of Fig. 27 and Fig. 28, that, since the coils are 90 degrees apart, the voltage of phase 1 is a maximum when the voltage of phase 2 is zero and vice versa. Both voltages are, however, of the same effective value. That is, a voltmeter placed across B_1E_1 would read the same as when placed across B_2E_2.

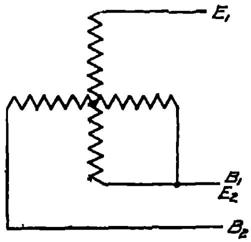


Fig. 38 — Two-Phase Three-Wire Connection

Due to the fact that the two windings are connected together, the two E. M. F.'s E_1 and E_2 add together, but not directly because they are out of phase Fig. 39(a). The effect is the same as two forces in mechanics acting at 90° with each other. To find

the voltage across $B_2 - E_1$, draw $E_{B_1E_1}$ Fig. 39(b) to scale to equal 100 volts in this case, and draw $E_{B_2E_2}$ 90° behind $E_{B_1E_1}$ also equal to 100 volts. Complete a square by drawing $E_{B_1E_1}E_{LL_1}$ and $E_{B_2E_2}E_{LL_1}$. The line voltage E_{LL_1} which is the diagonal of the square may be found, either by scaling the drawing or by calculation, to equal $\sqrt{100^2 + 100^2} = 141$ volts. That is, the voltage across the two outside wires of a two-phase three-wire system is equal to 1.41 times the voltage from the middle to either outside wire.

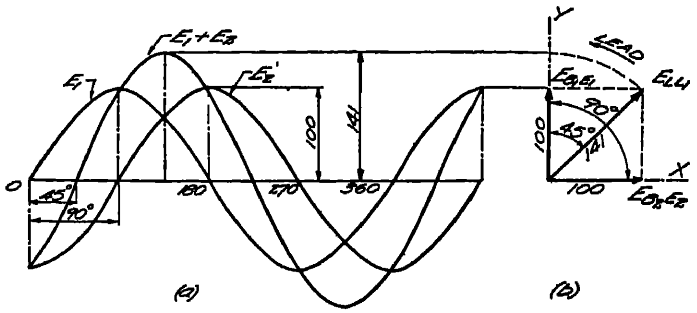


Fig 39 — Method of Showing that Voltage across Outside Lines of Two-Phase 3-Wire System is 1.41 Times the Voltage from Middle to Either Outside Line

(*d*) Three-Phase "Star" (Y). The terms "star" and "Y" are used to denote the same form of three-phase connection. In this connection, the ends of the three windings are connected together at a common point. The three remaining ends form the terminals of the machine.

For the present, a study of Fig. 40 will, perhaps, best give the information needed for an understanding of the principles involved in making the star connection. If ordinates be taken at, say, every 10° along the horizontal line or axis of abscissas, and measured carefully, it will be found that at every place the ordinate of one curve will be equal to the sum or difference of the ordinates of the other two curves. Thus, $E_{1-1} = E_{3-1} + E_{2-1}$ and $E_{2-2} = E_{3-2} + E_{1-2}$. In other words, the voltage of one phase is, at every instant, balanced by the voltages of the other two

phases. If the ordinate be taken at the point where E_2 is maximum, then Fig. 40 shows clearly that both E_1 and E_3 are equal to each other and opposite to E_2, and that their numerical sum

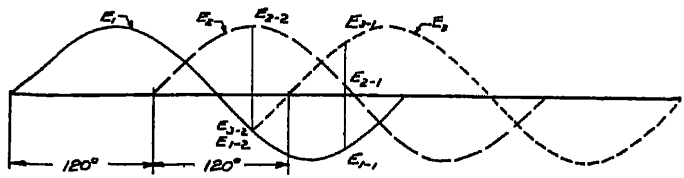


Fig. 40 — Curves Showing that when Three E M. F's are 120° Apart the E M F of One Phase is Balanced by the Other Two

is equal to E_2. This fact gives a simple and practical rule for properly connecting the three windings of a three-phase machine in star.

Place the machine so that the conductors of phase 2 are under the centers of a pole Phase 1 will lie at the left of phase 2 and phase 3 will lie at the right of phase 2. Mark the direction of the induced voltage in each phase. Connect so that if the E M. F. in the phase which is under the center of the pole acts towards the common connection, the E. M. F's of the other two phases will act away from the common connection.

Figure 41 shows this rule applied where the coils are spaced 120 electrical degrees apart.

In Fig. 42 the coils are placed in the slots exactly as in Fig. 41.

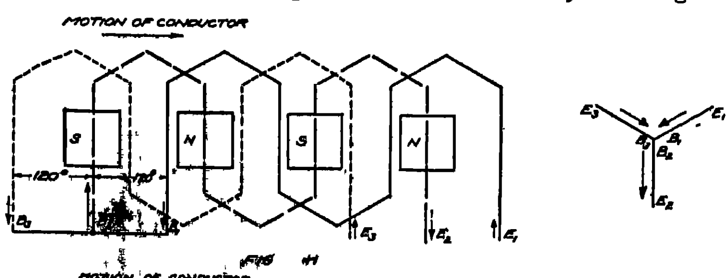


Fig. 41. —Method of Connecting "Y" When Coils are Spaced 120° Apart.

If desired, they may be connected in star by taking them in order as they come on the armature which will make them 60 degrees apart instead of 120 degrees apart. In this case one coil must be reversed before connecting to the common center. The rule stated above, however, applies. Figure 42 shows a machine with the coils taken 60 degrees apart and properly-connected three-phase star.

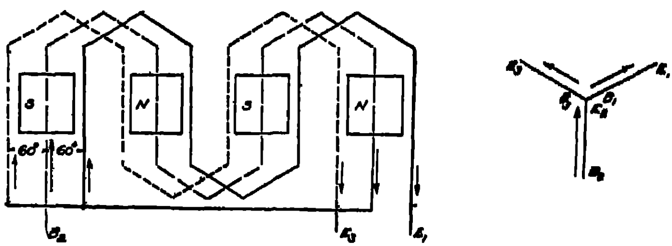


Fig 42 — Method of Connecting "Y" When Coils are Spaced 60° Apart.

(*e*) Three-Phase "Delta" (Mesh). In the delta- or mesh-connection, the three windings in the schematic representation, are connected in the form of an equilateral triangle or the Greek letter "delta." The lines are taken off at the corners of the delta. Inspection of Fig 43 shows that the current in each line wire is made up of the currents in the two windings that join at the corner of the delta. The line current is not the arithmetical sum of the two currents in the windings, because these two currents are not in phase with each other.

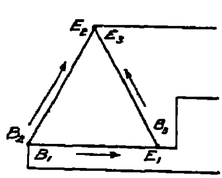


Fig 43 — Method of Connecting Three Windings in Delta.

For the purpose of making the delta-connection, a scheme similar to that used in making the star-connection will make the method clear. Refer again to Fig. 40 and consider that the voltages E_{3-1}, E_{2-1} and E_{1-1} are causing currents to flow. Then E_{1-1} will be balanced by E_{3-1} and E_{2-1} Take a point on the curve E_2 at which E_2 is maximum. E_1 and E_3 will be equal to each other and will balance E_2. To connect in delta then,

Place the machine so that the conductors of phase 2 are under the center of the pole Phase 1 will lie at the left of phase 2, and phase 3 will lie at the right of phase 2. Mark the direction of the induced E. M. F. in each phase. Connect the windings in the form of triangle so that if the E. M F which is maximum tends to cause current to flow around the triangle clockwise, the E M.F.'s in the other two phases will tend to cause current to flow counter-clockwise.

Having become familiar with windings shown spread out flat, next consider a winding as it would appear if viewed from the end of the armature. Figure 44 shows the coils of a revolving-

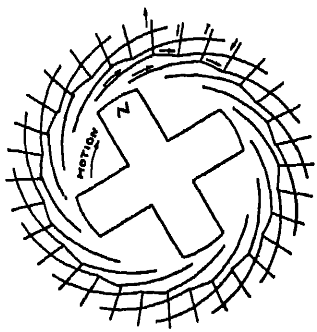


Fig. 44. — Coils of a Revolving-Field Type Alternator Viewed from End

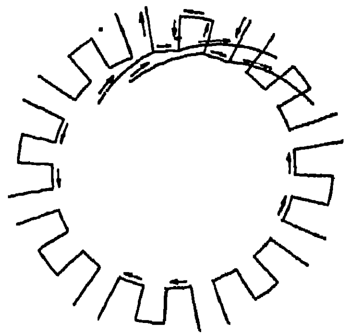


Fig. 45. — Method of Forming Pole Groups.

field type machine as they would appear in place before they are connected up. Suppose that the coils are to be connected to form a four-pole two-phase machine. There are 16 coil., so there would be $16 \div 2 = 8$ coils per phase. The first step in laying out the winding would be to form the pole groups. With 8 coils per phase there would be $8 \div 4 = 2$ coils per pole. Connect two coils in series to form the first pole group, then connect the next two coils similarly and continue until all 16 coils are connected in groups of two, Fig. 45. The next step is to connect the pole groups to form the phases, with poles alternately **N** and **S.** Mark arrows on pole groups alternately clock-wise and counter-clock-wise, and connect phase 1 then phase 2. In Figs. 46 and 47, B_1 E_1 will be the terminals of phase 1 and B_2 E_2 will be the terminals of phase 2.

Example of a Three-Phase "Y"-Connected Armature. Figure 48(a), (b) and (c) shows the various steps in connecting a 24-coil armature which is to have four poles and be connected three-phase "Y."

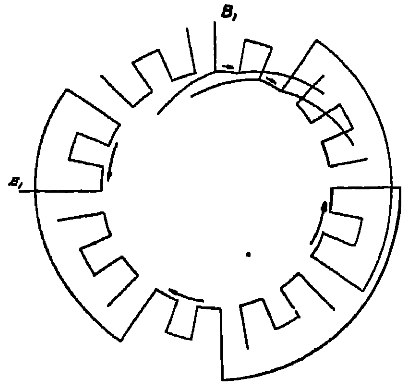


Fig 46 — Method of Forming One-Half of Pole Groups into Phase 1.

Figure 48(a) shows the winding simplified by omitting the sides of the coils that go to the slots. For the purpose of study, the method of showing the coils without the sides going to the slots is satisfactory as it saves work and makes the drawing less complicated.

In Fig. 48(b) the coils are grouped into poles and then the phases are formed. Since there are 24 coils and 3 phases, there will be 24 ÷ 3 = 8 coils per phase There are to be 4 poles in each phase so there will be 8 ÷ 4 = 2 coils per pole. The coils of each pole are connected together the same as in Fig. 45, and then the four poles are connected in series in such a manner as to make the poles of a phase alternately **N** and **S**. In Fig. 48(b), the poles are connected together in a different manner from that of Fig. 46 although the effect electrically is the same The method of Fig 48(b) gives six long leads of one length and 3 short leads of another length, to

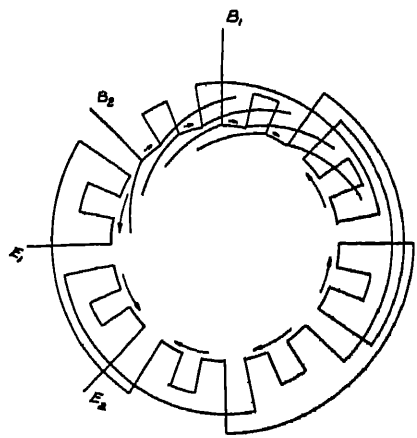


Fig 47 — Method of Forming Remaining Coil Groups into Phase 2.

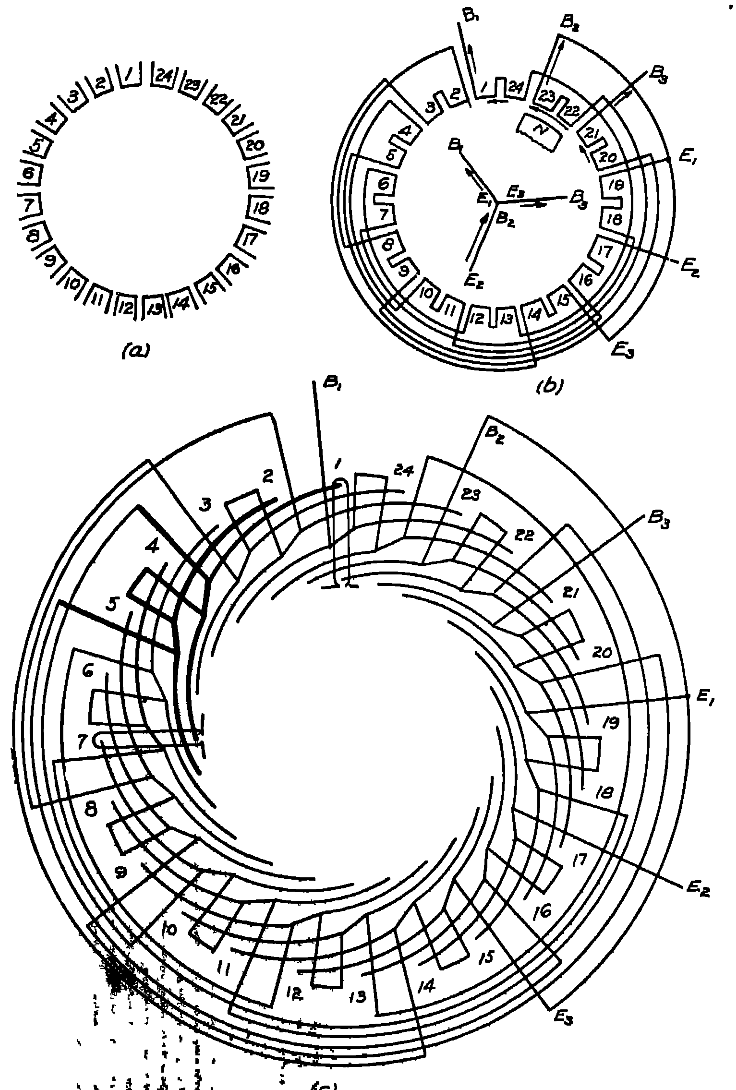


FIG. [illegible]. — Steps in Making "Y" Connection

make up the phase connections. This is a method preferred by some for a four-pole machine By tracing through the circuits of Fig. 48(b), it will be seen that the three phases are connected exactly alike. They are, however, spaced 60 electrical degrees apart on the armature

In a 3-phase machine the windings are to be 120 electrical degrees apart, so it will be necessary to reverse one winding before connecting to the other two. Draw one of the poles of the revolving field under coils 22 and 23 (phase 2). Mark a long arrow under coils 22 and 23. This arrow denotes the direction that E. M. F. will be induced in coils 22 and 23. Coils 1 and 24 (phase 1) and coils 20 and 21 (phase 3) are under the influence of the N pole and will have E. M F.'s induced in the same direction but of less value than the E. M F. in coils 22 and 23. The E. M F.'s in coils 1–24 and 20–21 will be equal to each other as can be seen by reference to Fig 40.

Apply the rule given on page 31 and connect the coils as shown by the drawing within the armature of Fig. 48(b). The terminals E_1, B_2, and E_3 are connected together B_1, E_2, and B_3 form the line terminals of the machine B_1, E_2, and B_3 could just as well have been connected together and E_1, B_2, and E_3 used for the line terminals.

Figure 48(c) shows the complete winding connected three-phase "Y"

Rating of Alternators. The output of an alternator is limited by the heating of its armature conductors. The heating is proportional to I^2R, so doubling the current would make the heating four times as large for a given resistance. However, as the resistance of copper increases with increase of temperature, the heating will actually be more than four times as much if the current is doubled.

Obviously, an alternator can carry current proportional to the size of its armature conductors. A machine designed with copper large enough to carry 100 amperes and insulation of such a character that it will safely withstand 2200 volts, would carry 100 amperes at 2200 volts continuously, and would be rated as a

220 kv-a. machine. $\left(\frac{100 \times 2200}{1000} = 220.\right)$

At 100% power factor such a machine would carry a load of

$$P = \frac{E \times I \times P F}{1000} = \frac{2200 \times 100 \times 1\ 00}{1000} = 220 \text{ kilowatts}$$

At 80% power factor it would only carry

$$P = \frac{E \times I \times P F.}{1000} = \frac{2200 \times 100 \times .80}{1000} = 176 \text{ kilowatts}$$

If we tried to make it carry 220 kilowatts at 80% power factor, we should have to increase the current to 125 amperes, for

$$I = \frac{P \times 1000}{E \times P.F} = \frac{220 \times 1000}{2200 \times .80} = 125 \text{ amperes}$$

This current would overheat the machine.

A machine rated at a given number of kilovolt amperes (kv-a.) will carry the same number of kilowatts (kw) or its maximum load, only at 100% power factor. At any other power factor, the kilowatts it will carry will be in the same ratio to its maximum load, as the power factor is to 100% power factor.

Effects of Load on Voltage of an Alternator — Regulation. The field set up by the current in the armature of an alternator produces an effect on the main field from the poles, somewhat similar to that produced by the field from the armature current in a separately-excited direct-current generator. The effect of load on an alternator is usually to produce distortion of the field. In case the current lags the voltage, the field set up by the armature demagnetizes the main field and thereby reduces the terminal voltage In case the armature current leads the voltage, the armature field actually increases the main field and raises the terminal voltage. The effect varies with the current and the angle of lag or lead, that is, with the power factor. The change in voltage from full load to no load, expressed as a percent of the full load voltage is called the regulation of an alternator. Expressed as a formula

$$\text{Regulation} = \frac{\text{No load volts} - \text{full load volts}}{\text{full load volts}} \times 100 \qquad (7)$$

Armature Reaction. The armature currents in an alternator produce fluxes in the armature that react on the main flux. The effects of the armature fluxes depend on the phase relation of armature voltage and current.

If the armature voltage and current are in phase, then the effect of the armature flux is principally distortion. The field is shifted in the direction of rotation, and the flux crowded into the trailing pole tips, as in a direct-current machine. Distortion of the field results in a reduction of terminal voltage. The reason for this is that, as the field is distorted, its magnetic length becomes greater and so its reluctance is increased The reluctance is further increased by the crowding of lines of force into the pole tips, working them at a density more nearly saturation. Here the permeability is less and therefore the reluctance greater. The net effect of distortion is a reduction in voltage as the load comes on.

If the nature of the load is such that the armature current lags behind the voltage, the flux set up by the armature current does not reach its maximum until the armature has turned several degrees from the position it would occupy if the current were in phase. The current with its flux is, in effect, carried under the next pole, and is then in such a direction that it demagnetizes the pole. This of course results in a drop in terminal voltage.

If the current leads the voltage, the flux due to the armature current reaches a maximum when the conductors are in such relation to the poles that the armature flux assists the main flux. The result is that with a leading current the terminal voltage may actually rise as the load comes in.

The effects of the armature flux on the main field flux are shown graphically by Figs. 49 and 50.

Operation with Lagging Current. Figure 49 (a) to (h) shows schematically the behavior of an alternator with current in phase with voltage and also with current lagging 30°, 60° and 90°.

At (a) the armature circuit contains only a resistance R. The coil is shown on the axis of the field poles and is running clockwise In the position shown, the generated electromotive force is

maximum because the coil is cutting squarely across the lines from the field, or at a maximum rate Since there is only resistance in the circuit, the current is in phase with the electromotive force and therefore maximum also. The current in the armature coil sets up a field acting downward. The effect is the same as if the main field were represented by a force ϕ_f and the armature field by a force ϕ_a at right angles to ϕ_f as shown at (b). The resultant field is shifted to ϕR_0 or towards the trailing pole tip. This shifting of the flux concentrates it in the small area of the pole tip and if the pole tip becomes saturated, the ampere-turns on the field are not sufficient to keep up the flux at this high density of saturation, so it drops off and therefore the voltage of the machine drops as well.

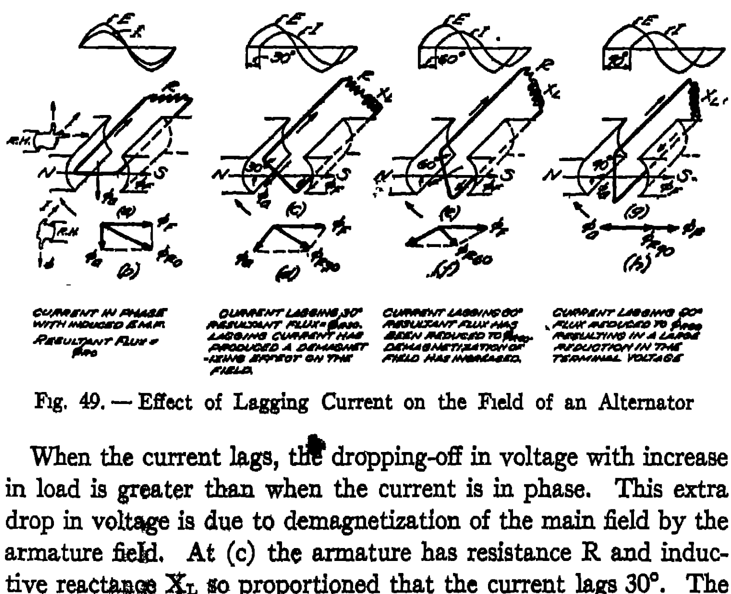


Fig. 49. — Effect of Lagging Current on the Field of an Alternator

When the current lags, the dropping-off in voltage with increase in load is greater than when the current is in phase. This extra drop in voltage is due to demagnetization of the main field by the armature field. At (c) the armature has resistance R and inductive reactance X_L so proportioned that the current lags 30°. The current will reach its maximum value 30° later than when the current and electromotive force are in phase. It will reach its maximum when the coil has turned 30° from the axis of the poles or to the position shown by (c). The flux will be as at ϕR_{30} (d).

In sketch (e) the ratio of X_L to R has been changed so that the current lags 60°, the resultant field is shown by ϕR_{60} at (f). Sketch (g) shows a load of such a nature that the current lags 90°. Here the coil must turn 90° from the position shown at (a) before the current reaches its maximum. At the position shown by (g) the armature field directly opposes the main field and the resultant field is ϕR_{90} as shown at (h).

Operation with Leading Current. Figure 50 (a) to (h) show conditions in an alternator when the current is leading. The analysis is the same as in Fig. 49 except that with leading current, the current will reach its maximum 30°, 60°, and 90° before the voltage reaches its maximum, or before the coil reaches the position in line with the axis of the poles as shown at (a). Diagrams (d), (f), and (h), Fig. 50, show that the resultant flux and terminal voltage increase with a leading current.

Parallel Operation of Alternators — Synchronizing. Alternating current generators may be operated in parallel and thereby

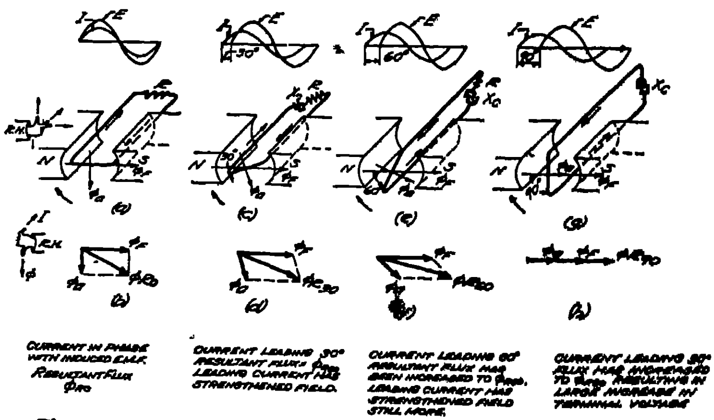


Fig. 50 — Effect of Leading Current on the Field of an Alternator.

supply a set of buses with power equal to the combined power output of the separate generators.

Assuming that one machine is running and on the buses, the machine that is to be paralleled with it should deliver to the

switch, through which it is to be paralleled to the other machine, a voltage of the same value and frequency as the bus voltage. Further, the two voltages must be in phase, that is, they must have their zero values occurring at the same time and their maximum values occurring at the same time and in the same direction. A voltmeter can be used to determine when the two machines have the same voltage, and either ordinary lamps or a synchroscope to determine when the machines have their voltages in phase. Only the lamp method will be described at present.

Let A_1, Fig. 51, be the alternator which is running and supplying power to the buses, and let A_2 be the alternator which is to be paralleled with it or synchronized as it is called. L_1 and L_2 are two lamps each with a voltage rating equal to the voltage of one machine (one lamp with a voltage rating or twice the machine voltage could be used).

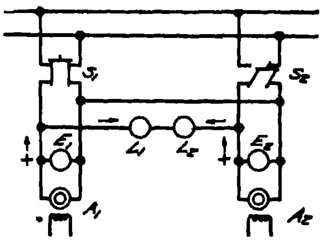


Fig. 51 — Two Alternators Connected for Parallel Operation.

Assume that machine A_1 is running and on the buses and that the speed of A_2 has been adjusted so that its frequency is the same as A_1 and its voltage the same as A_1. If the two machines are in the same phase relation to each other, so that the left-hand lead is for example plus, at a given instant, then both A_1 and A_2 will try to send current through the lamps, but the two voltages will balance and the lamps will be dark. If the machines are not in the same phase relation the voltage of one machine will be different from the other and current will flow through the lamps. In case the machines should happen to be directly opposite in phase, the lamps would be subjected to twice the machine voltage and unless two were used or one of a voltage double that of one machine, the lamps would burn out.

As the machines are brought in synchronism with each other by control of the prime mover on the one to be synchronized, the lamps will flicker. As the machines gradually approach synchronism the periods of brightness and darkness gradually

lengthen. When the lamps are dark, the switch S_2 on the incoming machine may be closed.

Synchronizing is discussed further under synchronous motors.

PROBLEMS

1. Follow the method indicated by Fig 37 and lay out a developed single-phase winding with 16 coils and 8 poles.

2 Lay out a developed 4-pole 2-phase winding with 16 coils.

Follow the method of Figs 48(a), (b) and (c), in the following problems:

3 Lay out a 4-pole single-phase armature winding with 12 coils

4 Lay out a 32-coil, 8-pole, 2-phase armature winding with 4 line wires

5. Connect the armature of Fig. 48(a) 4-pole, 2-phase, 3-wire.

6. Connect the armature of Fig 48(a) 4-pole, 3-phase delta

7. Lay out a three-phase winding as follows: Number of coils 48, number of poles 8, coils per pole per phase 2.

8 Lay out a 6-pole 3-phase armature with 36 coils Connect the armature delta

CHAPTER III

INDUCTANCE

Counter Electromotive Force. Self and Mutual Inductance. A coil produces a choking effect upon a varying current greater than the choking effect of the resistance of the coil as measured in ohms by the ordinary direct-current drop of potential method. If an iron core be inserted into the coil, this choking effect is much increased. The impressed electromotive force is opposed by a counter electromotive force that reduces the current in a manner similar to that by which the counter electromotive force in a direct-current motor armature reduces the armature current. There is the difference, however, that with a varying or alternating current, the current is made to lag behind the voltage that produces it. This lag is caused by a property of the circuit called inductance. That property of a circuit, by virtue of which an electromotive force is induced by varying lines of force caused by varying current is called the inductance of the circuit.

If the varying lines of force induce an electromotive force in the circuit itself, the inductance is called self-inductance, if the lines of force induce an electromotive force in a neighboring circuit, the inductance is called mutual inductance. Inductance exists in a circuit even though no iron is present, but since iron is a magnetic substance it forms a much better path for the lines of force than air or wood, so the choking effect is much increased, that is, the inductance is greater.

Familiar Examples of Inductance. Anyone who has experimented with shunt-wound generators has probably noticed the rather large arc that is drawn when the field circuit is opened. This arc is caused by the lines of force of the field magnets closing in, as the current falls, and generating an electromotive force in the field winding. The voltage dissipates itself by causing an arc

at the switch. If the switch is opened wide very quickly, the voltage will build up to a high value and may jump to ground or otherwise break down the insulation of the machine. A large machine usually has a "field discharge resistance" which is thrown in by an auxiliary contact on the switch which closes as the main contacts are opened thus allowing the voltage to discharge through a suitable resistance. Field switches not equipped with discharge resistances should be opened slowly, allowing the voltage to draw out an arc and thus dissipate itself.

A very simple illustration of the effect of inductance, consists of connecting a lamp in series with a coil containing an iron core that can be moved in and out. When an alternating current is sent through the circuit the lamp will be dim when the core is in but will brighten up as the core is withdrawn. The inductance is greater when the core is in because the iron forms a better path for the magnetic lines of force than the air (roughly 1000 times better) and so there are more lines cutting the coil with the core in and therefore a greater counter E. M. F. and a smaller current

Other illustrations of inductance are, induction motors and transformers running light, choke coils and synchronous motors with low excitation on the fields.

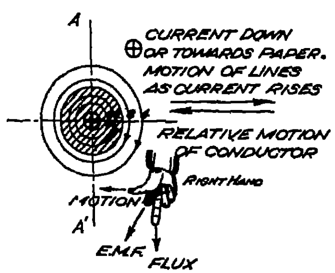


Fig. 52 — Generation of Counter E M F. by Moving Lines of Force

How Inductance Causes a Counter Electro-motive Force. In Fig. 52 consider that you are looking at the end of a conductor that is carrying current away from you and that this current starts at a very low value and rises to, say, 20 amperes. Since the current flows away from you, the lines of force set up by the current will encircle the conductor in a clockwise direction. Further, they may be thought of as starting at the center of the conductor and expanding outward as the current rises. That is, when

the current is small, we can think of the lines as at 1, and when it is large, as having expanded outward to 4. In moving from 1 to 4, the lines cut the section of the conductor at the right of AA′ from left to right. Considering the lines as stationary, and the conductor moving (in order to apply the three-finger rule), the conductor may be considered as moving to the left. Apply the rule to the part of the conductor to the right of the line AA′. Let the first finger, which shows direction of flux, point downward, the thumb which shows direction of motion of conductor, to the left, then the middle finger which shows direction of induced electromotive force, will point upward from the paper. The induced electromotive force will oppose the impressed electromotive force.

Consider next, that the current is at its full value and steady and the lines of force are at position 4. No electromotive force is induced as long as the lines are steady because there is no cutting action. If now the circuit is broken, as in the case of the shunt generator previously mentioned, the lines of force will close in. Considering again the section of the conductor to the right of AA′, the motion of the lines is from right to left, or the relative motion of conductor is from left to right. Applying the three-finger rule the induced electromotive force is towards the paper or adds itself to the impressed electromotive force.

In the case of the field previously mentioned, the high voltage appearing at the switch, when it is opened, results from this cutting action of the field upon the many turns of the winding. From the above, it is evident that as the current rises the electromotive force of self induction opposes the flow of current and as the current falls it assists its flow.

Lag of Current due to Inductance. The effect of inductance is to cause the current to lag behind the electromotive force that produces it. If it were possible to make up a circuit entirely of inductance, with no resistance, the current would lag 90° behind the impressed E. M. F. Commercial circuits have currents lagging much less than 90°.

To understand why inductance causes a lagging current, con-

sider that an alternating current flowing in a coil causes the flux to rise and fall according to a sine law as curve ϕ in Fig. 53

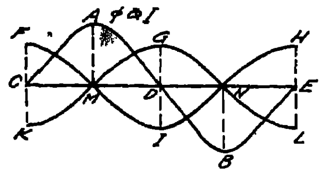


Fig 53 — Curves of Flux Current and E. M F

The current and flux are in phase so that curve ϕ may also be used with a different scale to represent current I as well

As explained earlier, the changing flux induces an electromotive force in the conductors which it cuts. The electromotive force is highest when the flux is changing most rapidly, and lowest when the flux is changing least rapidly. Inspection of Fig 53 shows that at points A and B the flux is changing least rapidly (for an instant at A or B it is constant) and that at points C, D and E it is changing most rapidly, because the slope of the curve is steepest. From the above we see that the E. M. F. will be maximum at points F, G and H or K, I and L and zero at M and N. Taking intermediate points and plotting curves we get curves E_c and E_{imp}, Fig 54, both of which satisfy the condition of maximum voltage when the flux is zero and minimum voltage when the flux is maximum. It remains only to determine which of these curves represents the actual counter E. M. F.

In Fig. 54 let the current be rising according to a sine law as shown at (b). Lines of force start at the center of the conductor

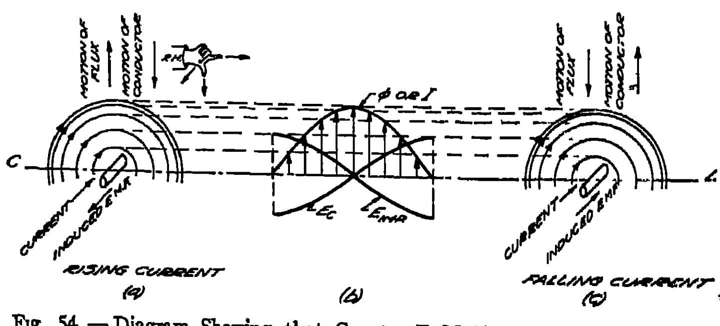


Fig 54. — Diagram Showing that Counter E M. F. Wave Lags Flux and that Impressed E M. F Wave Leads Flux.

and expand outward encircling the conductor clockwise as shown at (a).

Above the horizontal center line C–L, the lines of force are moving upward, which is the same, relatively, as considering the lines of force stationary and the conductor moving downward. By applying the three-finger rule, the direction of induced E. M. F. is opposite to the current. That is, while the current is rising as plotted at (b), E_0 must be plotted below the horizontal line C–L. As the current nears its maximum value, the counter E. M. F. becomes less and less until when the current has reached its maximum, there is no cutting action so that the E_0 becomes zero. At (c) the current has begun to fall and the lines of force to close in. By applying the three-finger rule, the direction of the induced E. M. F. is found to be the same as the current or must be plotted above the horizontal line C–L. Reference to (b) shows that E_0 lags I and ϕ by 90°. The impressed E. M. F., E_{imp}, must balance E_0 at every instant and is represented by the curve E_{imp} which is 180° from E_0. In other words, the flux lags the impressed voltage; is in phase with the current and is 90° ahead of the counter-electromotive force.

Unit of Inductance. Development of a Formula. Thus far the effect of inductance has been illustrated but no method outlined for calculating the numerical value of inductance. The unit of inductance is the henry. The symbol is "L." A circuit has an inductance of one henry when current, changing at the rate of one ampere per second, induces an E. M F. of one volt across the terminals of the circuit. To illustrate, Fig. 55 shows an ideal circuit containing inductance. The resistance of the coil is shown as if it were a separate resistance, so that, diagrammatically, "L" consists of inductance only. When the switch S is closed the current rises to a value $I = \frac{E}{R} = \frac{1}{1} = 1$ ampere, but does not rise instantly due to the lag caused by the inductance. If a volt-

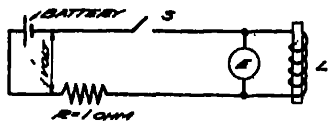


Fig 55. — Circuit Showing Effect of Inductance

meter be connected across at E, the needle will be found to "kick" over a few volts as the circuit is closed and to "kick" in the opposite direction when the circuit is opened, due to the induced voltage. This voltage is due to the lines of force set up by the current cutting the turns of the coil in one direction as the current rises and cutting the turns in the other direction as it falls. If now the construction of the coil L is such that there are just the right proportions of turns and iron so that, if the current rises from 0 to 1 ampere in 1 second, and there is induced an electromotive force of 1 volt, the circuit has an inductance of 1 henry.

From the preceding, it will be evident that the induced voltage will be directly proportional to the inductance and the current but will be inversely proportional to the time in which the current change occurs. Using the field coil of the generator as an illustration again, the inductance of the coil is large because there are many turns of wire on an iron core. When the field switch is pulled out quickly, a fairly large current falls to zero in a short time, hence the induced voltage is high. If the switch is pulled out slowly, thereby drawing an arc, the time that the current is falling to zero is greater so that the induced voltage is less.

Expressed as a formula, the average volts induced in a circuit will be.

$$E_{av} = \frac{L\,I}{t} \qquad (8)$$

Where L = inductance in henrys
I = change in current in amperes
t = time in seconds in which current changes.

Inductive Reactance. As the formula $E_{av} = \frac{L\,I}{t}$ stands, it will be inconvenient to use it because the part $\frac{I}{t}$, or the rate at which the current changes, cannot be measured by ordinary instruments. In an alternating current circuit, the frequency is a measure of the rate of change of the current and by taking into account this fact, a formula can be developed that involves current, voltage and frequency and a new quantity called inductive reactance that is in

ohms. The symbol for inductive reactance is X_L. It may be used like R is used in the algebraic statement of Ohm's law. The method of doing this will be explained under Series and Parallel Circuits. It may also be used to compute the inductance L

In terms of the usual quantities measured in alternating current circuits

$$X_L = 2\pi fL \quad (9) \qquad \text{and} \qquad E = 2\pi fLI \quad (10)$$

Where X_L = inductive reactance in ohms
$2\pi = 6.2832$
f = frequency
L = inductance in henrys
I = effective current in amperes.

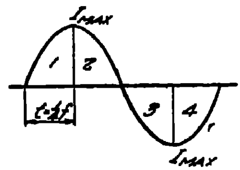


Fig 56 — Curve Showing Four Changes of Current per Cycle

The derivation of the formula is as follows: In Fig. 56 it is seen that the current changes from 0 to I_{max}, I_{max} to 0, 0 to $-I_{max}$ and $-I_{max}$ to 0 or 4 times per cycle. The time of one cycle $= \frac{1}{f}$ seconds. The time of one change $= \frac{1}{4f}$ seconds

The rate of change is obtained by dividing amperes by time, or,

$$I_{max} \div \frac{1}{4f} = 4fI_{max} \text{ amperes per sec.}$$

Now $\qquad E_{av} = \frac{LI_{max}}{t} = L \times \text{rate of change}$

so $\qquad E_{av} = L4fI_{max}$

also from (5) $\qquad E_{av} = E_{max} \times .636$

so $\qquad E_{max} \times .636 = 4fLI_{max}$

and $\qquad E_{max} = \frac{4fLI_{max}}{.636} = 6.28fLI_{max}$

also from (4) $\qquad E_{max} = E_{eff} \div .707$

$\qquad I_{max} = I_{eff} \div .707$

so $\qquad \frac{E_{eff}}{.707} = \frac{6.28fLI_{eff}}{.707}$

or $\qquad E_{eff} = 6.28fLI_{eff}$

$$= 2\pi fLI_{eff} \qquad (11)$$

Development of a Formula for a Coil. In order to develop a formula that will give quantitative relations between inductance, current, turns of wire and dimensions of a coil, a brief review of the fundamental relations between magnet poles and electric currents will be desirable.

A unit pole may be defined as a pole of such strength, that it will repel a similar pole one centimeter away from it in air, with a force of one dyne. The definition gives the idea of force between poles but does not directly express a relation between pole strength and electric current. It is more convenient for the purpose of developing a formula, involving current, to picture a unit pole as a point in space that radiates lines of magnetic force equally in all directions. If such a point be enclosed in a hollow sphere of 1 centimeter radius, as shown by Fig 57, all the lines of force from the pole will cut the surface of the sphere and cut it uniformly. If the pole is of such strength that one line, and only one line, cuts each square centimeter of the surface of the sphere, the pole is a unit pole. Since the surface of a sphere is $4\pi r^2$, the area of the surface of a sphere of 1 cm. radius is 4π sq. cm. or 12.6 sq cm. A unit pole will therefore give out 4π lines or 12.6 lines *

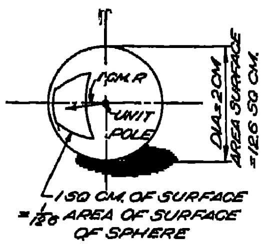


Fig 57. — Pictorial Representation of a Unit Pole

If such a pole be brought near a conductor carrying a current, force will be exerted on the pole by the lines of force from the current in the conductor. If the pole be at the center of a single turn of wire carrying current, every line of force set up by the current will act on the pole. This will be apparent from Fig. 58. If the pole be moved around the conductor, as for instance, along the dotted line "a," Fig. 59, every line of force from the pole would

* A "line" of force is understood to be that unit of magnetic force that acts on a square centimeter of surface and not an actual line in the ordinary sense of the word The difficulty of imagining 12 6 lines can be overcome by substituting "units."

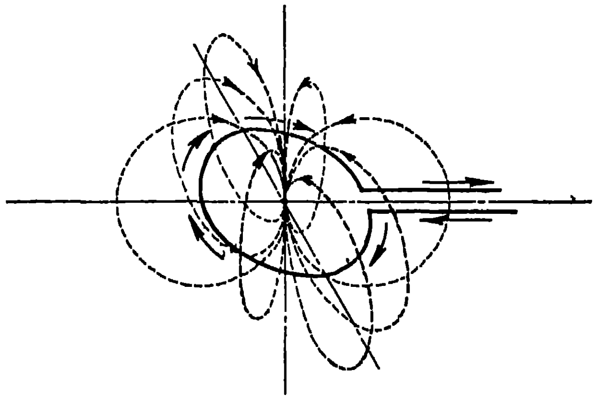


Fig 58 — Pole at Center of Turn of Wire Is Acted Upon by All Lines of Force from Current in Wire.

cut the conductor. Similarly if the pole were stationary, and the current were built up from zero to a certain value, every line of force from the current, in expanding outward would react on the pole. In either case force would be required, in the first, mechanical force to move the pole around the conductor and in the second, electromotive force to build up the current.

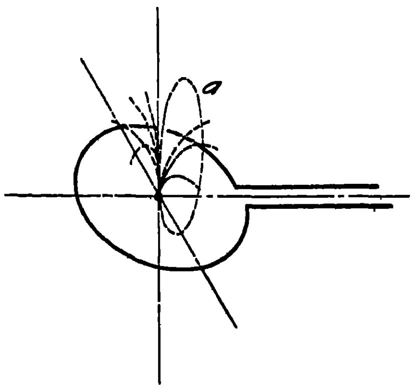


Fig 59. — All Lines of Force from a Pole Cut a Turn of Wire When Pole Is Moved Around the Wire.

The pole and current react to oppose the force that tries to effect a change.

The actual number of lines of force from a conductor threading or linking with any part of the circuit, for one absolute unit of current, is denoted by "1" which is called the coefficient of induction of the circuit, or the inductance. The inductance, in a sense, is a measure of the magnetic quality of the circuit,

since it gives an idea whether a large or small number of lines of force are set up by a given current. For example, if the space between the pole and coil of Fig. 58 had been of better magnetic material than air, say of iron, so that one unit of current would have produced twice as many lines of force as with air, the circuit would have been twice as good magnetically and this would have appeared by the coefficient of induction or inductance being twice as large. From the above it will readily follow that the inductance and current have reciprocal relations, that is, l varies as $\frac{1}{i}$. l will evidently be greater with an increase in flux ϕ or with an increase in turns with a given current because increasing either will increase the lines threading the circuit.

So l varies directly as the flux and turns and inversely as the current. Or

$$l = \frac{\phi n}{i} \tag{12}$$

The practical unit of inductance is 10^9 as large as the absolute unit just described and is called the henry.

If the inductance given by the formula $l = \frac{\phi n}{i}$ were to be expressed in henrys there would be only $\frac{\phi n}{i} \times \frac{1}{10^9}$ as many henrys as absolute units. Further, if the current had been measured in amperes, the amperes would have been 10 times as great because the ampere is only $\frac{1}{10}$ of the absolute unit of current.

So

$$L = \frac{\phi n}{i \times 10^9} \times 10 = \frac{\phi n}{10^8 I} \tag{13}$$

Where L = the inductance in henrys
ϕ = the total flux
I = the current in amperes
n = the number of turns of wire.

In a magnetic circuit

$$\text{Flux} = \phi = \frac{\text{magnetomotive force}}{\text{reluctance}} = \frac{\text{M. M. F.}}{\mathcal{R}} = \frac{4\pi nI}{10\,\mathcal{R}} \tag{14}$$

and $$\mathcal{R} = \frac{l_0}{\mu\, A_0} \tag{15}$$

Where n = the number of turns of wire
I = the current in amperes
$\mathcal{R}$ = the reluctance in oersteds
μ = the permeability (a numerical number)
l_0 = the length of the coil in centimeters
A_0 = the area of the core in square centimeters.

Since from (14)

$$\text{Flux} = \phi = \frac{4\pi nI}{10\mathcal{R}} = \frac{4\pi nI}{10\ \frac{lc}{\mu A_0}} = \frac{4\pi nI\mu A_0}{10 l_0}$$

and since from (13)

$$L = \frac{\phi\, n}{10^8 I}$$

$$L = \frac{4\pi nI\mu A_0}{10 l_0} \times \frac{n}{10^8 I}$$

$$= \frac{4\pi n^2\mu A_0}{10^9 l_0} \tag{16}$$

The above equation does not hold strictly true for coils of all shapes but expresses the fundamental relations between inductance, turns of wire, permeability, area of core of the coil and length of the coil. It shows that the inductance varies as the square of the number of turns, directly as the permeability and area, and inversely as the length.

PROBLEMS

1 Mention one important piece of electrical apparatus that depends for its operation on mutual inductance. Explain the operation

2. Mention one piece of apparatus that depends for its operation on self-induction Explain

3. Which has the greater inductance, a horseshoe electro-magnet with the keeper or armature on or off? Why? Which way would it draw the greater alternating current?

4. What is the inductance of a circuit in which the current in falling from 10 amperes to 0 in 1 second induces 5 volts?

5. Calculate the reactance of a circuit whose inductance is .2 henry, if the frequency is 60 cycles.

6. What will be the voltage across the circuit of Prob. 5 if the current is 10 amperes?

7. A coil has 3000 turns of wire and is wound on a wooden core the area of which is 20 square centimeters. If the length of the coil is 30 centimeters what is its inductance?

8 About how much would the inductance of the coil of Prob. 7 be increased if an iron core were substituted for the wooden core?

CHAPTER IV

CAPACITY

Condenser. If two metal plates are separated by a very thin dielectric, such as mica or paper, and the positive side of a fairly high-voltage direct-current circuit be connected to one plate and the negative side to the other plate, a sensitive ammeter connected in the circuit will momentarily deflect as voltage is applied, showing that current flows in the circuit Such an arrangement of plates separated by sheets of dielectric is called a condenser. The electricity that flows when the circuit is closed is called the charge.

When the voltage is removed, the condenser sends out current in a reverse direction. The reason for this is that when the voltage is applied the dielectric is under a stress, the nature of which is such that the plates receive a charge. As soon as the electric pressure is removed, the stressed dielectric tries to send out the charge that has been forced upon the plates. A cylinder C, with a rubber diaphragm D, across it, as shown by Fig. 60, constitutes a mechanical model of a condenser The two pipes P_1 and P_2 that feed into the cylinder may be thought of as the two wires connecting to the plates. Each pipe connects with a vessel containing water which may be thought of as the battery or generator that supplies the voltage. It is clear that as the height of the water in A is increased to A_1 the pressure on the diaphragm will be increased and the diaphragm will tend to move over to D_1, allowing water to flow into the cylinder. The amount of water that flows into the cylinder corresponds to the charge of electricity in coulombs that flows into a condenser.

Charge in a Condenser. A study of Fig. 60 will show that the greater the pressure on the diaphragm, the greater will be the flow, and the greater the quantity or charge the cylinder will re-

ceive. Further, if the size of the cylinder and diaphragm be increased, a given pressure will allow a greater charge to be stored up in the condenser than with the small cylinder and diaphragm.

In an electric condenser the capacity of the condenser is denoted by C, the voltage by E, and the charge it receives by Q. As in Fig. 60, the capacity that the cylinder will receive for a

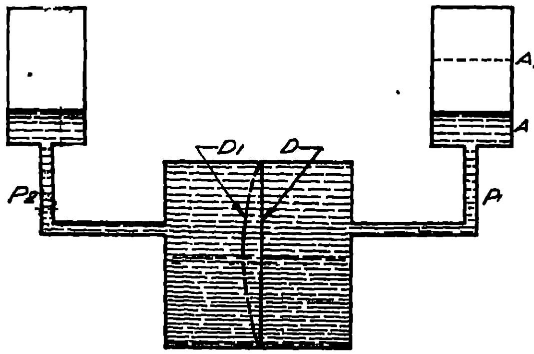


FIG 60. — Mechanical Model of a Condenser.

given pressure depends upon the size and elastic quality of the material which forms the diaphragm, also on the thinness of the diaphragm. So in an electric condenser, the capacity depends on the area of the dielectric stressed; the dielectric properties of the dielectric, and the thinness of the dielectric. In a condenser it is desirable to have a very thin dielectric just as in Fig. 60, it is desirable to have a thin diaphragm. A dielectric too thin, however, will break down just as too thin a diaphragm will break when pressure is applied

The unit by which condensers are measured is called the farad. A condenser has a capacity of one farad when it is so constructed that if it has a pressure of one volt applied to its terminals it will allow one coulomb of electricity to flow into it. Expressed another way: If it is entirely discharged and one coulomb of electricity flows into it, the voltage across the terminals will rise to one volt.

In symbols: $C = \dfrac{Q}{E}$ (17)

Where C = capacity of the condenser in farads
Q = the charge in coulombs
E = the E. M. F. in volts

From above: $Q = CE$, (18) $E = \dfrac{Q}{C}$ (19)

A smaller unit than the farad is used in measuring condensers. This unit is known as the microfarad. A microfarad is one-millionth of a farad.

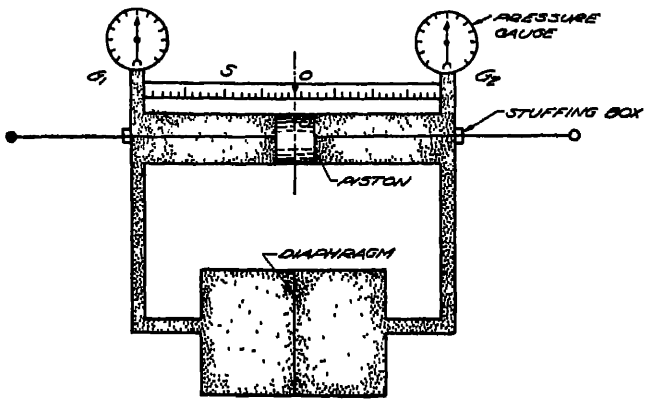


Fig 61. — Mechanical Model of a Condenser Connected in an Alternating Current Circuit.

Behavior of Condensers on Alternating-Current Circuits. If the two vessels in Fig. 60 be replaced by a cylinder with a piston as in Fig. 61 and the piston be moved back and forth, the con-

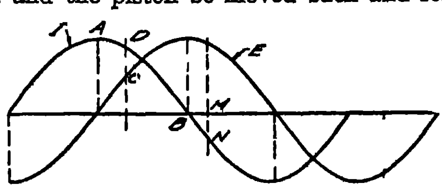


Fig. 62 — Diagram Showing that Current Leads E. M. F in a Circuit Containing a Condenser.

denser will be charged first in one direction and then in the opposite direction. This condition is similar to that in an alternating-current circuit. If a record of pressures be kept by gauges G_1 and G_2, and a record of piston displacements be read from the scale S, two curves can be obtained similar to Fig. 62, one of

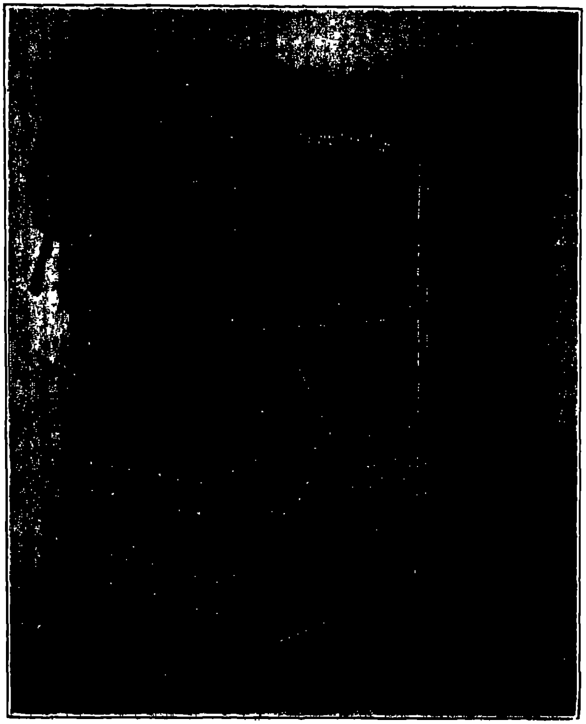


Fig. 63 — 300 kv-a 2300-Volt Static Condenser. (Westinghouse Electric & Mfg. Co.)

which corresponds to the current flow (quantity) in a condenser and the other to the voltage (pressure) The current curve will lead the voltage curve.

Referring to the model of Fig. 61 it is clear that the greatest flow will occur when the diaphragm just starts to move, or when

the pressure is least. Also that the least flow will occur when the diaphragm is fully stretched or when the pressure is greatest. These facts establish points A and B on the curve I, Fig. 62. Further study of Fig. 62 shows that when the pressure has risen to C, the flow is smaller than at A, but is still in the direction of the pressure. Similarly, after E has reached its maximum and

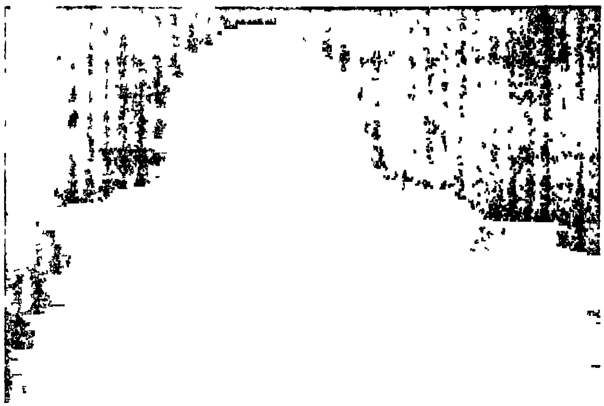


Fig. 64. — 2800 H P 6600-Volt Synchronous Condenser.
(Electric Machinery Mfg Co)

starts to fall, the elasticity of the diaphragm forces current out and it flows in the opposite direction as at MN.

In a circuit containing a condenser the current leads the impressed E. M. F. In a circuit containing an inductance, the current lags the impressed E. M. F. Commercial circuits containing lightly loaded induction motors, transformers or other pieces of apparatus which behave like inductance and cause a lagging current may have their currents brought more nearly in phase with their voltages by connecting condensers in the circuits. One of these condensers for a three-phase circuit is shown by Fig. 63.

A type of alternating-current motor that has a direct-current

field and is known as a synchronous motor may be used instead of a condenser. When the field of such a machine is strongly excited, the machine will draw a leading current. Sometimes these machines are used only for power-factor correction and do not carry mechanical loads; in such cases they are called synchronous condensers. Such a machine is shown by Fig. 64.

As explained under power factor, it is desirable from an operating standpoint, to have the current and voltage in phase, as less current, for a given amount of power, will be carried by the lines and apparatus. The smaller current will, of course, give smaller losses.

Calculation of Capacity Reactance. The capacity of a condenser is measured in farads, but in order to calculate circuits containing condensers it is desirable to have a formula that expresses the capacity effect in ohms. A parallel case exists with inductance: the inductive effect is measured in henrys, but the choking effect of inductance may be expressed in ohms by the development of simple relations between the inductance and the frequency of the circuit. Both inductance and capacity oppose the flow of current in somewhat the same manner as resistance. Resistance opposes it directly, inductance, in such a way that the current lags behind the voltage that produces it, while capacity causes the current to lead the impressed voltage.

The capacity effect is denoted by X_c called capacity reactance and is in ohms. $X_c = \frac{1}{2\pi fC}$ From which $E = IX_c = I \times \frac{1}{2\pi fC}$.

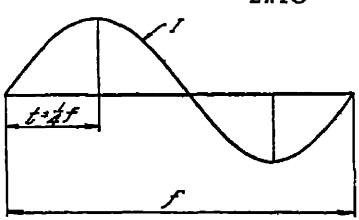


Fig 65 — Sine Wave of Current

I is the effective current, E the effective voltage, and C the capacity of the circuit in farads.

The expression $E = \frac{I}{2\pi fC}$ is derived as follows: In Fig. 65, let I be a current that rises and falls according to a sine law. The current passes from 0 to I maximum twice a cycle and from I maximum to 0 twice a cycle, or there are 4 changes per cycle.

The time of one change, $t = \frac{1}{4f}$ seconds. The current that flows is the average current I_{av} between 0 and I maximum which for a sine wave is .636 I maximum.

So the quantity of electricity that flows:

$$Q = I_{av} \times t = I_{max} \times 636 \times t \tag{20}$$

From the condenser formula, (18)

$$Q = CE_{max}$$

and from (20)

$$Q = I_{max} \times .636 \times t$$

and since $t = \frac{1}{4f}$

$$Q = I_{max} \times .636 \times \frac{1}{4f}$$

$$= \frac{I_{max}}{\frac{4f}{.636}} = \frac{I_{max}}{6.28f}$$

$$= \frac{I_{max}}{2\pi f}$$

Also, since $Q = CE_{max}$ and $Q = \frac{I_{max}}{2\pi f}$

$$CE_{max} = \frac{I_{max}}{2\pi f}$$

or $E_{max} = \frac{I_{max}}{2\pi fC}$

To get the equation in terms of effective values, multiply both sides by .707,

thus, $E_{max} \times .707 = \frac{I_{max}}{2\pi fC} \times .707$

and since $E_{max} \times .707 = E_{eff}$ and $I_{max} \times .707 = I_{eff}$

we obtain $E_{eff} = \frac{I_{eff}}{2\pi fC} = I_{eff} \times \frac{1}{2\pi fC}$

the quantity $\frac{1}{2\pi fC}$ is denoted by X_c and called the Capacity Reactance. X_c is in ohms.

Hence
$$E_{eff} = \frac{I_{eff}}{2\pi fC} = I_{eff}X_c \tag{21}$$

Condensers in Series. When condensers are in series, the E. M. F. across the combination is the sum of the separate E M. F.'s across the condensers. Thus if several condensers C_1, C_2, C_3, etc., are in series

$$E = E_1 + E_2 + E_3 + \ldots$$

Now
$$E_1 = \frac{Q_1}{C_1}, \quad E_2 = \frac{Q_2}{C_2}, \quad E_3 = \frac{Q_3}{C_3}$$

The charge C_1 induces an equal charge in C_2, and C_2 an equal charge in C_3, etc., so that $Q_1 = Q_2 = Q_3$

Hence
$$E = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \ldots$$

dividing by Q,
$$\frac{E}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

But
$$\frac{E}{Q} = \frac{1}{C} \quad \text{(p. 57)}$$

So
$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

$$C_s = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \tag{22}$$

Where C_s = Capacity of condensers in series.

Example:

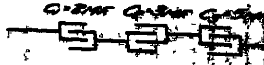


Fig. 66 — Condensers in [illegible]

In Fig. 66 let 3 condensers of 2, 3, [illegible] microfarads respectively [illegible] in series.

The capacity of the combination is

$$C_S = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}} = \frac{1}{\frac{1}{2}+\frac{1}{3}+\frac{1}{5}}$$
$$= .968 \text{ mf. Ans.}$$

Condensers in Parallel. When condensers are placed in parallel, the effect is the same as increasing the number of plates, viz., the capacity of the circuit is increased. When several condensers are in parallel the total capacity is the sum of the separate capacities. That is, $C_P = C_1 + C_2 + C_3 + \ldots$ (23)

Example:

Let Fig. 67 represent 3 condensers of 2, 3 and 5 microfarads respectively connected in parallel.

The capacity of the combination is

$$C_P = C_1 + C_2 + C_3$$
$$= 2 + 3 + 5$$
$$= 10 \text{ mf. Ans.}$$

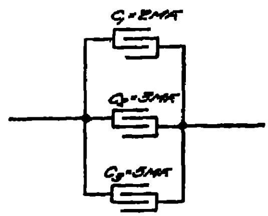


Fig. 67. — Condensers in Parallel.

It will be noted that the formulas for condensers in series or parallel are similar to those for resistance in series and parallel except that they are interchanged.

Condensers in Parallel-Series. When condensers are in a parallel-series combination, as in Fig. 68, find the capacity of the parallel part first and treat this as one condenser, then find the capacity of this imaginary condenser in series with the other condenser.

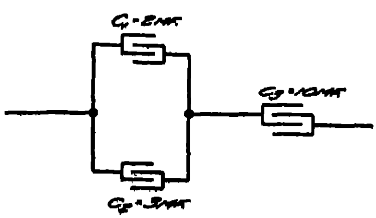


Fig. 68. — Condensers in Parallel Series.

Example:

$C_P = C_1 + C_2 = 2 + 3 = 5$ mf.

$$C_{PS} = \frac{1}{\frac{1}{C_P}+\frac{1}{C_3}} = \frac{1}{\frac{1}{5}+\frac{1}{10}}$$
$$= 3.33 \text{ mf. Ans.}$$

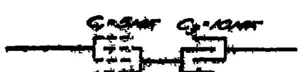


Fig. 69. — Equivalent Circuit of Fig. 68.

When arranged as in Fig. 70, solve series part first.

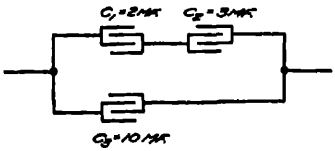


Fig 70 — Two Condensers in Series Paralleled With a Third Condenser

Thus

$$C_S = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{1}{\frac{1}{2} + \frac{1}{3}}$$
$$= 1.2 \text{ mf.}$$
$$C_{SP} = 1.2 + 10$$
$$= 11.2 \text{ mf.} \quad \text{Ans.}$$

PROBLEMS

1. What will be the charge on a condenser whose capacity is 100 microfarads, when put across a circuit whose voltage is 110?

2 What voltage would have to be put across a condenser of 200 microfarads capacity to allow 1 ampere to flow? The frequency of the circuit is 60 cycles

3 Would a condenser draw more or less current on a 60-cycle circuit than a 25-cycle circuit, the voltage in each case being the same? Explain

4 How large would a condenser have to be to draw 1 ampere on a 110-volt 60-cycle circuit?

5 Calculate the capacity of 3 condensers of capacities of 2, 5 and 10 microfarads each, when connected in series.

6 Calculate the capacity of the three condensers of Prob 5 when they are connected in parallel

7 How much current would flow with the arrangement of Prob. 5 if the voltage across the condensers is 110 and the frequency 60 cycles?

8 How much current would flow with the arrangement of Prob. 6 if the voltage is 110 and the frequency 60 cycles?

CHAPTER V

SERIES CIRCUITS

In general, Ohm's law cannot be applied as simply to alternating-current circuits as to direct-current circuits, on account of the fact that where there is inductance or capacity in an alternating-current circuit, the current is thrown out of phase with the voltage that produces it

This phase displacement makes it necessary to modify the usual formula $I = \frac{E}{R}$ to take into account the effect of inductance and capacity, as well as resistance. In developing the formula for a circuit containing inductance, use is made of inductive reactance which takes into account the inductance and the frequency of the circuit. Inductive reactance is in ohms, as explained in the chapter on Inductance. The symbol is X_L and it is equal to 2π times the frequency times the inductance in henrys, viz., $X_L = 2\pi fL$.

The E. M. F. impressed upon a circuit containing both resistance and inductance in series as in Fig. 71 may be thought of as

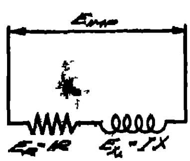


Fig. 71. — Circuit with Resistance and Inductance.

Fig. 72. — Relations of Current, Resistance Drop and Reactance Drop.

of two parts E_R and E_{XL}. As the current rises and falls, the resistance opposes it directly and at every instant the resistance-drop $E_R = IR$. Similarly the E. M. F. set up by the inductance is $E_X = IX_L$. It was shown under Inductance that the rising and

falling current in an alternating-current circuit set up lines of force in phase with the current and that these varying lines of force induced an E. M. F. in the circuit 90 degrees behind the current. The E M F. to overcome the induced E. M. F. would have to directly oppose it or be 180 degrees from it. Hence the E. M. F. to overcome the effect of inductance would be 90 degrees ahead of the current. The relations are as in Fig. 72.

Now $E_{XL} = IX_L$ and $E_R = IR$ so we can change the diagram of Fig 72 to Fig. 73. If now we combine IR and IX we shall get the total E. M. F. E_{imp} which is necessary to send the current through the circuit. Its direction and magnitude will be OE_{imp}.

From Fig 73 it will be seen that the impressed E. M. F., E_{imp} is ahead of the current. That is, the current lags the E. M. F. by an angle ϕ.

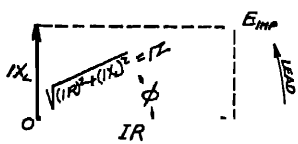


Fig 73 — Diagram Showing Relations of Resistance Drop, Inductive Reactance Drop and Impedance Drop

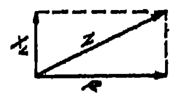


Fig 74. — Relation of Resistance, Inductive Reactance and Impedance.

The line OE_{imp} represents an E. M. F. whose value is

$$\sqrt{\overline{IR}^2 + \overline{IX_L}^2}$$

In order to form an equation containing I, let $OE_{imp} = IZ$. When Z is a quantity measured in ohms and called "impedance,"

then
$$OE_{imp} = IZ = \sqrt{\overline{IR}^2 + \overline{IX_L}^2}$$

$$I^2Z^2 = I^2R^2 + I^2X_L^2$$
$$Z^2 = R^2 + X_L^2$$
$$Z = \sqrt{R^2 + X_L^2} \qquad (24)$$

It will be seen from Fig. 73 that each side of the triangle contains I, so if we divide through by I we shall get a similarly-shaped triangle whose sides R, X and Z will be as in Fig. 74.

When capacity is present as well as inductance, the capacity reactance must be laid off in the opposite direction from the inductive reactance, because the effect of capacity is to make current lead, while inductance tends to make it lag. That is, we lay off inductive reactance upward or 90° ahead of R, and the capacity reactance downward or 90° behind R, as in Fig. 75. Figure 76 shows two methods of finding impedance, when there are inductance reactances and capacity reactances in a circuit.

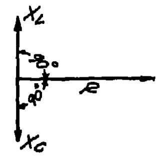


Fig. 75 — Relations of Resistance, Inductive Reactance and Capacity Reactance.

Inspection of Fig. 76 will show that current will lag behind the voltage when the inductive reactance is larger than the capacity reactance and that current will lead the voltage when the capacity reactance is larger than the inductive reactance.

Fig 76. — Methods of Finding Impedance in a Circuit Containing Inductive Reactance and Capacity Reactance

Figure 76 shows also that when inductive reactance and capacity reactance are equal, the current will be in phase with the voltage and the resistance will equal the impedance.

Effective Resistance. By effective resistance is meant the resistance that a circuit offers to alternating current. It may vary with the voltage, frequency or current. Effective resistance is not the same as impedance, because impedance includes the reactance of the circuit while effective resistance only includes that increase over the true ohmic resistance due to the effect of the alternating E. M. F. and current.

Briefly, when alternating current flows in a conductor, it is forced toward the surface of the conductor, giving what is known as the "skin effect." The net area of copper carrying current is

less than if the current were evenly distributed as with direct current, so an increase in resistance results from this cause.

Further, the alternating flux set up by the current cuts any conductors in or near the circuit and induces eddy currents in them. These eddy currents heat the conductors and require that more voltage be applied to the circuit to keep up the current. Similarly, any magnetic material in or near the circuit is cut by the flux and in being magnetized and demagnetized requires energy to overcome the hysteresis loss in the magnetic material.

Further, the E. M F. exerts a stress on the insulation, and as this stress is applied first in one direction and then in the other, a heat loss occurs in the insulation.

The effect of these losses is to require extra voltage to keep up the same current that would flow with a unidirectional voltage, in other words, increase the "resistance."

All of the losses will appear as watts if a wattmeter be connected in the circuit.

Therefore, if we measure carefully by means of a wattmeter the watts used in a coil carrying direct current, and then measure the watts with the same value of current using alternating current, we should expect the reading of watts with alternating current to be greater. In the case of direct current,

$$P_{dc} = I_{dc}^2 R \qquad \text{or} \qquad R = \frac{P_{dc}}{I_{dc}^2} \tag{25}$$

R is the true ohmic resistance.

In the second case, when we use alternating current, the wattmeter will read the true watts used in the circuit, but they will be greater than for the direct current for the reasons previously mentioned; viz., skin effect, eddy current losses, hysteresis loss and dielectric losses.

With alternating current,

$$P_{ac} = I_{ac}^2 R_{eff} \qquad \text{or} \qquad R_{eff} = \frac{P_{ac}}{I_{ac}^2} \tag{26}$$

R_{eff} is the effective resistance.

Graphically, the ohmic resistance, effective resistance, reactance and impedance may be illustrated by a modification of the well-known triangle, Fig. 77.

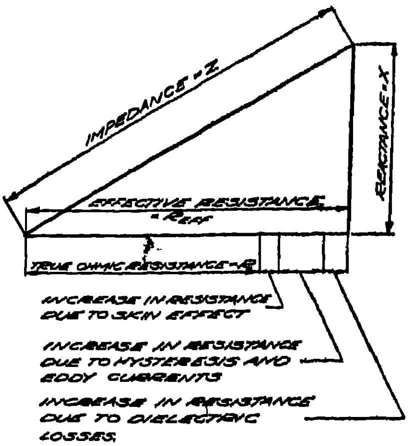


Fig. 77. — Diagram Illustrating Effective Resistance.

Analysis of Series Circuits. —

I. **Resistance only.**

$$I = \frac{E}{R} \tag{27}$$

E = effective volts
I = effective current
R = resistance in ohms

The current is in phase with the E. M. F.
The power factor is 100%.

Problem. Volts 110, resistance 55 ohms. Required the current.

$$I = \frac{E}{R}$$

E = 110 volts
R = 55 ohms

Subs. $I = \frac{110}{55} = 2$ amp. *Ans.*

Fig. 78. — Solution Diagram, Resistance Only.

II. Resistance and Inductance only.

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + X^2}} = \frac{E}{\sqrt{R^2 + (2\pi fL)^2}} \tag{28}$$

E = effective volts
I = effective current
R = resistance in ohms
Z = impedance in ohms
$= \sqrt{R^2 + X^2} = \sqrt{R^2 + (2\pi fL)^2}$
X_L = reactance in ohms
$= 2\pi fL$
f = frequency
2π = 6 2832
L = inductance in henrys

Current lags
The power factor is less than 100%

$$P.F. = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X^2}} = \frac{R}{\sqrt{R^2 + (2\pi fL)^2}} \tag{29}$$

Angle of lag is the angle whose tangent is $\frac{X}{R}$ or $\frac{2\pi fL}{R}$. (30)

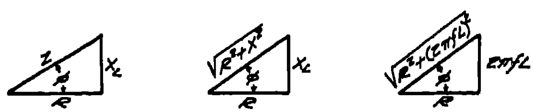


Fig 79 — Diagram, Resistance and Inductance Only

Problem. Volts 108, resistance 60 ohms, inductive reactance, 90 ohms (*a*) Make a drawing showing the circuit, (*b*) Find current, (*c*) Find power factor, (*d*) Find angle of lag, (*e*) Make diagram showing quantities.

Solution·
(*a*)

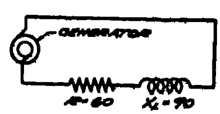


Fig. 80. — Circuit with Resistance and Inductive Reactance.

(*b*) $I = \frac{E}{\sqrt{R^2 + X^2}}$

E = 108 ohms
R = 60 ohms
X_L = 90 ohms

$$\text{Subs. } I = \frac{108}{\sqrt{60^2 + 90^2}} = \frac{108}{108}$$

$$= 1 \text{ amp. } \textit{Ans.}$$

(c)

$$\text{P.F.} = \frac{R}{Z}$$

$$R = 60 \text{ ohms}$$

$$Z = 108 \text{ ohms}$$

$$\text{Subs.} \quad = \frac{60}{108} = .555 = 55.5\,\%$$

(d) Angle of lag is the angle whose cosine is $\frac{R}{Z} = 555 = 56°$ approximately. Ans.

(e)

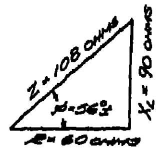


Fig 81. — Solution Diagram, Resistance and Inductive Reactance.

III. Resistance and Capacity only.

E = effective volts

R = resistance in ohms

Z = impedance in ohms

$$= \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}$$

X_C = reactance in ohms

$$= \frac{1}{2\pi fC}$$

$2\pi = 6.2832$

f = frequency

C = capacity in farads

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + X_C^2}}$$

$$= \frac{E}{\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}} \qquad (31)$$

The current leads the E. M. F. The power factor is less than 100%.

$$\text{P.F.} = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{R}{\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}} = \cos\phi \qquad (32)$$

Angle of lead is the angle whose tangent is $\frac{X_C}{R} = \frac{\frac{1}{2\pi fC}}{R}$ or whose cosine is

$$\frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{R}{\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}}$$

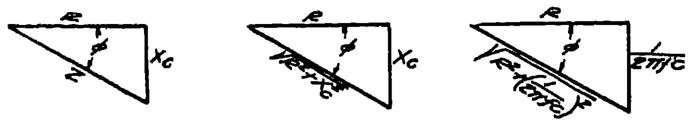


Fig 82 — Diagram, Resistance and Capacity.

Problem. Volts 110, resistance 10 ohms, capacity .000265 farad, frequency 60, required (a) current and (b) power factor.

Solution:

$$E = 110 \text{ volts}$$
$$R = 10 \text{ ohms}$$
$$f = 60 \text{ cycles}$$
$$C = 000265 \text{ farads}$$

(*a*)

$$I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}}$$

Subs.

$$I = \frac{110}{\sqrt{10^2 + \left(\frac{1}{6.2832 \times 60 \times .000265}\right)^2}} = \frac{110}{14.1} = 7.8 \text{ amp. Ans.}$$

(*b*)

$$\text{P.F.} = \text{Cos}\,\phi = \frac{R}{Z} = \frac{10}{14.1} = .709 = 70.9\,\%. \quad \text{Ans.}$$

Note Capacity is usually measured in *microfarads*. Divide microfarads by 1,000,000 before substituting in formulas.

IV. Resistance, Inductance and Capacity.

E = effective volts

R = resistance in ohms

Z = impedance in ohms

$$= \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

X_L = inductive reactance
X_C = capacity reactance
2π = 6.2832
L = inductance in henrys
C = capacity in farads
f = frequency

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}} \qquad (33)$$

Current may lag or lead or be in phase with E. M. F. The power factor will be less than 100% if the current lags or leads. Power factor will be 100% when current is in phase with E. M. F.

$$P.F = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}$$

$$= \cos \phi \qquad (34)$$

The angle between E. M. F. and current is the angle whose tangent is $\dfrac{X_L - X_C}{R}$ or $\dfrac{2\pi fL - \frac{1}{2\pi fC}}{R}$

or whose cosine is

$$\frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}$$

When X_L	is greater than X_C	current lags
or $2\pi fL$	" " " $\frac{1}{2\pi fC}$	" "
When X_C	" " " X_L	current leads
or $\frac{1}{2\pi fC}$	" " " $2\pi fL$	" "
When X_C	is same value as X_L	current is in phase
or $\frac{1}{2\pi fC}$	" " " " $2\pi fL$	" " " "

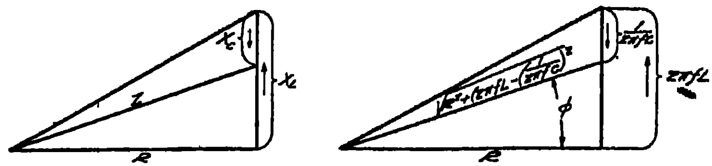


Fig 83 — Diagram, Resistance, Inductance and Capacity.

Problem. Resistance 100 ohms, inductance 265 henry, capacity .0000295 farads, frequency 60 cycles, volts 110 Required: (*a*) current, (*b*) power factor, (*c*) diagram properly lettered.

Solution

E = 110 volts
R = 100 ohms
f = 60 cycles
L = 265 henry
C = .0000295 farads

(*a*) $$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}$$

Subs.

$$= \frac{110}{\sqrt{100^2 + \left(6\ 2832 \times 60 \times 265 - \frac{1}{6.2832 \times 60 \times .0000295}\right)^2}}$$

$$= \frac{110}{\sqrt{100^2 + (100 - 90)^2}}$$

$$= \frac{110}{\sqrt{100^2 + 10^2}} = \frac{110}{\sqrt{10,100}}$$

$$= \frac{110}{100.5} = 1\ 09 \text{ amp.} \quad \text{Ans.}$$

(*b*)

$$\text{P.F.} = \frac{R}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}$$

$$= \frac{100}{\sqrt{10,100}} = .994 = 99.4\% \quad \text{Ans.}$$

(*c*)

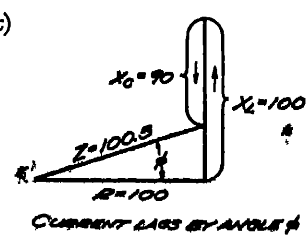


Fig. 84. — Solution Diagram, Resistance, Inductance and Capacity.

Resonance in a Series Circuit.

In the formula $I = \dfrac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$

When $X_L = X_C$, $I = \dfrac{E}{\sqrt{R^2 + 0^2}} = \dfrac{E}{R}$

That is, when the inductive reactance equals the capacity reactance, the current is equal to $\frac{E}{R}$, the same as in direct-current circuits, or in alternating-current circuits containing no inductance or capacity.

When $X_L = X_C$, or $2\pi fL = \frac{1}{2\pi fC}$, the circuit is said to be in resonance. Resonance may be caused in a circuit by a change in L, f, or C. When the resistance is low and the voltage fairly high a resonant condition in a circuit will allow a very large current to flow; often sufficient to cause damage. The following example will illustrate how changing the inductance will cause resonance and a very large current.

A circuit has a resistance of 1 ohm; an inductance of .0601 henry and a capacity of .001 farad. If 220 volts at 60 cycles are impressed on the circuit the current will be,

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}$$

$$= \frac{220}{\sqrt{1^2 + \left(2 \times 3.14 \times 60 \times .0601 - \frac{1}{2 \times 3.14 \times 60 \times .001}\right)^2}}$$

$$= \frac{220}{\sqrt{1^2 + (22.65 - 2.65)^2}}$$

$$= \frac{220}{\sqrt{1^2 + 20^2}} = \frac{220}{\sqrt{401}} = \frac{220}{20.0} = 11 \text{ amperes}$$

If the inductance be changed to .00704 henry the current will be,

$$I = \frac{220}{\sqrt{1^2 + \left(2 \times 3.14 \times 60 \times .00704 - \frac{1}{2 \times 3.14 \times 60 \times .001}\right)^2}}$$

$$= \frac{220}{\sqrt{1^2 + (2.65 - 2\ 65)^2}} = 220 \text{ amperes}$$

Resonance may be caused by a change in frequency when both inductance and capacity remain constant. The condition for resonance is that,

$$2\pi fL = \frac{1}{2\pi fC}$$

Multiplying both sides by $2\pi f$

$$(2\pi f)^2L = \frac{1}{C}, \quad \text{or} \quad (2\pi f)^2 = \frac{1}{LC}$$

from which $$2\pi f = \sqrt{\frac{1}{LC}}$$

or $$f = \frac{1}{2\pi\sqrt{LC}} \tag{35}$$

PROBLEMS

1. What current will flow in a circuit containing 25 ohms resistance and 42 ohms inductive reactance in series if the voltage across the circuit is 220?

2. What voltage will be required to send a current of 12 amperes through a series circuit which has a resistance of 16 ohms and an inductance of .4 henry? The frequency of the circuit is 60 cycles per second.

3. What will be the power factor of the circuit of problem 2?

4. A circuit has a resistance of 12 ohms and an inductance of .08 henry. The voltage is 110 and the frequency 60. Calculate (*a*) current, (*b*) drop across resistance and inductance, (*c*) power factor, (*d*) angle of lag

5. A series circuit contains a resistance of 90 ohms, an inductance of .2 henry and a condenser of a capacity of .00002 farads. The voltage across the circuit is 110. Find (*a*) the current, (*b*) the power factor and (*c*) the angle of lag or lead.

6. A voltage of 120 volts at 60 cycles was impressed across a choke coil. The current was 6 amperes. What was the impedance?

7 A wattmeter placed in the circuit of problem 6 read 480 watts. What was (*a*) the power factor? (*b*) the inductive reactance and (*c*) the inductance at the frequency and current at which the coil was tested?

8 What will be the angle of lag or lead and the current in a circuit containing 20 ohms resistance. 10 ohms inductive reactance and 12 ohms capacity reactance, if 110 volts are impressed on the circuit?

9 How large a condenser must be placed in series with a resistance of 5 ohms and inductance of 2 henry to bring the current in phase with the voltage if the frequency is 60? What current will flow if 100 volts are impressed on the circuit?

10 What will be the effect on the current in a circuit containing resistance and inductance if the frequency be increased?

11. What will be the current in a circuit containing 10 ohms resistance and a condenser of .00002 microfarads if 200 volts at 60 cycles are impressed on the circuit? What will be the current if the frequency be changed to 25 cycles?

12 What is the effect on the power factor of a series circuit containing inductance, of increasing the resistance?

CHAPTER VI

PARALLEL CIRCUITS

In an alternating-current circuit the current in any branch is equal to the voltage across the branch divided by the impedance of the branch. If, for instance, there are two branches and one contains resistance only and the other resistance and inductance, the current in the branch which has resistance only will be in phase with the voltage, and the current in the branch which has inductance will lag behind the voltage. The currents in the two branches will therefore be out of phase with each other.

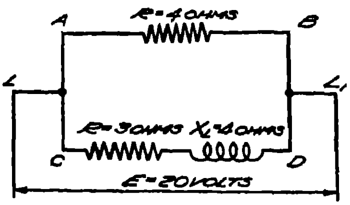


Fig. 85. — Typical Parallel Circuit.

The solution of a simple problem of this type will make this clear. In Fig. 85 the branch AB has a non-inductive resistance of 4 ohms. The branch CD has a resistance of 3 ohms and an inductive reactance of 4 ohms in series, 20 volts alternating are impressed across the circuit.

In branch AB,

$$I = \frac{E}{R} = \frac{20}{4} = 5 \text{ amperes in phase with voltage.}$$

In branch CD,

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + X^2}} = \frac{20}{\sqrt{3^2 + 4^2}} = \frac{20}{5} = 4 \text{ amperes which lags}$$

behind the voltage E by the angle whose tangent is $\frac{X}{R} = \frac{4}{3} = 53$ degrees.

To find the relation of the two currents to the voltage and to each other, draw the arrow or vector E = 20, Fig. 86. Since

I_{AB} neither lags or leads E_{AB}, I_{AB} can be drawn in the same direction as E_{AB}. In the branch CD the current lags behind the voltage by 53 degrees and has a value of 4 amperes, so draw I_{CD} 53 degrees behind E_{AB} and of a value of 4 to scale. The line current is the sum of the branch currents, taking into account both their magnitudes and directions, so if we combine vectors

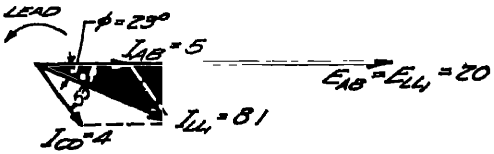


Fig. 86. — Graphical Method of Finding Line Current in a Parallel Circuit

I_{AB} and I_{CD} we shall obtain I_{LL_1} which lags behind E_{LL_1} by 23 degrees and whose value scales off 8.1 amperes.

The method of solution outlined above could be carried out for any problem, except that when the angles are small, a considerable error is likely to enter in, due to the difficulty of laying off and scaling the vectors accurately. The method next outlined enables us to use arithmetical computations instead of scaling lengths and angles from a drawing.

The method involves the use of three new terms, admittance, conductance and susceptance. Admittance may be defined as the reciprocal of impedance; the symbol is Y. That is, $Y = \frac{1}{Z}$. Just as in a series circuit the total voltage across a combination of resistance and inductive reactance is made up of an energy component and a reactive component, so in a parallel circuit, the total current through a resistance and reactance in parallel is made up of an energy component and a reactive component. Since $Y = \frac{1}{Z}$ and $I = \frac{E}{Z}$, $I = EY$. In a parallel circuit the energy component is denoted by "g" called conductance and the reactive component denoted by "b" called susceptance. If the two sides of a right-angle triangle be noted by "g" and "b" and the hypothe-

nuse by Y, then $Y = \sqrt{g^2 + b^2}$, or $EY = \sqrt{\overline{Eg}^2 + \overline{Eb}^2}$. The conductance "g" corresponds to the side of the triangle marked R in a series circuit and the susceptance "b" to the side marked X in a series circuit

Figure 87 shows the quantities Z, R and X properly marked on a triangle and Fig 88 shows the quantities Y, g and b on a similar triangle, $\phi_1 = \phi$.

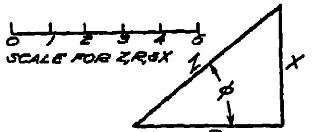


Fig. 87. — Triangle Showing Relation of R, X and Z

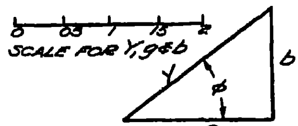


Fig 88. — Triangle Showing Relation of g, b and Y.

From geometry, $\frac{g}{Y} = \frac{R}{Z}$

Hence $g = \frac{RY}{Z} = \frac{R}{Z} \times \frac{1}{Z} = \frac{R}{Z^2}$

From geometry, $\frac{b}{Y} = \frac{X}{Z}$

Hence $b = \frac{XY}{Z} = \frac{X}{Z} \times \frac{1}{Z} = \frac{X}{Z^2}$

From the above,

When R = resistance in ohms of branch considered,
X_L = reactance in ohms of branch considered due to inductance,
X_C = reactance in ohms of branch considered due to capacity,
Z = impedance in ohms,

Then $$Y = \frac{1}{Z} \tag{36}$$

$$g = \text{conductance} = \frac{R}{Z^2} \tag{37}$$

$$b = \text{susceptance} = \frac{X}{Z^2} \tag{38}$$

Consider the problem shown in Fig. 85 by the "admittance method"

$R_{AB} = 4$, $Z_{AB} = 4$, $X_{AB} = 0$, hence,

$$Y_{AB} = \frac{1}{Z} = \frac{1}{4} = .25$$

$$g_{AB} = \frac{R}{Z^2} = \frac{4}{4^2} = .25$$

$$b_{AB} = \frac{X}{Z^2} = \frac{0}{4^2} = 0$$

$R_{CD} = 3$, $Z_{CD} = 5$, $X_{CD} = 4$, hence,

$$Y_{CD} = \frac{1}{Z} = \frac{1}{5} = .2$$

$$g_{CD} = \frac{R}{Z^2} = \frac{3}{5^2} = .12$$

$$b_{CD} = \frac{X}{Z^2} = \frac{4}{5^2} = .16$$

also $Y^2 = g^2 + b^2$. from (4

Next construct a triangle Fig. 89 putting the proper value on each side. From Fig. 89, $Y_{LL_1} = 403$. Hence $I = E \times Y = 20 \times .403$ $= 8\ 06$ amperes which lags behind the line voltage by an angle whose tangent is $\frac{.16}{.37}$ or 23°–30′ approximately.

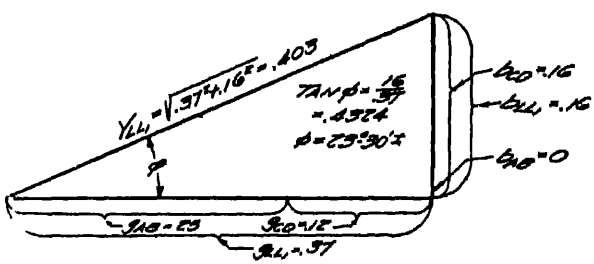


Fig. 89. — Solution Diagram, Parallel Circuit with Resistance and Inductive Reactance.

Where there are many branches, the work may be done systematically by making a table of the various quantities. The whole procedure may be summarized as follows:

To solve a problem by the "admittance method" find R, X, Z, g and b for each branch. Make a table of these values. Lay off to scale on a horizontal line all the "g's" and at the right-hand end of this line draw a vertical line. Lay off the "b's" on this vertical line. Lay off the "b's" due to inductance upward on this vertical line, and the

"b's" due to capacity downward At the point you reach when you lay off the last "b" draw a line connecting with the left-hand end of the horizontal line, thus forming a right-angle triangle. The hypothenuse thus obtained will be the admittance Y of the entire circuit. The total current will be EY.

Analysis of Parallel Circuits. —

I. Resistance Only.

Problem. A circuit has two branches. One branch contains a resistance of 10 ohms and the other a resistance of 20 ohms The voltage across the circuit is 100. Find (*a*) the total current, (*b*) the current in each branch, (*c*) the power factor of the circuit.

Solution:

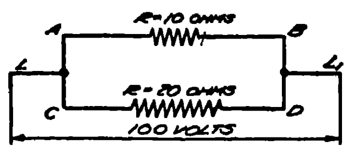


Fig. 90. — Circuit with Resistances in Parallel.

SYMBOL	CIRCUIT	
	A – B	C – D
R	10	20
X	0	0
Z	10	20
g	.1	05
b	0	0

$R_{AB} = 10$

$X_{AB} = 0$

$Z_{AB} = 10$

$$g_{AB} = \frac{R}{Z^2} = \frac{10}{10^2} = .1$$

$$b_{AB} = \frac{X}{Z^2} = \frac{0}{10^2} = 0$$

$R_{CD} = 20$

$X_{CD} = 0$

$Z_{CD} = 20$

$$g_{CD} = \frac{R}{Z^2} = \frac{\cancel{10}\ 20}{20^2} = \cancel{.025}\ 0.05$$

$$b_{CD} = \frac{X}{Z^2} = \frac{0}{20^2} = 0$$

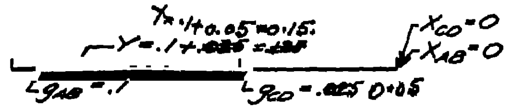


Fig. 91. — Solution Diagram, Resistances in Parallel.

(a) $I_{LL_1} = EY$ 0.15 $E = 100$ $I_{LL_1} = I_{AB} + I_{CD}$
$= 100 \times \cancel{.125}$ $Y = \cancel{.125}$ 0.15 $= 10 + 5 = 15$ amp.
$= \cancel{12.5}$ amp.   *Ans.* 15 amp.

(b) $I_{AB} = \dfrac{E_{AB}}{Z_{AB}}$ $E_{AB} = 100$, $Z_{AB} = 10$

$= \dfrac{100}{10}$

$= 10$ amp.   *Ans.*

$I_{CD} = \dfrac{E_{CD}}{Z_{CD}}$ $E_{CD} = 100$, $Z_{CD} = 20$

$= \dfrac{100}{20}$

$= 5$ amp.

(c) P.F. $= \dfrac{g}{Y} = \dfrac{\cancel{.125}\ 0.15}{\cancel{.125}\ 0.15}$ $g = .1\cancel{2}5$, $Y = .1\cancel{2}5$

$= 1$

$= 100\%$ *Ans.*

II. Resistance and Inductance.

Problem. A circuit has two branches. One branch contains a resistance of 12 ohms and the other branch an inductive reactance of 10 ohms. The voltage of the circuit is 100. Find (*a*) total current, (*b*) current in each branch, (*c*) power factor of the circuit.

Solution:

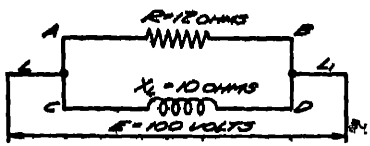


Fig. 92. — Circuit with Resistance and Inductive Reactance in Parallel.

SYMBOL	CIRCUIT	
	A – B	C – D
R	12	0
X	0	+10
Z	12	10
g	083	0
b	0	1

$R_{AB} = 12$
$X_{AB} = 0$
$Z_{AB} = 12$
$$g_{AB} = \frac{R}{Z^2} = \frac{12}{12^2} = 083$$
$$b_{AB} = \frac{X}{Z^2} = \frac{0}{12^2} = 0$$

$R_{CD} = 0$
$X_{CD} = 10$
$Z_{CD} = 10$
$$g_{CD} = \frac{R}{Z^2} = \frac{0}{10^2} = 0$$
$$b_{CD} = \frac{X}{Z^2} = \frac{10}{10^2} = .1$$

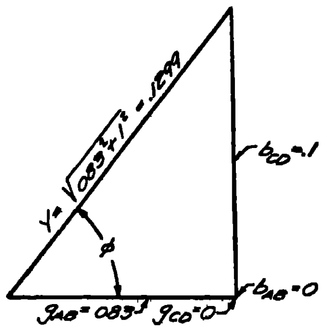


Fig. 93. — Solution Diagram, Resistance and Inductive Reactance in Parallel.

(a) $I_{LL_1} = EY$
$= 100 \times 1299$
$= 12.99$ amp. *Ans.*

$E = 100$
$Y = .1299$

(*b*) $$I_{AB} = \frac{E_{AB}}{Z_{AB}} \qquad E_{AB} = 100, \quad Z_{AB} = 12$$

$$= \frac{100}{12}$$

$$= 8.33 \text{ amp.} \qquad \textit{Ans.}$$

$$I_{CD} = \frac{E_{CD}}{Z_{CD}} \qquad E_{CD} = 100, \quad Z_{CD} = 10$$

$$= \frac{100}{10}$$

$$= 10 \text{ amp.} \qquad \textit{Ans.}$$

(*c*) $$\text{P.F.} = \frac{g}{Y} \qquad g = .083, \quad Y = 1299$$

$$= \frac{.083}{.1299}$$

$$= .64$$

$$= 64\% \qquad \textit{Ans.}$$

III. Resistance and Capacity.

Problem. A circuit has two branches. One branch contains a resistance of 25 ohms and the other a capacity reactance of 20 ohms. The voltage is 100. Find (*a*) the total current, (*b*) the current in each branch, (*c*) the power factor of the circuit.

Solution:

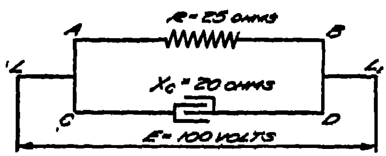


Fig. 94. — Circuit with Resistance and Capacity Reactance in Parallel.

SYMBOL	CIRCUIT	
	A – B	C – D
R	25	0
X	0	–20
Z	25	20
g	04	0
b	0	– 05

$R_{AB} = 25$ $\qquad$ $R_{CD} = 0$

$X_{AB} = 0$ $\qquad$ $X_{CD} = 20$

$Z_{AB} = 25$ $\qquad$ $Z_{CD} = 20$

$$g_{AB} = \frac{R}{Z^2} = \frac{25}{25^2} = .04 \qquad g_{CD} = \frac{R}{Z^2} = \frac{0}{20^2} = 0$$

$$b_{AB} = \frac{X}{Z^2} = \frac{0}{25^2} = 0 \qquad b_{CD} = \frac{X}{Z^2} = \frac{20}{20^2} = .05$$

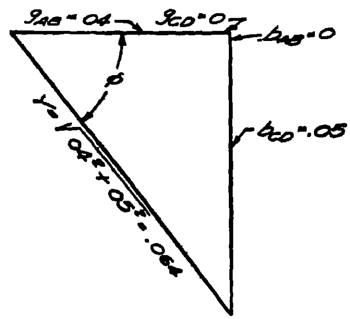


Fig 95. — Solution Diagram, Resistance and Capacity Reactance in Parallel.

(a) $\quad I_{LL_1} = EY$ $\qquad$ $E = 100$

$\quad = 100 \times 064$ $\qquad$ $Y = .064$

$\quad = 6.4$ amp. $\quad$ *Ans.*

(b) $$I_{AB} = \frac{E_{AB}}{Z_{AB}} \qquad E_{AB} = 100, \quad Z_{AB} = 25$$

$$= \frac{100}{25}$$

$$= 4 \text{ amp.}$$ *Ans.*

$$I_{CD} = \frac{E_{CD}}{Z_{CD}} \qquad E_{CD} = 100, \quad Z_{CD} = 20$$

$$= \frac{100}{20}$$

$$= 5 \text{ amp.}$$ *Ans.*

(c) $$\text{P.F.} = \frac{g}{Y} = \frac{.04}{.064} = .625$$

$$= 62\,5\%$$ *Ans.*

IV. Resistance, Inductance, Capacity.

Problem. A circuit has three branches. The first branch contains a resistance of 25 ohms, the second an inductive reactance of 33.3 ohms, the third a capacity reactance of 12 5 ohms. Find (a) the total current, (b) the current in each branch, (c) the power factor of the circuit. The voltage across the circuit is 100.

Solution:

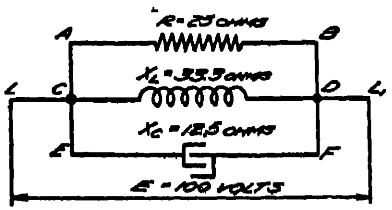


Fig. 96 — Circuit with Resistance, Inductive Reactance and Capacity Reactance in Parallel.

SYMBOL	CIRCUIT		
	A – B	C – D	E – F
R	25	0	0
X	0	+ 33 3	– 12 5
Z	25	33 3	12 5
g	04	0	0
b	0	03	– 08

$$R_{AB} = 25$$
$$X_{AB} = 0$$
$$Z_{AB} = 25$$
$$g_{AB} = \frac{R}{Z^2} = \frac{25}{25^2} = 04$$
$$b_{AB} = \frac{X}{Z^2} = \frac{0}{25^2} = 0$$

$$R_{CD} = 0$$
$$X_{CD} = 33\ 3$$
$$Z_{CD} = 33.3$$
$$g_{CD} = \frac{R}{Z^2} = \frac{0}{33\ 3^2} = 0$$
$$b_{CD} = \frac{X}{Z^2} = \frac{33\ 3}{33\ 3^2} = 03$$

$$R_{EF} = 0$$
$$X_{EF} = 12\ 5$$
$$Z_{EF} = 12.5$$
$$g_{EF} = \frac{R}{Z^2} = \frac{0}{12.5^2} = 0$$
$$b_{EF} = \frac{X}{Z^2} = \frac{12\ 5}{12.5^2} = .08$$

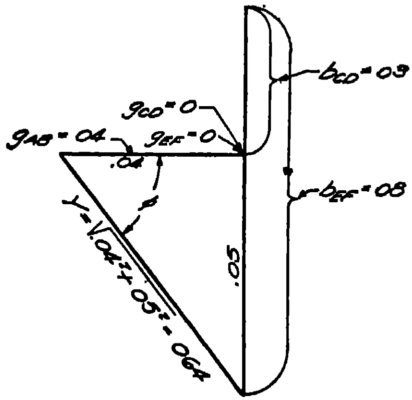


Fig 97 — Solution Diagram, Resistance, Inductive Reactance and Capacity Reactance in Parallel.

(*a*) $I_{LL_1} = EY$ $\qquad E = 100$

$= 100 \times .064$ $\qquad Y = .064$

$= 6.4$ amp *Ans.*

(*b*) $I_{AB} = \dfrac{E_{AB}}{Z_{AB}}$ $\qquad E_{AB} = 100$, $Z_{AB} = 25$

$= \dfrac{100}{25}$

$= 4$ amp. *Ans.*

$I_{CD} = \dfrac{E_{CD}}{Z_{CD}}$ $\qquad E_{CD} = 100$, $Z_{CD} = 33.3$

$= \dfrac{100}{33\ 3}$

$= 3$ amp. *Ans.*

$I_{EF} = \dfrac{E_{EF}}{Z_{EF}}$ $\qquad E_{EF} = 100$, $Z_{EF} = 12.5$

$= \dfrac{100}{12\ 5}$

$= 8$ amp. *Ans.*

(*c*) $\text{P.F.} = \dfrac{g}{Y}$ $\qquad\qquad g = .04$, $Y = .064$

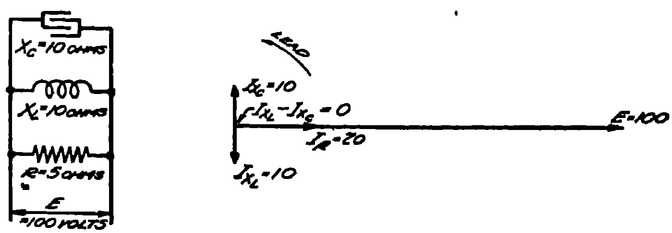
$$= \frac{04}{.064}$$
$$= .625$$
$$= 62.5\% \quad \textit{Ans.}$$

Parallel Resonance. It was shown that for a circuit containing resistance, inductive reactance and capacity reactance in series, the line current became maximum when $X_L = X_C$ or $2\pi fL = \dfrac{1}{2\pi fC}$. The reason for this is that in the equation

$$I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$$

the term $(X_L - X_C) = 0$ or the inductive and capacity reactive effects neutralize each other, leaving only the ohmic resistance to oppose the flow of current. If the ohmic resistance is small the current becomes very large.

In the case of a parallel circuit, the line current becomes minimum when the circuit is in resonance, as the following analysis will show.



(a)

Fig. 98 — Circuit and Diagram Illustrating Parallel Resonance.

In Fig. 98 let an alternating voltage of 100 volts be impressed across a circuit which contains a resistance $R = 5$ ohms, an inductive reactance $X_L = 10$ ohms and a capacity reactance $X_C = 10$ ohms, all in parallel.

The currents will be as follows:

$$\text{In R,}\quad I_R = \frac{100}{5} = 20 \text{ amperes in phase with E}$$

$$\text{In } X_L,\ I_{XL} = \frac{100}{10} = 10 \text{ amperes, lagging E by } 90^\circ$$

$$\text{In } X_C,\ I_{XC} = \frac{100}{10} = 10 \text{ amperes leading E by } 90^\circ$$

Inspection of Fig. 98 will show that I_R, the current that flows when $X_L = X_C$, is the smallest current that can flow for a given voltage of 100 volts, with the frequency kept constant.

If, for instance, X_L had been 12 ohms and X_c had been 8 ohms, then I_{XL} would have been $\frac{100}{12} = 8.33$ amperes, lagging behind the voltage E by 90°, and I_{XC} would have been $\frac{100}{8} = 12.5$ amperes, leading E by 90° There would have been a component of current along the line I_{XC}, Fig 98 (b), equal to 12 5 − 8.33 = 4.17 amperes. The resultant line current would have been $I' = \sqrt{4.17^2 + 20^2} = 20.4$ amp which is greater than I_R. Further, this current I' would lead the voltage E by an angle whose tangent is $\frac{4.17}{20} = .2085$ or 11° − 50′ approximately.

Similarly, if X_c had been greater than X_L, then the line current I' would have been larger than I_R but would have lagged behind I_R.

PROBLEMS

1. A circuit with three branches has a resistance of 22 ohms in the first branch, an inductive reactance of 55 ohms in the second branch and a capacity reactance of 27 5 ohms in the third branch. Find (*a*) total current, (*b*) current in each branch, (*c*) power factor. Make a diagram showing the relation of the quantities found

2. A circuit has two branches. One branch contains a resistance of 2 ohms and the other contains a resistance of 4 ohms in series with a capacity reactance of 3 ohms. The voltage of the circuit is 100. Find (*a*) total current, (*b*) current in each branch, (*c*) power factor. Make a diagram showing the relation of the quantities found.

3. A circuit has two branches. One branch contains a resistance of 160 ohms and the other contains an inductance of .1 henry in series with a capacity reactance of 100 microfarads The voltage of the circuit is 100 volts at 60 cycles Find (*a*) total current, (*b*) current in each branch, (*c*) power factor of the circuit

4. Find the total current and the current in each branch of a circuit of two branches one of which contains a resistance of 20 ohms in series with an inductance of .1 henry and the other contains a resistance of 25 ohms in series with an inductance of .05 henry. Voltage is 110 and frequency 60.

5. Find the total current and the current in each branch of the circuit shown below. Make a diagram showing the relation of the quantities found. Suggestion Find impedance of branched part of circuit first.

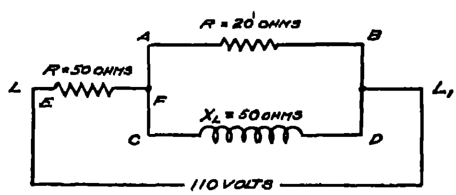


Fig 99 — Series-Parallel Circuit.

CHAPTER VII

VECTORS

In electrical work a vector is an arrow which rotates counter clockwise as a radius, about a point. The arrow may be used to denote either E. M F or current. The length of the arrow represents the magnitude of the E. M. F. or the current, and the angle the arrow makes at a given instant with a line of reference or another vector, is its phase displacement from that line of reference or vector. Vectors show the same relations between E. M. F.'s and currents that waves of E M F. and current show. Vectors, however, are much simpler to draw, and to use in numerical problems than the actual waves. In many cases vectors can be drawn to scale, and desired results obtained by scaling the drawing, no numerical calculations being necessary. In other cases solution of problems can be made by simple trigonometry.

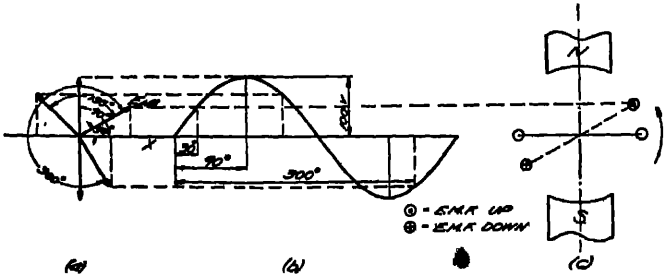


Fig. 100. — E. M. F Vector.

In Fig. 100(a) $E_{AB} = 100$ represents a vector of E. M. F. whose maximum value reaches 100 volts. E_{AB} rotates counter clockwise at a uniform rate. Let the angle that E_{AB} makes with the horizontal line OX be the angle that the coil which generates the E. M. F. has turned from the neutral plane at the given instant as shown at (c), then if the wave of the E. M. F. be plotted

as shown at (b), in Fig. 100, it will be seen that a line dropped from the point of the arrow E_{AB} to the horizontal will be the value of the E. M. F. for that particular position of the coil.

Hence when the maximum value of an E. M. F. is denoted by a vector, the length of a line dropped from the end of the vector to the horizontal line represents the instantaneous value of the E. M. F for the phase angle shown.

Relation Between E.M.F. and Current Shown by Vectors. A vector may be used to indicate the current that flows in a circuit. Referring to Fig. 100, assume that the 100 volts indicated by E_{AB} are impressed across a circuit containing 1 ohm resistance and 1 ohm inductive reactance. The relation of reactance and resistance is such in this case that the current of 70.7 amperes will lag the E. M. F. by 45°. The vectors would then be drawn as in Fig. 101, making I_{AB} 45° behind E_{AB}. The full lines Fig. 101 show one position of E_{AB} and I_{AB} The dotted lines show another position of E_{AB} with I_{AB} 45° behind it.

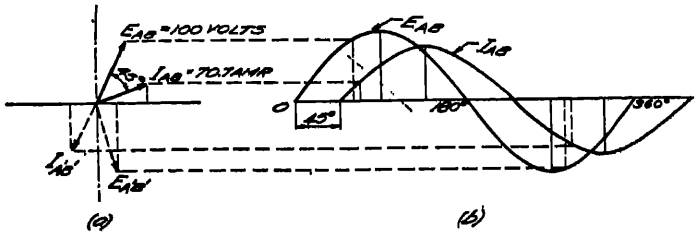


Fig 101. — E. M. F. and Current Vectors.

By following the projection lines from the vectors to the curves, the values shown by the vectors and curves are seen to be the same.

The vector E_{AB} shown by the full line represents the fact that the coil is just approaching the center of one pole. The vector $E_{A'B'}$ shown dotted represents the condition when the coil has turned somewhat past the center of the next pole. At (b) Fig. 101 the complete curves of E. M. F. and current are shown.

Addition of Vectors. Vectors of the same kind may be added just as, in mechanics, forces may be added. Two methods of ad-

dition will be considered, the crank-phase method and the topographic method.

By the crank-phase method, all the vectors to be added are drawn radiating from a point. Figure 102 shows three E. M F.'s of values 100, 80 and 75 volts respectively with phase relations to the reference line OX of 30°, 60°, and 90°.

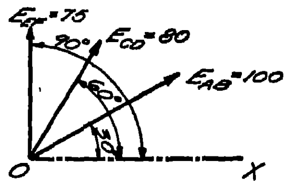


Fig 102. — Crank-Phase Method of Showing Vectors.

To add vectors shown by the crank-phase method, form a parallelogram using any two vectors as two sides and find the diagonal of this parallelogram. Take the diagonal thus formed as one side of a new parallelogram and one other vector as the adjacent side, and form another parallelogram. Find the diagonal of this parallelogram, etc., and continue until all vectors have been used. The length of the last diagonal found will be the value of the sum of the vectors. The angle between this diagonal and the line of reference will be the phase relation of the sum or resultant, to the line of reference, and the angle between the resultant and any one of the vectors will be the phase relation of the resultant to the particular vector chosen

In Fig. 103, from E_{AB} as a center and with E_{CD} as a radius draw an arc mn, and with E_{CD} as a center and E_{AB} as a radius draw another arc pq, cutting arc mn. Connect the point of intersection of mn and pq with E_{AB} and E_{CD} forming a parallelogram. Draw the diagonal $E_{AB} \oplus E_{CD}$. With the point of intersection of mn and pq as a center and a radius equal to E_{EF}, draw the arc rs. With E_{EF} as a center and a radius equal to $E_{AB} \oplus E_{CD}$ draw the arc tv. Form a parallelogram and draw the diagonal $E_{AB} \oplus E_{CD} \oplus E_{EF}$. This diagonal represents by its length, the vector sum of E_{AB}, E_{CD} and E_{EF}. It leads the line of reference OX by the angle α. By scaling the drawing, the line $E_{AB} \oplus E_{CD} \oplus E_{EF}$ is found to be 232 volts and the angle α is found to be 56° 40′.

If the results are needed to a high degree of accuracy the length of diagonals and value of angle α should be obtained by trigonometry using the formulas given in Chap. XIII.

To add by the topographic method, draw the vectors so that the head of one arrow touches the tail of the next, continuing until all vectors to be added are so drawn. The angle between any two vectors is the angle obtained by extending the first vector, and laying off the second vector from this extended vector, clockwise or counter clockwise as the case may be The sum of the vectors is a line drawn from the tail of the first vector to the head of the last vector. Figure 104 shows the vectors of Fig. 103 added by the topographic method. Draw E_{AB}, Fig. 104,

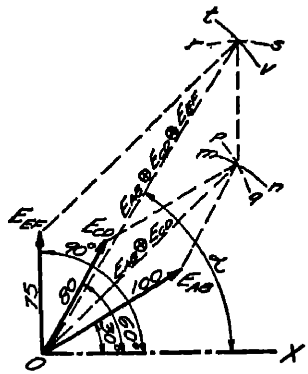


Fig. 103 — Crank-Phase Method of Adding Vectors.

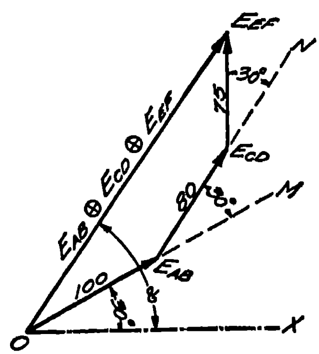


Fig. 104. — Topographic Method of Adding Vectors

and extend it to M. At E_{AB} draw E_{CD} making an angle of 30° with E_{AB} extended. E_{CD} is drawn leading E_{AB} by 30° because E_{CD} leads OX by 60° and E_{AB} leads OX by 30°. The difference of 30° is in a leading direction. Extend E_{CD} to N and draw E_{EF} in a leading direction from E_{CD} of 30° (90°–60°). Draw OE_{EF}. It will be seen that OE_{EF} has the same value and direction as $E_{AB} \oplus E_{CD} \oplus E_{EF}$ in Fig. 103.

Note that when the topographic method of adding vectors is used, the vectors to be added point around the polygon in one direction and the resultant in the opposite direction.

Subtraction of Vectors. To subtract one vector from another, reverse the subtrahend and proceed as in addition. In Fig. 105,

the vector E_{AB} leads the reference line OX by 30° and has a value of 80 volts. E_{OD} has a value of 60 volts and leads OX by 75°.

To subtract E_{OD} from E_{AB}, reverse E_{OD} as shown by the dotted line (E_{OD} reversed is denoted by E_{DO}). From the end of vector E_{DO} with E_{AB} as a radius draw an arc mn and with E_{AB} as a center and E_{DO} as a radius draw arc pq. The vector $E_{AB} \oplus E_{DO}$ represents in magnitude and direction the difference between E_{AB} and E_{OD}. In Fig 106, E_{OD} is subtracted from E_{AB} using the topographic method of representation.

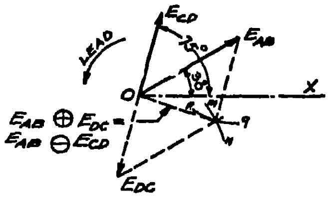


Fig. 105. — Subtraction of Vectors.

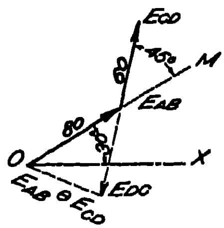


Fig. 106 — Subtraction of Vectors, Topographic Method.

E_{AB} is first drawn making an angle of 30° with OX. E_{OD} is drawn from the point of arrow E_{AB} making an angle of 45° with E_{AB} extended. E_{CD} which is the subtrahend is reversed and is shown by a dotted line extending to E_{DO}. OE_{DO} is the resultant.

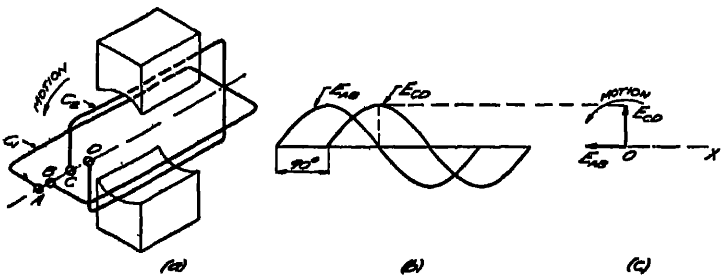


Fig. 107. — Two-Phase Machine with Waves and Vectors of E. M F.

Use of Vectors in a Two-Phase Circuit. Let C_1 and C_2 Fig. 107 (a) be the coils of a two-phase machine placed 90° apart on the armature. Let the voltage E_{AB} of coil C_1 be 100 and voltage

E_{CD} of coil C_2 be 100 also. Since the coils are 90° apart on the armature and coil C_1 is ahead of C_2 in the direction of rotation, the E. M. F. waves will be as at (b) and the vectors as at (c).

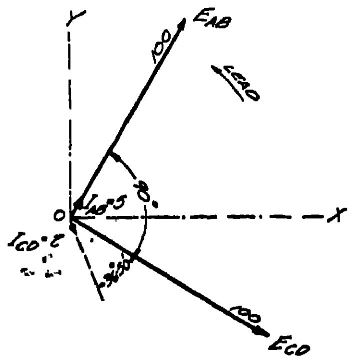


Fig. 108.—E. M. F.'s and Currents in a Two-Phase Circuit Phase AB Contains Only Resistance Phase CD Contains Resistance and Inductive Reactance.

Slip rings A and B may be connected to one circuit and rings C and D to another. Each armature winding may supply lamps or other load.

Assume that slip rings A and B are connected to a circuit containing a resistance of 20 ohms and that the slip rings C and D are connected to a circuit containing a resistance of 40 ohms and an inductive reactance of 30 ohms. The current in the circuit AB will be $\frac{100}{20} = 5$ amperes in phase with E_{AB} and the current in CD will be $\frac{100}{\sqrt{40^2 + 30^2}} = 2$ amperes. The 2 amperes will lag behind E_{CD} by an angle whose tangent is $\frac{30}{40}$, which is 36° 50′. The vectors for E. M. F. and current will be as shown by Fig 108.

Interconnection of Phases. It is common to interconnect two phase windings so that three wires may be taken off instead of four. In order to get the voltage relations clearly in mind, it is best to think of each winding as on a separate armature of a two-pole machine. Consider that the armatures are set 90° apart. Then if they are wound alike, the same E. M. F.'s will be generated in each. These E. M. F.'s will be in the same direction in each of the windings as they pass a given pole, but the two E. M. F.'s will be 90° apart. The E. M. F. of the windings which is ahead in the direction of rotation will "lead" the other E. M. F.

The two-phase 3-wire connection is illustrated by Fig 109. Load in this illustration is across the outside lines only.

Machine #1 generates a voltage E_{AB} acting from A to B, and Machine #2 generates a voltage E_{CD} acting from C to D. The voltage tending to send current through the external circuit or load from B′ to D′ is $E_{B'D'}$ which is the difference between the two voltages E_{AB} and E_{CD}. As these voltages are not in phase with each other, $E_{B'D'}$ is the vector or geometrical difference, and not the algebraic difference, because the directions as well as magnitudes of voltages must be taken into account.

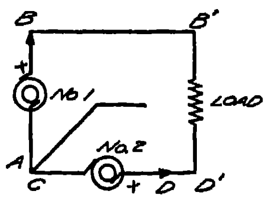


Fig 109 — Two-Phase 3-Wire Circuit with Load across Outside Line Wires

In order to keep vectors in their proper relation to each other, in circuits similar to Fig. 109 and other more complicated circuits, it is best to mark arrows on the circuit pointing outward or inward if the windings or loads are connected at a common point, and around the circuit clockwise or counter clockwise if the windings or loads are connected in a loop or mesh. These arrows denote what is known as "positive direction through the circuit." They simply indicate in which direction an E. M. F would have to act to send current through the circuit in the direction that the arrow points. These arrows must not be confused with instantaneous values of E. M. F. or current.

Having placed arrows on a circuit pointing, say, outward as in Fig. 109, an equation may be formed as follows: Start at a point such as B′ and read around the circuit. If you read in the direction the arrow is pointing, call the quantity plus. If you read against the arrow, call the quantity minus. Thus, in going from B′ to D′ if we put an arrow on the load pointing toward D′, we should read this $E_{B'D'}$. In going from D′ to the common point AC we go against the arrow, so we call this value minus or $-E_{CD}$. In going from A to B we go with the arrow or $+E_{AB}$.

Hence, in the form of an equation:

$$E_{B'D'} = -E_{CD} + E_{AB}$$
$$= E_{AB} - E_{CD}$$

The arrows AB and CD might have been put on pointing toward AC.

$$\text{Then } E_{B'D'} = E_{DC} - E_{BA}$$
$$= E_{AB} - E_{CD}$$

Reference to Fig. 110(*a*) to (*h*) will show that when windings which are similar have their corresponding ends connected together, the E. M. F. across the other two ends is the vector difference of the E. M. F.'s of the separate windings

(*a*) Shows two windings in the same slot, viz., 0° apart. Application of the three-finger rule shows that the E M. F.'s are up in both conductors which are under the south pole. Since these E. M F.'s are both alike for the position of the coils shown, the E. M. F. across B′ and D′ is zero. If we mark arrows on coils AB and CD to represent direction of induced E. M. F., then in going through the winding from D to B we will go against E. M. F. E_{CD} and with E M F. E_{AB}, that is, $E_{DB} = -E_{CD} + E_{AB}$ A voltage E_{DB} which tends to send current through the windings from D′ to B will tend to send current through an external circuit from B′ to D′ so $E_{DB} = E_{B'D'}$ and $E_{DB} = E_{B'D'} = E_{AB} - E_{CD}$. The vectors E_{AB} and E_{CD} are shown at left of (a). Since it is evident from the drawing that the E. M. F across B and D is zero, then the vector E_{CD} must be reversed to get a resultant of zero, or $-E_{CD}$ is drawn downward from 0.

At (*b*) coil CD is advanced 30° by putting in slots farther along the surface of the armature. E_{CD} then moves ahead as shown by vectors at left of (b) and $-E_{CD}$ is directly opposite to E_{CD} The resultant is now $E_{B'D'}$.

(*c*) shows the relations when coil CD is placed 60° ahead of AB. (*d*) shows the conditions when coil CD is placed 90° ahead of coil AB. This is the spacing in a two-phase machine. It is clear from arithmetic that $E_{B'D'} = 1.41\ E_{AB}$ or E_{CD}.

(*e*) shows the coils 120° apart, which is the spacing for a 3-phase machine. In this case $E_{B'D'}$ is 1.73 either E_{AB} or E_{CD}.

(*g*) shows that as coil CD is placed nearly 180° from coil AB, that the voltage is nearly equal to twice either E_{AB} or E_{CD}.

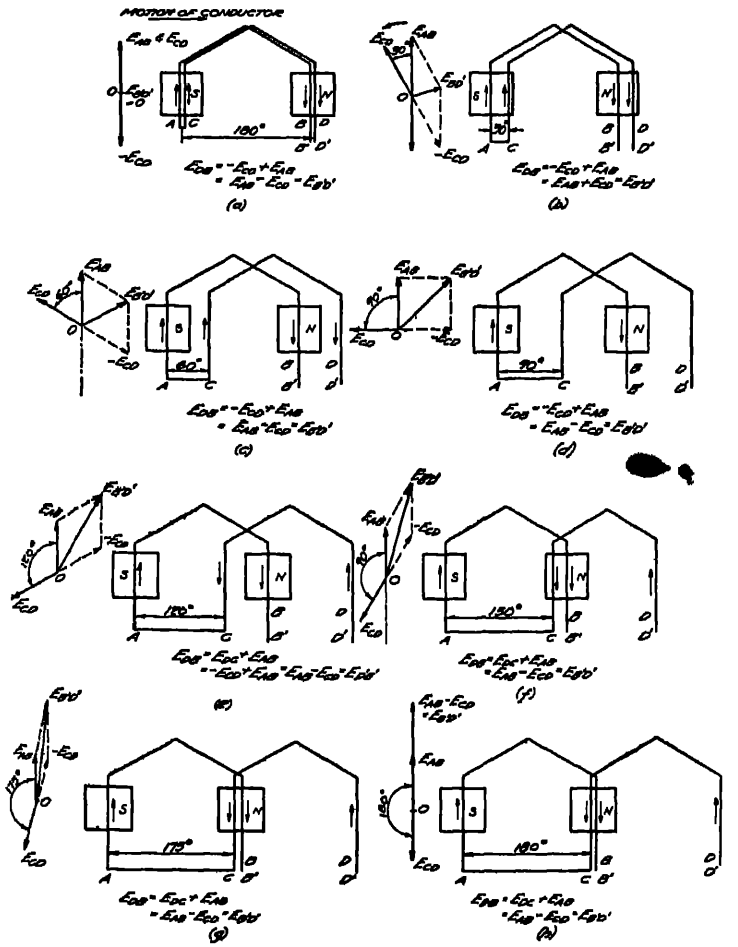


Fig 110. Vectors Showing Effect of Different Coil Spacings.

(*h*) shows that when coil E_{CD} is moved over still farther and coil side C comes in the same slot as coil side B that the coils are 180° apart and the voltage across the terminals is twice E_{AB} or E_{CD}, or is obtained by taking the vector difference of E_{AB}

and E_{CD} 180° apart This is the same as the arithmetical sum of E_{AB} and $-E_{CD}$, viz, $E_{B'D'}$, shown at left of (h).

Voltage Relations in a Simple Two-Phase 3-Wire Circuit. Let AB and CD, Fig. 111, be the windings of a two-phase generator, with the beginnings of the two windings connected at a common point. Their voltages act outward from A to B and C to D. Since power flows from generator to the load, we can mark the direction of the flow of power with arrows pointing to the load. Consider that the load is across B′D′ only. It is desired to find the voltage B′D′ and its direction.

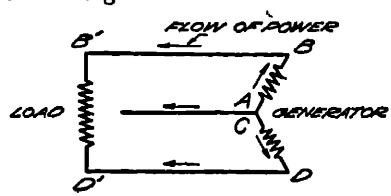


Fig. 111 — Two-Phase 3-Wire Circuit

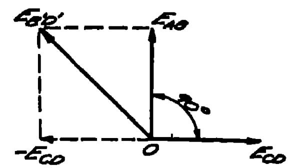


Fig. 112. — Vectors of E M F for 2-Phase 3-Wire Circuit of Fig. 111.

Refer to the diagram of connections Fig. 111 and go around the circuit counter clockwise starting at B′.

$$E_{B'D'} = -E_{DD'} - E_{CD} + E_{AB} + E_{BB'}$$

Since the line is supposed to be of negligible resistance in the illustration, $E_{DD'} = 0$ and $E_{BB'} = 0$

So $$E_{B'D'} = 0 - E_{CD} + E_{AB} + 0$$
$$= E_{AB} - E_{CD}$$

Draw E_{AB} and E_{CD} to scale Fig. 112, to represent the voltages of windings AB and CD. In this illustration, E_{AB} leads E_{CD} 90°. Next solve the vector equation:

$E_{B'D'} = E_{AB} - E_{CD}$ as follows:

Reverse E_{CD}, Fig. 112, to get $-E_{CD}$ and combine $-E_{CD}$ and E_{AB} getting $E_{B'D'}$ which is the value and direction of the voltage sending current from B′ to D′.

If the windings had been connected as in Fig. 113,

$$E_{B'C'} = E_{AB} - E_{DC},$$

or $E_{B'C'}$ would have direction and value shown by Fig. 114.

It will be seen that the actual voltage across the load is the same in each case but its phase relation has changed 90°.

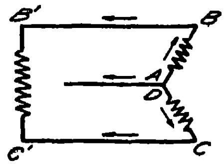


Fig 113 — Two-Phase 3-Wire Circuit with Beginning of Generator Winding No. 1. Connected to End of Winding No 2.

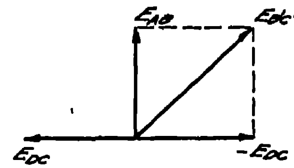


Fig 114. — Vectors for the Circuit of Fig 113

From arithmetic, if E_{AB} and E_{CD} are each 100 volts, then the voltage across the outside lines ($E_{B'D'}$ or $E_{B'C'}$) is 141 volts That is, in a balanced two-phase 3-wire circuit the voltage across the outside lines is 1.41 times the voltage of the generator windings.

Current Relations in a Two-Phase 3-Wire Circuit with Non-Inductive Load. In Fig. 115 the generator of Fig. 107 is connected three-wire by connecting slip rings B and C together. A non-inductive load of three branches A'B', B'D' D'A' is connected to the generator. The resistances of the loads are such that A'B' draws 2 amperes, B'D' 5 amperes and D'A' 10 amperes.

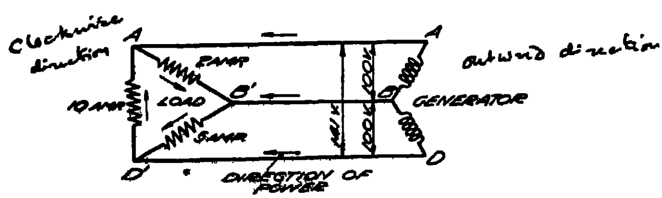


Fig. 115. — Two-Phase 3-Wire Circuit with Non-Inductive Loads.

Put arrows on the load pointing around the circuit clockwise. Mark the direction of the flow of power from generator to load. In going around the circuit, $E_{A'B'} = -E_{AB}$ or $E_{AB} = -E_{A'B'} = E_{B'A'}$ and $E_{B'D'} = -E_{BD}$, or $E_{BD} = -E_{B'D'} = E_{D'B'}$.

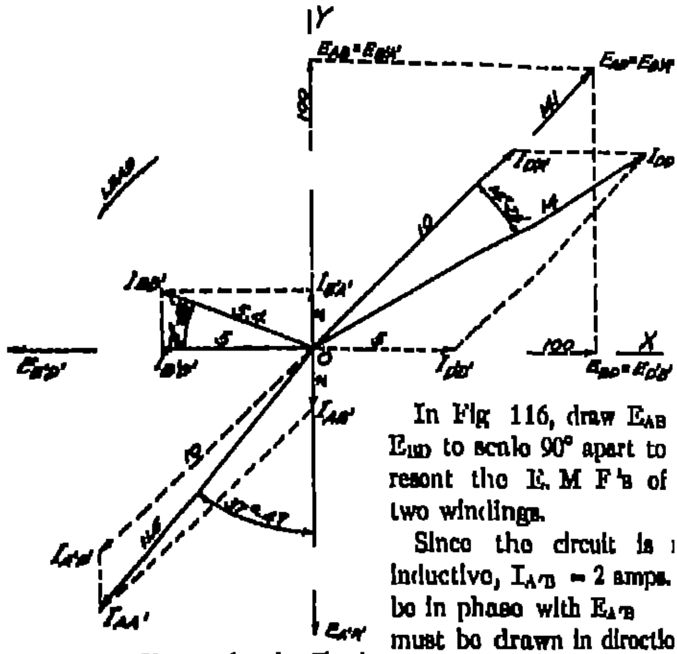


Fig 116. — Vectors for the Circuit of Fig. 115

In Fig 116, draw E_{AB} E_{BD} to scale 90° apart to resent the E. M F's of two windings.

Since the circuit is inductive, $I_{A'B'} = 2$ amps. be in phase with $E_{A'B'}$ must be drawn in directio $E_{D'A'}$ reversed because I_{A} $- I_{B'A'}$ Extend $E_{B'A'}$ the O and lay off $I_{A'B'} = 2$ Draw $I_{B'D'}$ on $E_{D'B'}$ reversed I $I_{D'A'} = 10$ on $E_{D'A'}$

By inspection of circuit Fig 115 the following facts appear

(1) $I_{AA'} = I_{AB} - I_{DA} = I_{A'B'} + I_{A'D'}$
(2) $I_{BB'} = I_{B'D'} - I_{A'B'} = I_{BD'} + I_{D'A'}$
(3) $I_{DD'} = I_{D'A'} - I_{BD'} = I_{D'A'} + I_{D'B'}$

In writing the above equations $I_{A'B'}$ is read plus because w through the circuit with the arrow that denotes positive direc $I_{D'A'}$ minus because we go through the circuit against the arrow

The next step will be to perform the vector additions indic by equations (1), (2) and (3)

In Fig 116, combine $I_{A'B'}$ and $I_{A'D'}$ getting $I_{AA'}$ whose v scales 11 5 amps. and which lags $E_{A'B'}$ by 37° 47′

Combine $I_{B'D'}$ and $I_{BA'}$ getting $I_{BB'}$ whose value scales 5.4 amps. and which lags $E_{BD'}$ by 21° 48′

Combine $I_{D'A'}$ and $I_{D'B'}$ getting $I_{DD'}$ whose value scales 14 amps. and which lags $E_{D'A'}$ by 14° 28′

Figures 117 and 118 show that the same results could have been obtained if the positive direction around the circuit had been assumed in a counter clockwise direction.

From Fig 117 the equations are:

$$I_{AA'} = I_{A'D'} - I_{BA'} = I_{A'D'} + I_{A'B'}$$
$$I_{BB'} = I_{B'A'} - I_{D'B'} = I_{BA'} + I_{B'D'}$$
$$I_{DD'} = I_{D'B'} - I_{A'D'} = I_{D'B'} + I_{D'A'}$$

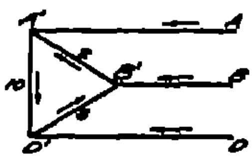


Fig 117 — Positive Direction Around Circuit Assumed Counter Clockwise.

The vector diagram becomes as in Fig 118.

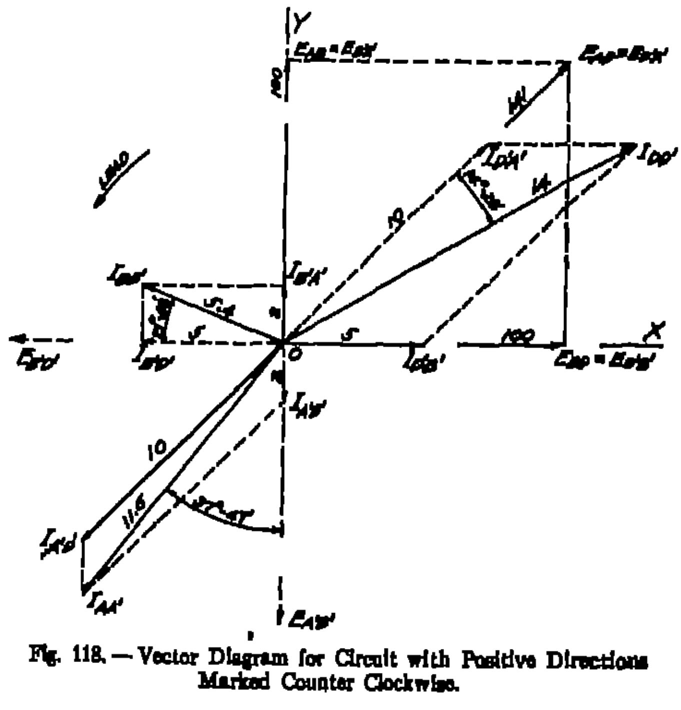


Fig. 118. — Vector Diagram for Circuit with Positive Directions Marked Counter Clockwise.

Current Relations in a Two-Phase Three-Wire Circuit wit Inductive Loads. Use the same machine as in Fig 107, but cor sider that branch A'B' Fig 119 has 3 ohms resistance and 4 ohm inductive reactance The current $I_{A'D'}$ will be $\frac{100}{\sqrt{3^2+4^2}} = 20$ amp lagging $E_{A\,B}$ by an angle whose tangent is $\frac{4}{3} = 1.33 = 53° \; 8$ Consider B'D' has 6 ohms resistance and 4 ohms inductive reacl ance. The current $I_{B'D'}$ will be $\frac{141}{\sqrt{6^2+4^2}} = 19\,6$ amperes lag ging $E_{B'D'}$ by an angle whose tangent is $\frac{4}{6} = 667 = 33°\text{–}42$ Consider D'A' has 5 ohms resistance and 5 ohms inductive react ance The current $I_{D'A}$ will be $\frac{100}{\sqrt{5^2+5^2}} = 14\,1$ amps., laggin $E_{D'A'}$ by an angle whose tangent is $\frac{5}{5} = 1 = 45°$

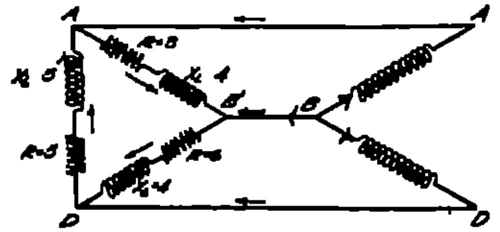


Fig. 119 — Two-Phase 3-Wire Circuit with Inductive Loads.

From the circuit diagram

$$
\begin{aligned}
&(1)\quad I_{AA'} = I_{A'B'} - I_{D'A'} = I_{A'B} + I_{A'D'}\\
&(2)\quad I_{BB'} = I_{B'D'} - I_{A'B'} = I_{B'D'} + I_{B'A'}\\
&(3)\quad I_{DD'} = I_{D'A} - I_{B'D'} = I_{D'A'} + I_{D'B'}
\end{aligned}
$$

The partial vector is shown by Fig 120, the complete vecto by Fig 121

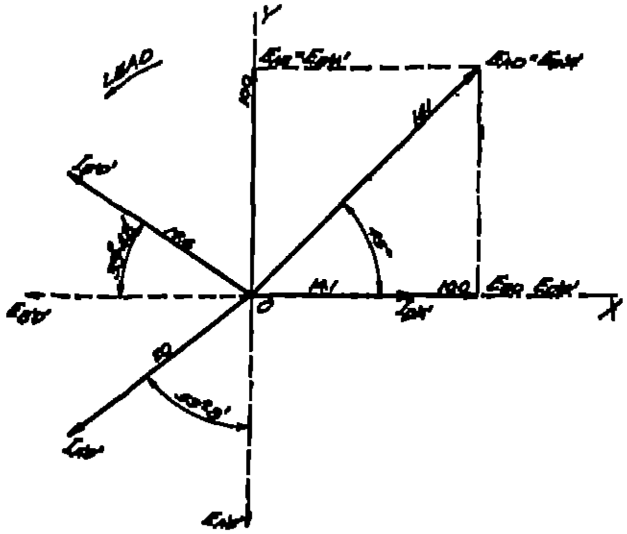


Fig. 120. — Partial Vector Diagram for a Two-Phase, Three-Wire Circuit with Inductive Loads.

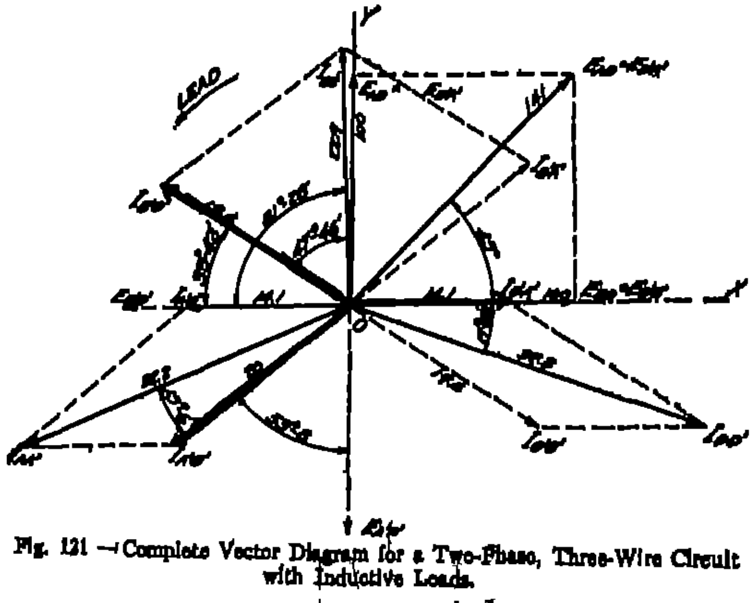


Fig. 121 — Complete Vector Diagram for a Two-Phase, Three-Wire Circuit with Inductive Loads.

Three-Phase Connections. There are three common methods of connecting three-phase circuits The "delta" connection, the "open delta" connection, and the "Y" connection. In the delta connection the three windings or loads are connected in the form of a triangle The name "delta" comes from the fact that the triangular-shaped figure thus formed resembles the Greek letter "delta" (Δ) When the windings on an alternator are 120 electrical degrees apart, the end of the first winding is connected to the beginning of the second, the end of the second to the beginning of the third, and the end of the third to the beginning of the first The line wires are taken off at the points of connection thus formed, that is, at the corners of the triangle.

The open delta connection is similar to the regular delta or closed delta connections, except that only two of the three windings or loads are used The third side of the triangle is left out, or left "open." Three line wires are used, one at the end of the first winding or load, one at the junction of the first and second windings or loads, and the third at the end of the second winding or load

In the "Y" (wye) connection, the three windings or loads are connected in the form of the letter Y When the windings of an alternator are 120 electrical degrees apart on the armature, the beginnings of the windings may be connected together and the ends connected to the line wires, or vice versa.

Sometimes a fourth wire is taken off from the common connection at the center of the Y and carried out as a line wire. Motors may be connected across the three regular "outside" line wires, and lamps from any outside wire to this fourth wire which goes to the center of the Y The wire which goes to the center of the Y is called the "neutral." When the loads are balanced no current flows in this common or neutral wire

On account of the similarity of the open delta connection to the two-phase 3-wire connection already discussed, the open delta connection will be considered next.

Vector Relations in an Open Delta Circuit. Figure 122 shows part of two of the windings of a three-phase alternator These

windings can be connected open delta by connecting B and C together as shown at (a) The topographic vectors will be as shown at (b)

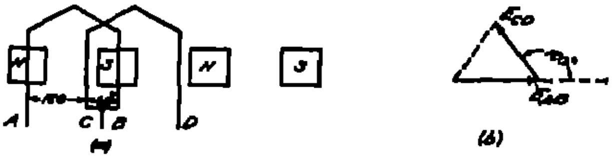


Fig 122. — Connections of Windings in Open Delta with Vectors of E. M F

It will be noted that with the open delta connection it will take a resultant of a length equal to either E_{AB} or E_{CD} to complete the polygon shown at (b) In other words, if a tap be taken off at CB, the E M F's across A–CB, CB–D and A–D will all be equal, and a three-phase circuit results, using but two windings.

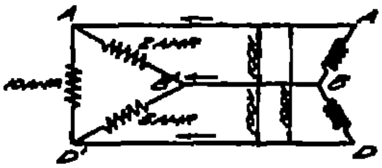


Fig. 123. — Three-Phase Open Delta-Connected Generator with Non Inductive Loads.

The diagram of connections will be as in Fig 123 It is similar to Fig 115 and 119 It should be remembered, however, that the two windings are 120 degrees apart instead of 90 degrees

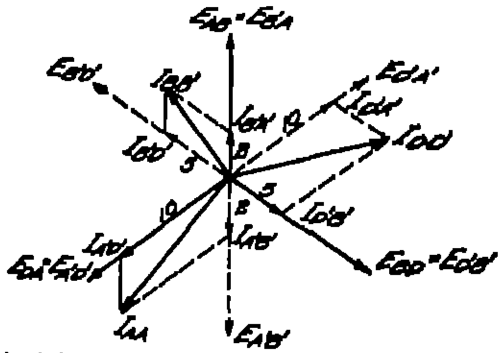


Fig. 124. — Vector Diagram for a 3-Phase Open Delta Circuit with Non Inductive Loads.

The loads for this illustration will be assumed to be the same those in Fig 115

Considering the generator first, it will be seen from the vecto of Fig 122 that the line voltages will all be equal, viz, 100 Dra the voltage vectors E_{AB}, E_{BD}, E_{DA}, etc., Fig 124 Since $I_{A'B'}$ in phase with $E_{A'B'}$ it will be drawn along $E_{A'B'} = 2$, $I_{B'D'}$ will l drawn along $E_{B'D'} = 5$, and $I_{D'A'}$ along $E_{D'A'} = 10$ Then fro the diagram of connections, Fig 123,

$$I_{AA'} = I_{A'B'} - I_{D'A'} = I_{A'B'} + I_{A'D'}$$
$$I_{BB'} = I_{B'D'} - I_{A'B'} = I_{B'D'} + I_{B'A'}$$
$$I_{DD'} = I_{D'A'} - I_{B'D'} = I_{D'A'} + I_{D'B'}$$

Combining vectors according to the method previously outlin for a two-phase 3-wire circuit we obtain the currents $I_{AA'}$, $I_{BB'}$ and $I_{DD'}$ in magnitude and direction

Vector Relation in a 3-Phase Δ-Connected Circuit. In t three-phase delta-connected circuit, the three windings or loa are connected so that the end of the first connects with the b ginning of the second, the end of the second connects with t beginning of the third and the end of the third connects with t beginning of the first The line wires are taken off at the poin of connection

The diagram of connections will be as in Fig 125

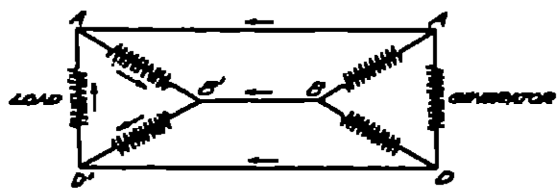


Fig. 125 —Diagram of Connections. 3-Phase Delta-Connected Circuit

To construct the vector diagram, draw the voltage vectors E_{AB}, E_{BD}, E_{DA} to scale 120 degrees apart. Figure 126

If the circuit is non inductive the phase currents $I_{A'B'}$, $I_{B'D'}$, $I_{D'A'}$ will be in phase with $E_{A'B'}$, $E_{B'D'}$, $E_{D'A'}$

From the diagram of connections

$$I_{AA'} = I_{A'B'} - I_{D'A'} = I_{A'B'} + I_{A'D'}$$
$$I_{BB} = I_{B'D'} - I_{A'B'} = I_{B'D'} + I_{B'A'}$$
$$I_{DD} = I_{D'A'} - I_{B'D'} = I_{D'A'} + I_{D'B}$$

Combine vectors $I_{A'B'}$ and $I_{A'D'}$ and obtain $I_{AA'}$

" " $I_{B'D'}$ and $I_{B'A}$ and obtain I_{BB}

" " $I_{D'A'}$ and $I_{D'B'}$ and obtain $I_{DD'}$

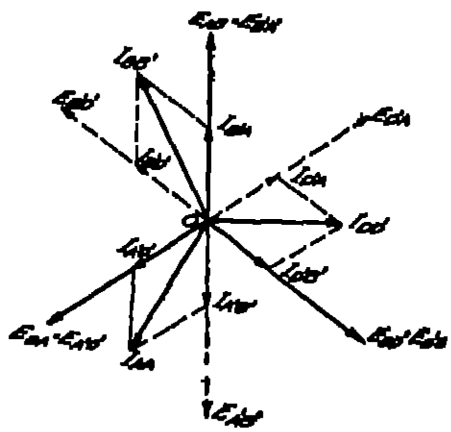


Fig. 126.—Vector Relations in a Balanced Three-Phase Delta Connected Circuit with Non Inductive Loads.

Vector Relations in a 3-Phase Y-Connected Circuit. In a three-phase Y-connected circuit, three windings spaced 120 electrical degrees apart have their three corresponding ends connected together The other three ends form the line terminals. Thus if AB, CD and EF are three windings spaced 120 degrees apart on the armature, A, C and E may be connected together and B, D and F used for the line. This connection may be thought of as 3 separate single-

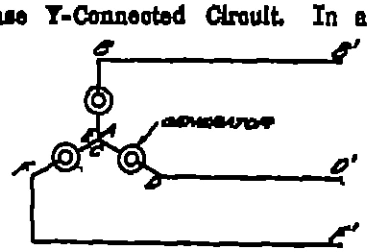


Fig 127—Illustration of a 3-Phase Y Connection.

phase machines with armatures spaced 120 degrees apart connected as in Fig 127

Each machine will generate the same E. M F The direction will be outward from 0 The instantaneous values will of course vary as the armatures turn, but relatively the E M F 's may be thought of as always 120 degrees apart. So in going from, say, D to B through the Y we go with one arrow and against another or the voltage from D to B is the vector difference of two equal E M F 's 120 degrees apart, that is, $E_{B'D'} = E_{AB} - E_{CD}$ Likewise $E_{D'F'} = E_{CD} - E_{EF}$ and $E_{F'B'} = E_{EF} - E_{AB}$

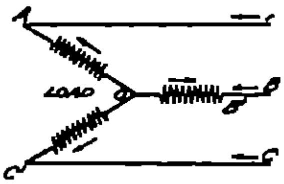


Fig. 128. — Diagram of Connections, 3-Phase Y-Connected Circuit.

Let the three loads be equal and connected at O Fig 128 Mark arrows on the Y pointing outward from O In reading through the circuit, $E_{AB} = E_{OA} - E_{OB'}$, $E_{BO} = E_{OB} - E_{OO'}$, $E_{OA} = E_{OO'} - E_{OA'}$ Draw the vectors $E_{OA'}$, $E_{OB'}$, $E_{OO'}$ to scale 120 degrees apart, Fig 129

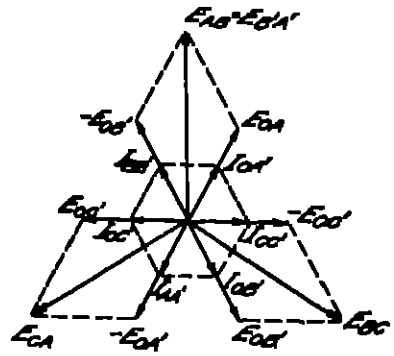


Fig. 129 — Vector Relations in a Balanced 3-Phase Y-Connected Circuit with Non-Inductive Loads.

Subtract $E_{OB'}$ from E_{OA} and obtain E_{AB}
" $E_{OO'}$ " $E_{OB'}$ " " E_{BO}
" $E_{OA'}$ " $E_{OO'}$ " " E_{OA}

If the circuit is non inductive $I_{OA'}$ is in phase with $E_{OA'}$, $I_{OB'}$ is in phase with $E_{OB'}$ and $I_{OC'}$ is in phase with $E_{OC'}$. Lay off these currents to scale along $E_{OA'}$, $E_{OB'}$ and $E_{OC'}$. It is clear from the diagram of connections Fig 128 that the line current is the same as the current in each branch of the Y. Also that one line acts as the return for the other two or,

$$I_{AA'} = I_{A'O} = I_{OB'} + I_{OC'}$$
$$I_{BB'} = I_{B'O} = I_{OA'} + I_{OC'}$$
$$I_{CC'} = I_{C'O} = I_{OA'} + I_{OB'}$$

PROBLEMS

1 Draw a vector of E M F of maximum value of 60 volts making an angle of 55° with horizontal reference line OX. Find graphically the instantaneous E. M. F for this phase angle.

2. Draw an E. M F vector E_{AB} of 25 volts at 45° with OX and a vector E_{CD} of 35 volts leading E_{AB} by $22\frac{1}{2}$° Find the vector sum of E_{AB} and E_{CD} and the phase angle with OX, by (*a*) crank method, (*b*) topographic method.

3. Draw two vectors $E_{AB} = 40$ and $E_{CD} = 60$. E_{AB} makes an angle of 60° with OX and E_{CD} lags E_{AB} by 40° Using crank method, subtract E_{CD} from E_{AB}. Find value and phase angle of resultant with OX.

4 Draw the vector diagram for a single-phase circuit containing a resistance of 16 ohms and an inductive reactance of 12 ohms. Find direction and value of E M F to send 4 amperes through the circuit. How much does current lag behind the E. M F?

5 Draw the vector diagram for a single-phase circuit containing a resistance of 8 ohms, an inductive reactance of 10 ohms and a capacity reactance of 4 ohms. Does the current lag or lead and by how much? What current will flow if 100 volts are impressed upon the circuit?

6. What will be the voltage of the coil group of the three-phase generator shown by the drawing, if each coil of the group gives 80 volts? Figure 130.

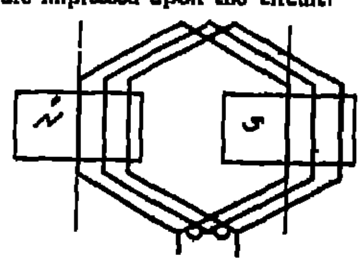


Fig. 130.—Coil Group for Generator

Make vector diagrams for circuits of Probs. 7, 8, 9, 10

7

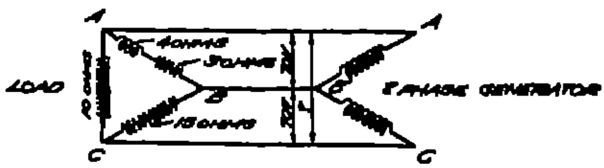


Fig. 131.

8.

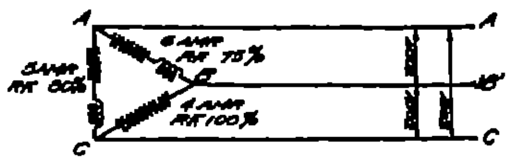


Fig 132

9

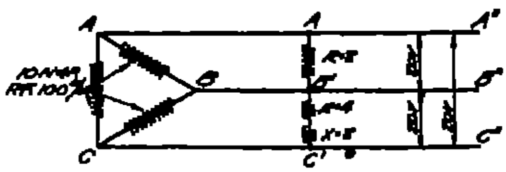


Fig. 133

10.

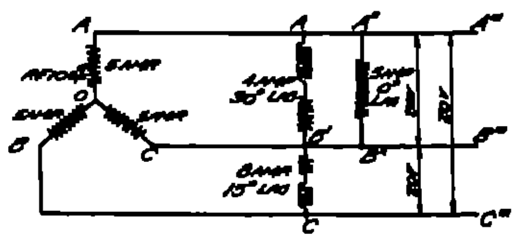


Fig. 134.

11. Show by vectors that the line current in a balanced 3-phase circuit is equal to 1 73 times the delta current and that it lags line E. M. F 30°

12 Show by vectors the effect on voltages of getting one phase reversed in connecting up a 3 phase Y-connected generator

13. Show by vectors that the power in a three-phase circuit is $P = \sqrt{3}\,EI$ (39)

CHAPTER VIII

TRANSFORMERS

Principle of the Transformer In Fig 135 let C_1 be a coil of insulated wire wound on an iron core and let C_2 be a similar coil, entirely separate from C_1 Connect a sensitive voltmeter to C_2 and connect a battery to C_1 through an adjustable rheostat, so that the current through C_1 may be varied by moving the handle of the rheostat

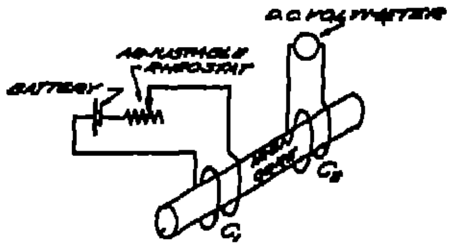


Fig 135 —Apparatus for Showing Principle of the Transformer

The following facts will be noticed upon varying the current through C_1, if the current be made to rise in C_1, the voltmeter will deflect in one direction, if the current be made to fall the voltmeter will deflect in the opposite direction If the current be held steady at any value the voltmeter will return to zero From the above, since there is no electrical connection between the coils, we conclude that the voltage in C_2 is generated by the lines of magnetic force set up by the current We conclude also that it is necessary to have the lines change in order to obtain a voltage in C_2. The two coils on the iron core in Fig 135 constitute a simple transformer A regular transformer is fed from an alternating-current generator instead of the arrangement of battery and rheostat.

The generation of an E M F in one coil by a varying current in an adjacent coil is by electro-magnetic induction, or as it is commonly called, by transformer action

The coil C_1 which is connected to the source of current is called the primary coil and the coil C_2 in which E. M. F is induced is called the secondary coil. The piece of apparatus itself is known as a static transformer

Standard Types of Transformers. Figures 136 to 139 illustrate the principal types of transformers in use at the present time In power and lightning work, transformers are made with what is known as closed cores The simple transformer shown by Fig 135 is an open-core type of transformer If the iron core were made in the form of a closed ring, the apparatus would become a closed-core transformer

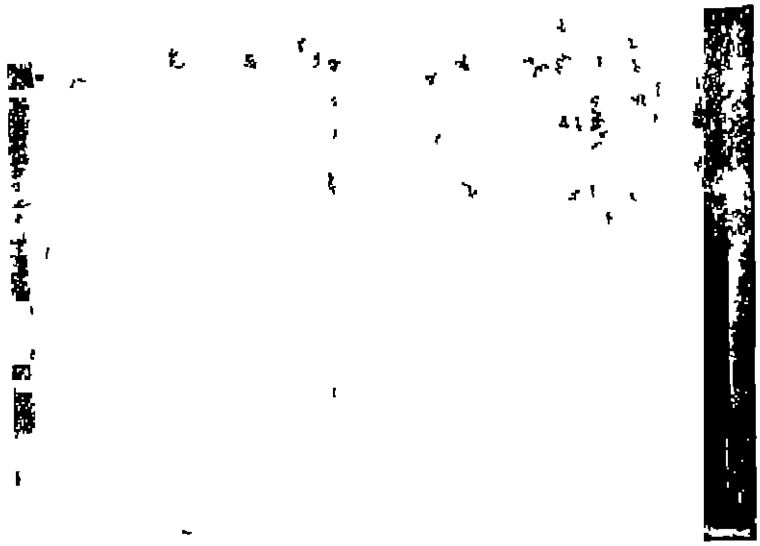


Fig. 136. — Core Type Transformer 8,333 Kv-a. 220,000 Volts. (General Electric Co.)

Figure 136 shows a single-phase power transformer rated at 220,000 volts on the high side and 11,000 volts on the low side

The particular arrangement of coils and core give it the name core type. Figure 137 shows another type of construction known as the shell type In the core type of transformer, the copper very largely surrounds the iron, and in the shell type, the iron

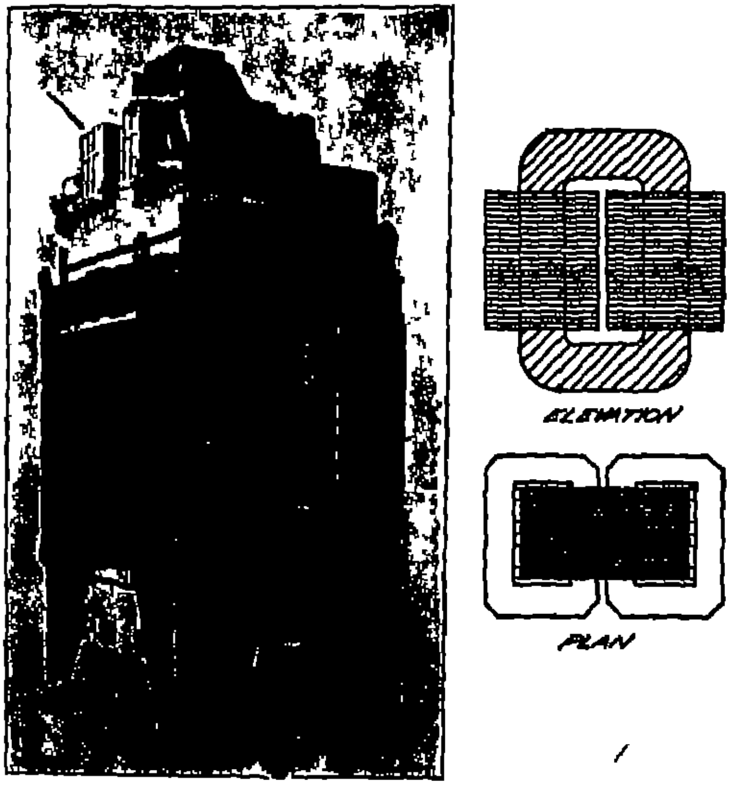


Fig. 137 — Shell Type Transformer, 25,000 Kv-a. 127,000 to 72,000 Volts. (Westinghouse Electric and Mfg. Co.)

largely surrounds the copper Figure 138 shows a shell type transformer used in lighting or distribution work. Due to the arrangement of the iron, this is called a distributed shell type. Figure 139 shows a three-phase transformer suitable for power work.

The core type of construction, in general, finds its best application in high-voltage work. The shell type of construction is

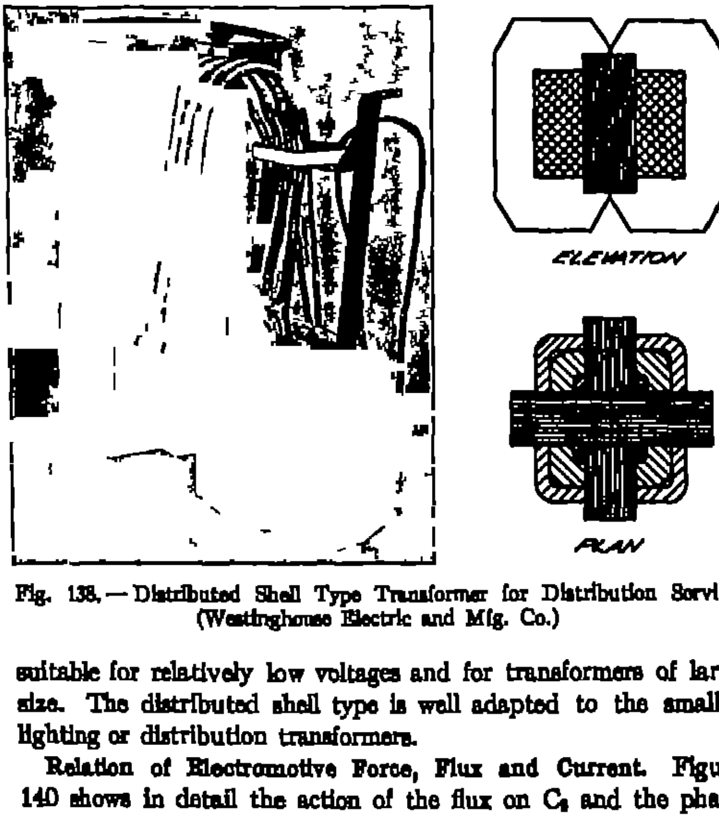


Fig. 139. — Distributed Shell Type Transformer for Distribution Service. (Westinghouse Electric and Mfg. Co.)

suitable for relatively low voltages and for transformers of large size. The distributed shell type is well adapted to the smaller lighting or distribution transformers.

Relation of Electromotive Force, Flux and Current. Figure 140 shows in detail the action of the flux on C_2 and the phase relations of current, flux, generated E. M. F. (counter E. M. F.) and impressed E. M. F. Sketch (*a*) is a section showing the core and one turn of coil C_1 and one turn of C_2. Let the current, at the instant considered, pass through the primary coil C_1 in a clockwise direction as viewed from the left end of the core, then the lines of force will encircle the conductor C_1 in a counter-clockwise direction. If the current be made to increase from a low value to maximum, the lines of force will expand outward

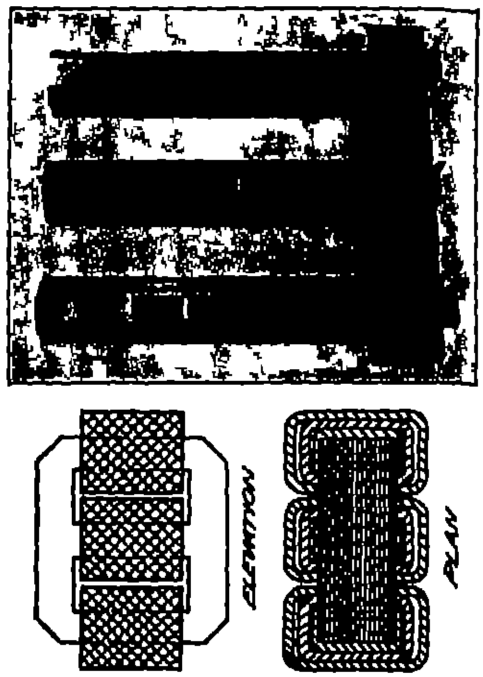


Fig. 139 — Three-Phase Core-Type Transformer.
(Westinghouse Electric and Mfg. Co.)

and cut conductor C_2 from left to right. This is the same as considering the lines of force stationary and moving C_2 to the left. Application of the three-finger rule shows that an E. M. F

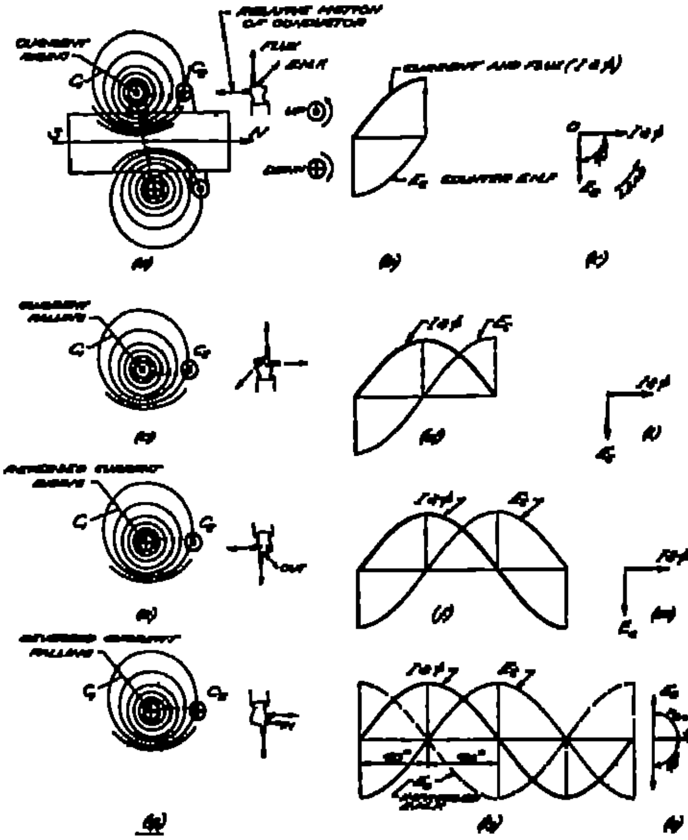


Fig. 140.—Relation of Electromotive Force, Flux and Current.

will be induced in C_2 which tends to send current through C_2 in the opposite direction from which it is flowing in C_1. Sketch (b) shows the wave of current and flux as it would rise in an alternating-current circuit and the induced E. M. F., E_c.

Sketch (*c*) shows the flux closing in or cutting C_2 from right to left. Application of the three-finger rule to (*c*) shows that the counter E. M. F. in C_2 is in the opposite direction from what it was acting at (*a*). (*d*) shows that as the current falls to zero the counter E. M. F in C_2 rises to a positive maximum. Sketches (*e*), (*f*), (*g*) and (*h*) of Fig. 140 show the relations of flux, current and counter E. M F. as the current reverses, passes to a negative maximum, and finally returns to zero

In order to establish the current in C_1, it is necessary to impress voltage on the coil This voltage, in the ideal transformer considered, will be directly opposite to the counter E M. F., E_c or will be the curve E_0 in (*h*).

A study of Fig 140 shows that the lines of force set up by the current in C_1 will cut C_1 as well as C_2 so that a counter E. M. F. will be induced in C_1 as well as C_2 The E M F induced in C_1 is called the counter E. M. F. of self induction, and the E. M. F. induced in C_2, the counter E. M. F of mutual induction.

From Fig. 140 the following appear:

(1) As current changes in a coil, lines of force cut the conductors in an adjacent coil and induce a counter E. M. F 90° behind the current and flux

(2) In order to set up a current and flux an E. M F. must be impressed on one of the coils. In an ideal magnetic and electric circuit without losses, this impressed E M F. will be 180° from the induced E. M. F.

Ratio of Transformation. Assume that coil C_1 in Fig. 135 has but one turn of wire and that C_2 has but one turn of wire also, then if no lines of force are lost by leakage, all the lines set up by the current in C_1 will cut C_2. If one volt is induced in C_2, then one volt will have to be impressed upon C_1 to set up current and flux necessary to generate the volt in C_2. In other words the volts per turn in the primary and secondary are equal. It follows, then, that if the primary has 10 times as many turns as the secondary that the primary impressed voltage will be 10 times the secondary voltage. The transformer is then called a step-down transformer. If the secondary has 10 times as many turns as

the primary, the transformer will step up the voltage in the ratio of 1 to 10

The above ratio holds strictly only when there is no leakage and where there is no load on the transformer It will be shown later, by means of a diagram known as the Transformer Diagram, that the ratios are slightly different when a transformer is loaded, or if the leakage of flux lines is large

Ratio of Currents If we neglect the losses, the output of a transformer will equal the input That is,

$$I_p E_p = I_s E_s$$

or

$$\frac{E_p}{E_s} = \frac{I_s}{I_p} \tag{40}$$

From which we see that the currents are in the inverse ratio of the voltages. That is, if the primary has a high voltage it will have a small current, the secondary will have a low voltage and large current.

Operation Under Load. Let Fig 141 be an ideal transformer

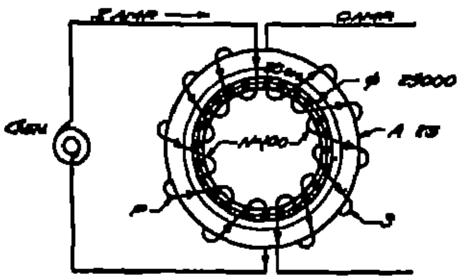


Fig. 141 — Core and Windings of an Ideal 1:1 Transformer with No Load.

or one in which there are supposed to be no losses and in whicl there is no leakage of magnetic lines of force. If voltage be ap plied to the primary, current will flow in accordance with the lav of the magnetic circuit

$$\phi = \frac{4\pi NI\mu a}{10l} \quad \text{or} \quad I = \frac{10l\phi}{4\pi N\mu a}$$

To get the illustration in concrete form, let the area of the core be, $a = 25$ sq cm., the number of turns, $N = 100$, the permeability, $\mu = 1000$, the length, $l = 25$ cm. and the flux, $\phi = 25{,}000$ lines of force.

$$\text{Then} \quad I = \frac{10 \times 25 \times 25{,}000}{4 \times 3\,1416 \times 100 \times 1000 \times 25} = \frac{25}{125} = 2 \text{ amperes, or}$$

1 ampere generates 12,500 lines The flux of 25,000 lines, which is shown by the shaded part of the drawing, threads the secondary S and generates a voltage in that coil.

Suppose that the secondary has the same number of turns as the primary, viz., a ratio of 1 to 1 and that it is closed through a resistance R as in Fig 142 and that a load of 50 amperes is taken

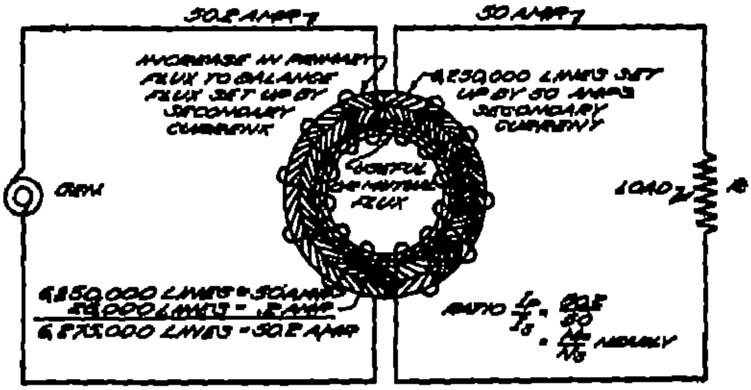


Fig. 142 —Ideal 1 1 Transformer with Loaded Secondary

from the secondary Each 1 ampere of secondary current will generate 12,500 lines, the same as in the primary or there will be $\frac{12{,}500 \times 50}{1} = 6{,}250{,}000$ lines opposed to the primary The result is that the primary immediately draws more current from the line, enough to balance these 6,250,000 lines of counter flux and to keep up the original flux of 25,000 necessary to magnetise the iron. The primary will then draw $2 + 50 = 50\,2$ amperes and set up total flux $6{,}250{,}000 + 25{,}000 = 6{,}275{,}000$ lines. It must not

be understood from the graphic representation of the counter flu and increase of primary flux, that the iron is worked to a densit equal to the total flux of 6,275,000 lines divided by the area (25 sq cm It is worked to a density corresponding to the origin flux of 25,000 lines shown by the heavily shaded part, divided b the area 25 sq cm. or 1000 lines per sq cm The counter flu and increased primary flux may be considered as two equal oppos ing forces resulting in no actual flow of counter-flux lines of forc around the circuit

From the above it is seen, that as the secondary is loaded th primary draws extra current to supply flux to balance the flux se up by the secondary current. The current necessary to suppl the useful flux is called the exciting current and is very smal in a well-designed transformer It is in the neighborhood o 5% of the full-load current, and may ordinarily be neglected i figuring the current ratio between primary and secondary S we may say for ordinary loads, that as the secondary curren increases, the primary current increases in practically the sam ratio

Mutual and Leakage Flux. In an actual transformer we d

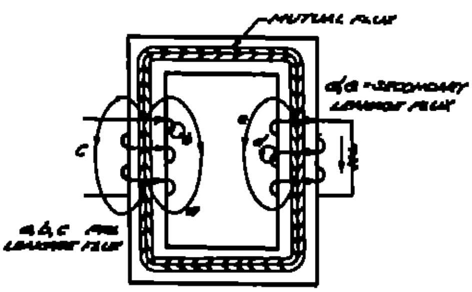


Fig. 143 —Mutual and Leakage Flux.

not get the ideal condition of Fig 142 because some of the lines of force, generated by the primary, short circuit and never reach the secondary, as for instance lines a, b, c, shown in Fig 143 Likewise some of the lines generated by the secondary, when it is carrying load, never reach the primary, as for instance lines d and

e. The flux set up by the primary that does not thread the secondary is called the primary leakage flux. The flux set up by the secondary that does not thread the primary is called the secondary leakage flux.

Effects of Leakage Flux. The effect of the secondary leakage flux is to cut down the secondary voltage just the same as if a reactance coil were connected in the secondary circuit. The drop in secondary voltage is $E_X = X_S I_S = 2\pi f L_S$ where L_S is the number of secondary leakage lines. The effect of the primary leakage lines is to use up some of the voltage impressed on the primary, the same as a reactance coil connected in the primary circuit. The amount of voltage drop caused by the primary reactance lines is $E_X = I_P X_P = 2\pi f L_P$ where L_P is the number of leakage lines.

Effect of Resistance of Windings on Voltages. In addition to a loss in voltage in primary and secondary, caused by leakage lines, there is a drop in the primary $E_R = I_P R_P$ due to the resistance of the primary winding, and a drop in the secondary $E_R = I_S R_S$ due to the resistance of the secondary winding. Both primary and secondary resistance drops make it necessary to add extra voltage to the primary to get the secondary voltage calculated from the ratio of turns This extra voltage to be added is not very great in a well-designed transformer at ordinary loads However the ratio $\frac{I_P}{I_S} = \frac{E_s}{E_p}$ does not hold strictly true for a loaded transformer. The method of calculating the amount by which the primary voltage must be increased over the amount calculated by ratio of turns, is shown by the study of the Transformer Diagram.

Transformer Diagram. The relations between currents and voltages in a transformer can best be shown by means of vectors. A vector diagram showing these relations is called a transformer diagram.

Referring to Fig. 140, it was seen that the secondary induced E. M F. was 90° behind the flux and current and the impressed E. M. F. was 90° ahead of the flux and current. For a transformer with

no losses these relations may be shown as in Fig. 144, which is a repetition of Fig. 140(n) with slightly different lettering.

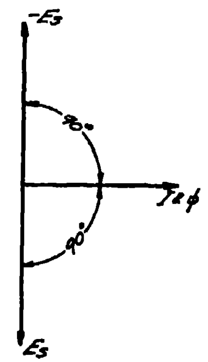


Fig 144 — Relation of Flux, Current and E M F 's in an Ideal Transformer

In order to take into account the losses in the core which are eddy current losses and hysteresis loss, redraw Fig. 144 and add a vector I_{h+e} in phase with $-E_S$, Fig. 145. I_{h+e} is the actual value of current that would be obtained if a wattmeter were connected in the primary circuit (with secondary open) and the power thus read divided by voltage. I_m is the current that would be required to magnetize an ideal core or one without losses. It is the current that would satisfy the equation of the magnetic circuit $I_m = \frac{10 l \phi}{4 \pi n \mu A}$, viz., the strictly magnetizing current. This would flow back and forth as the voltage rose and fell, but would not require energy. A mechanical analogy would

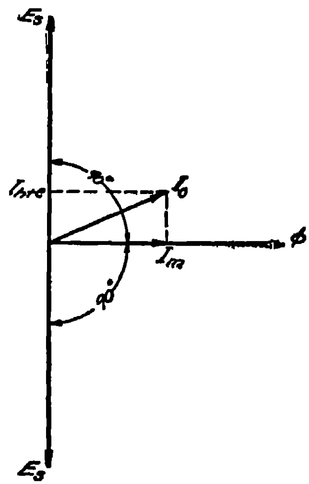


Fig 145 — Elementary Transformer Diagram Losses in Core Considered.

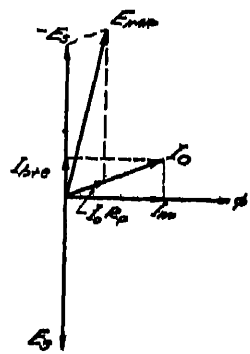


Fig. 146 — Transformer Diagram, for Unloaded Transformer.

be a spring without any friction. If I_{h+e} and I_m are combined vectorially, the actual exciting current will be I_o.

Figure 145 may further represent the complete diagram for a transformer with secondary open, or unloaded, if we draw I_oR_p along I_o to represent the resistance drop in the primary winding, and combine this E. M. F. with $-E_s$ obtaining E_{imp} the impressed voltage. This has been done in Fig. 146. It should be noted that the effect of the losses in the core is to make the impressed voltage E_{imp} and magnetizing current I_m slightly less than 90° out of phase.

The complete diagram for a loaded transformer is shown by Fig. 147. Starting with the elementary diagram of Fig 145, let I_s represent the secondary current. It should be drawn downward as it is usually somewhere near the vector E_s. Its exact position depends on the nature of the load on the transformer. I_s must have a component in the primary equal and opposite to it, which keeps I_s flowing. This component is $-I_s$. The actual primary current is the vector sum of $-I_s$ and I_o or I_p.

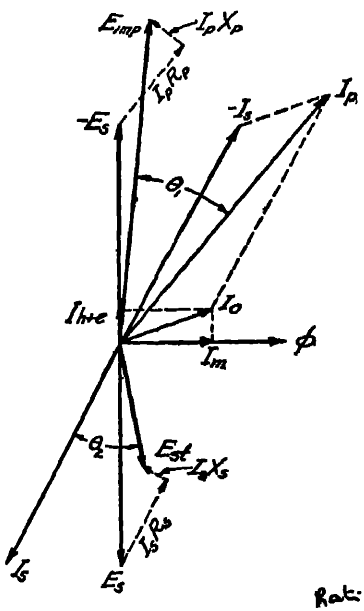


Fig. 147 — Complete Vector Diagram for a Loaded Transformer.

The secondary terminal voltage will be less than E_s on account of the drop due to the resistance of the secondary and the drop due to the reactance of the secondary. Since resistance drop is in phase with ~~voltage~~, current draw I_sR_s parallel to I_s and since reactance drop is at 90° with current, draw I_sX_s at 90° with I_s. E_{st} represents the actual secondary terminal voltage and cos θ_2 is the power factor of load.

Due to the primary resistance and reactance drops, $-E_s$ must

be increased in order to have sufficient voltage to keep up I Draw I_pR_p parallel with I_p and I_pX_p at right angles with I_p E_1 is the primary impressed voltage and the cosine of θ_1 is the pow factor of the loaded transformer, measured on primary side. should be noted that the power factors on the primary and se ondary sides are not necessarily alike

Losses in a Transformer The losses in a transformer are t iron losses in the core and the copper losses in the primary a secondary windings. The iron losses consist of the hystere loss and eddy-current loss The copper losses consist of the p mary copper loss which is equal to the primary current squar times the resistance of the primary winding, and the seconda copper loss which is equal to the secondary current squar times the resistance of the secondary winding

There is also a small eddy-current loss in the windings but tl is so small in an ordinary transformer that it may be neglect

Iron Losses. The molecules of a magnetic substance may thought of as little compass needles that try to line up in o direction when the magnetizing current flows around the coil o way, and try to line up in the opposite direction when the curr is reversed

Hysteresis may be thought of as "molecular friction", that a force tending to prevent the molecules being pulled around the magnetizing force, just as the mechanical friction of the pi of a compass needle and the friction of the air tend to prev it turning as a magnetic field near it is changed from one directi to the other

With alternating current, the magnetism is reversed v rapidly and the loss in "molecular friction" shows itself in t form of heat. The loss is called hysteresis loss The more stron the material is magnetized, viz, the greater the flux density, t farther the molecules have to be pulled around, hence the grea the loss. Likewise, the more rapidly they have to be pul around, viz., the higher the frequency of the magnetizing curr the greater the loss. The hysteresis loss depends also on t material, a hard steel will have a greater hysteresis loss th

soft iron It depends also on the volume of the iron, a large piece of iron will have a greater hysteresis loss than a small piece, provided it is magnetized to the same density

The following formula which takes into account the quality of the material, the volume, the frequency and the magnetic density has been developed from experiments.

$$P_h = \frac{KVfB^{1.6}_{max}}{10^7} \qquad (41)$$

Where P_h = watts lost
V = volume of iron in cu cm
f = frequency
B_{max} = maximum flux density in lines per sq cm,
K = a constant for the material varying from 001 to 006

The solution of this equation involves logarithms and considerable work, so the hysteresis loss P_h has been plotted in the form of curves for the frequencies commonly used, at the right hand side of the iron-loss curve sheet of Fig 149 To find the hysteresis loss, find density along the horizontal line and follow the vertical line that passes through the given density up to the curve that is marked with the proper frequency Follow the horizontal line from the point where the vertical line cuts the curve to the vertical center line of the chart, where the hysteresis loss for 100 cu cm of iron will be found

Thus the hysteresis loss in 100 cu. cm of iron worked to a density of 10,000 lines per sq cm at 60 cycles, is 3 watts.

Eddy Current Loss. The Eddy Current loss in the core is an I^2R loss caused by circulating currents induced by the magnetic lines of force that cut the core. The fact that voltages are induced in a core placed within a coil carrying alternating current can be determined by testing with a telephone receiver Touch the terminals to different parts of the core and a distinct click will be heard in the receiver, showing that there is a difference of potential between the points touched. This E M F causes currents

to flow in the core and heat it. A practical way of cutting down the eddy-current loss is to make the core of sheets and insulate the sheets by japan. This reduces the length of each effective conducting part of the core at right angles to the flux and therefore reduces the voltage on the section. This may be seen

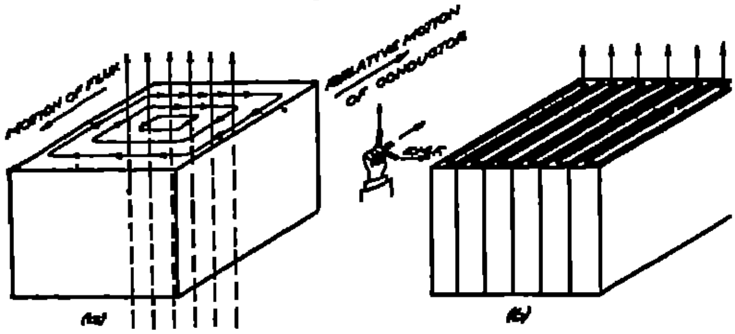


Fig. 148. — Core Cut into Sections to Lessen Eddy-Current Loss.

by reference to (a), Fig. 148, which represents a solid core in the path of flux. By cutting the core into, say, 6 slices as shown at (b) the number of lines cutting each slice is $\frac{1}{6}$ that with the solid core, so the current is cut down.

The following formula which has been developed from experiment takes into account the kind of material, the thickness of the plates, the frequency, and the density.

$$P_e = \frac{KVf^2 T^2 B^2_{max}}{10^{11}} \qquad (42$$

P_e = watts lost

K = a constant for the iron varying from 1.6 to 1.65

f = frequency

T = the thickness of sheets in centimeters

B_{max} = the maximum flux density in lines per sq. cm.

In order to lessen the work in computing eddy-current loss the curves on the left-hand side of the iron loss curve sheet, Fig. 149, have been plotted. These curves are plotted for $K = 1.6$ and a volume of 100 cu. cm. and different frequencies.

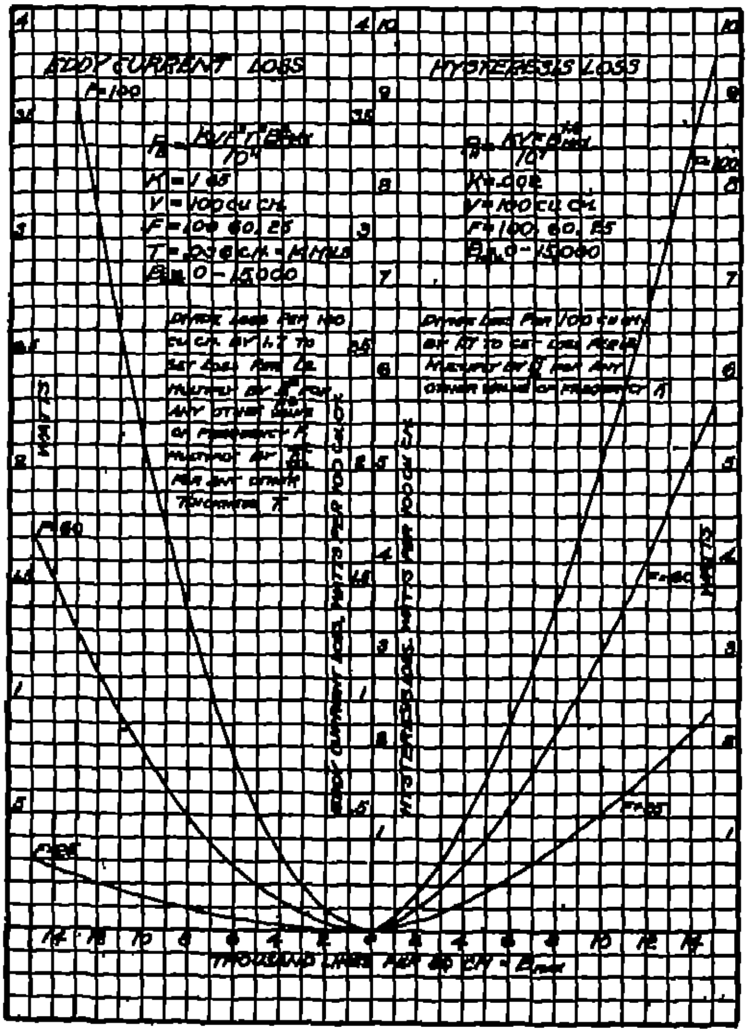


Fig 149 — Curves for Finding Iron Losses.

To find the eddy-current loss, find the density along the horizontal line and read up to the curve of proper frequency Next

read over to the vertical center line of the chart, where the wat lost per 100 cu cm. will be found

For example, the eddy-current loss at 60 cycles at a density 10,000 lines per square centimeter is 77 watts per 100 cu cm.

Table A Data on Distribution Transformers
(General Electric Company)

Type H, 60-cycle, single-phase, self-cooled, 460 volts to 115/230 volts

Kv-a. continuous 55° C. Rise	Core Loss Watts	Copper Loss Watts	Per Cent Efficiency				Per Cent Regulat	
			Full Load	3/4 Load	1/2 Load	1/4 Load	1 0 P F	8 P F
1 5	20	46	95 8	96 1	95 9	94 2	3 11	4 1
3	28	68	96 9	97 1	97 0	95 9	2 35	4 1
5	36	108	97 2	97 4	97 5	96 6	2 21	3 6
7 5	48	148	97 4	97 7	97 7	97 0	2 00	2 8
10	57	190	97 5	97 8	97 9	97 3	1 93	2 8
15	77	263	97 8	98 0	98 0	97 5	1 78	2 9
25	115	390	98 0	98 2	98 2	97 8	1 60	2 9
37 5	148	515	98 2	98 4	98 5	98 0	1 43	3 0
50	185	625	98 4	98 5	98 6	98 2	1 30	2 9
75	280	975	98 3	98 5	98 5	98 1	1 32	3 0
100	370	1200	98 4	98 6	98 6	98 2	1 25	2 9

Type H, 60-cycle, single-phase, self-cooled, 2300 volts to 115/230 volt

Kv-a continuous 55° C. Rise	Core Loss Watts	Copper Loss Watts	Per Cent Efficiency				Per Cent Regulation				
			Full Load	3/4 Load	1/2 Load	1/4 Load	1 0 P.F	9 P.F	8 P F	7 P F	P
1 5	20	46	95 8	96 1	95 9	94 2	3 11	3 95	4 10	4 12	4
3	28	68	96 9	97 0	97 1	95 9	2 35	3 75	4 10	4 38	4
5	36	108	97 2	97 4	97 5	96 6	2 21	2 31	3 60	3 75	3
7 5	48	148	97 4	97 7	97 7	97 0	2 00	2 50	2 58	2 60	2
10	57	190	97 5	97 8	97 9	97 3	1 91	2 50	2 60	2 65	2
15	77	263	97 8	98 0	98 0	97 5	1 80	2 80	3 05	3 20	3
25	115	390	98 0	98 2	98 2	97 8	1 60	2 65	2 90	3 10	3
37 5	148	515	98 2	98 4	98 5	98 0	1 43	2 65	3 00	3 25	3
50	185	625	98 4	98 5	98 6	98 2	1 30	2 60	2 95	3 20	3
75	280	975	98 3	98 5	98 5	98 1	1 32	2 65	3 05	3 3	3
100	370	1200	98 4	98 6	98 6	98 2	1 25	2 50	2 90	3 20	3
150	550	1875	98 4	98 5	98 6	98 2	1 31	2 65	3 01	3 29	3
200	800	2250	98 5	98 6	98 6	98 1	1 20	2 80	3 30	3 70	3

Table A (continued). Data on Distribution Transformers

(General Electric Company)

Type H, 25-cycle, single-phase, self-cooled, 460 volts to 115/230 volts

Kv-a continuous 55° C Rise	Core Loss Watts	Copper Loss Watts	Per Cent Efficiency				Per Cent Regulation	
			Full Load	3/4 Load	1/2 Load	1/4 Load	1 0 P F	8 P.F
1 5	22	65	94 5	95 0	95 1	93 4	4 4	5 5
3	33	92	96 0	96 4	96 4	95 1	3 2	5 2
5	43	150	96 3	96 7	98 8	95 9	3 1	4 5
7 5	62	205	96 5	96 9	97 0	96 1	2 8	4 4
10	70	250	96 9	97 2	97 4	96 7	2 7	4 2
15	102	320	97 2	97 5	97 6	96 8	2 2	3 9
25	147	505	97 4	97 7	97 7	97 0	2 1	3 1
37 5	235	670	97 4	97 7	97 7	97 0	1 8	3 1
50	320	1000	97 4	97 7	97 7	97 0	2 1	3 2
75	375	1665	97 4	97 7	97 9	97 5	2 3	3 4
100	405	2025	97 6	97.9	98 2	97 9	2 1	3 7

Type H, 25-cycle, single-phase, self-cooled, 2300 volts to 115/230 volts

Kv-a continuous 55° C Rise	Core Loss Watts	Copper Loss Watts	Per Cent Efficiency				Per Cent Regulation	
			Full Load	3/4 Load	1/2 Load	1/4 Load	1 0 P F	8 P F
1 5	27	70	93 9	94 4	94 3	92 2	4 8	5 9
3	38	112	95 2	95 7	95 7	94 3	3 8	4 6
5	50	172	95 7	96 2	96 3	95 3	3 6	4 6
7.5	65	225	96 3	96.7	96 8	95 9	3 1	4 0
10	77	305	96 3	96 8	97 0	96 3	3 1	4 3
15	110	410	96 6	97 0	97 2	96 4	2 8	4 2
25	150	545	97.2	97 5	97 6	96 8	2.3	3 5
37.5	265	700	97 2	97 5	97 6	96 8	2 0	3 0
50	335	1100	97.2	97 5	97 6	96 8	2 3	3.4
75	430	1700	97.2	97 6	97 7	97 2	2 4	3 9
100	535	2150	97 3	97.7	97 8	97 4	2 2	4 0
150	645	3250	97 4	97 8	98 0	97 7	2 2	4 1
200	770	3900	97.7	98 0	98 2	97 9	2 1	4 0

NOTE. Core loss, efficiency and regulation are based on rated volts and frequency using a sine wave.

Copper loss is based on copper loss by wattmeter method at or corrected to 75° C.

Table B. Average Values of Flux Density (B_m) (Still) *

	f = 25		f = 60	
Small lighting or distributing Transformers alloyed iron	sq in. 70000 to 85000	sq cm. 11000 to 13200	sq in 55000 to 70000	sq cm. 8500 to 11000
Power transformers alloyed iron	75000 to 90000	11600 to 14000	70000 to 90000	11000 t 14000

Table C Average Values of Current Density in Transformers. (Sti

	Amps per sq in	Cir mils per am
Standard lighting transformer (oil-immersed or self-cooled)	800 to 1100	1600–980
Transformers for use in central generating stations or substations (oil-cooled or air blast)	1100 to 1600	1160–800
Large, carefully designed transformers, oil-insulated with forced circulation of oil or with water cooling coils	1400 to 2000	900–650

* Table prepared from data in "Elements of Electrical Design" by Alfr Still

Calculation of the Number of Turns for a Transformer Tl formula for calculating the number of turns on a transformer

$$N = \frac{10^8 \times E_{eff}}{4.44 \times \phi \times f} \qquad (4.$$

N = the number of turns
E_{eff} = the effective volts
ϕ = the total flux
$= A \times B$
where A = actual area of core
and B = the flux density
f = the frequency

To calculate the number of turns on the primary, let E_{eff} equal the primary voltage. To calculate the number of turns on the secondary let E_{eff} equal the secondary voltage.

The formula is derived from the formula for a generator as follows:

$$E_{av} = \frac{2pVS\phi}{60 \times 10^8} \tag{44}$$

Where p = number of pairs of poles
V = revolutions per minute
S = the number of inductors on the armature
= 2N, if N is the number of coils
ϕ = the flux per pole

Now $$\frac{pV}{60} = f \qquad \text{from (1)}$$

and $$S = 2N$$

So $$E_{av} = \frac{2f \times 2N \times \phi}{10^8} = \frac{4fN\phi}{10^8}$$

For a sine curve $$\frac{E_{eff}}{E_{av}} = 1.11 \qquad \text{from (4) and (5)}$$

or $$E_{eff} = E_{av} \times 1.11$$

Multiply $$E_{av} = \frac{4fN\phi}{10^8} \text{ by } 1.11$$

Then $$E_{av} \times 1.11 = \frac{4fN\phi \times 1.11}{10^8}$$

or $$E_{eff} = \frac{4.44fN\phi}{10^8}$$

and $$N = \frac{10^8 \times E_{eff}}{4.44 \times \phi \times f} \tag{45}$$

An example will illustrate the application of the formula. Let Fig. 150 be the core of a transformer which is to have a primary winding such that it may be used on a 110-volt 60-cycle circuit. The core is made up of thin sheets of iron which are insulated from each other by a coating with japan. The insulation is to prevent eddy currents from circulating in the core. The gross area of the core is 5 × 5 cm. which equals 25 sq. cm., but on ac-

count of the japan on the iron about 90% of this area is iron, the actual or net area of the core is $25 \times 9 = 22.5$ sq cm

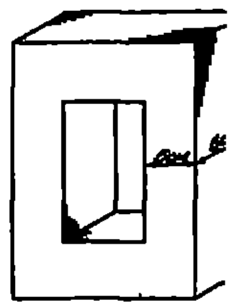


Fig. 150. — Core for Transformer

For a 60-cycle circuit we may assume a maximum density in the iron of 10,000 lines per sq cm. We have then,

$$E_{eff} = 110$$
$$f = 60$$
$$A = 22.5$$
$$B = 10,000$$

Substituting in $N = \dfrac{10^8 \times E_{eff}}{4.44 \times \phi \times f}$

we have $$N_p = \frac{10^8 \times 110}{4.44 \times 22.5 \times 10,000 \times 60} = 184 \text{ turns.}$$

To get the secondary turns we can substitute secondary volt in the formula, $$N_s = \frac{10^8 \times 55}{4.44 \times 22.5 \times 10,000 \times 60} = 92 \text{ turns.}$$

From the above it is seen that we could have obtained the s ondary turns, if we had known the primary turns, by multiplyi the primary turns by the ratio of secondary voltage to prim voltage. Or if we had known the secondary turns we could ha obtained the primary turns by multiplying the secondary tu by the ratio of primary to secondary voltages

The principles outlined in the study of the transformer will brought out clearly by checking through the design of a sm transformer The procedure can be summarized under six hea ings as follows

I. Given
   1. Kv-a. rating
   2. Primary volts
   3. Secondary volts
   4. Frequency

II. Assume
   1. Efficiency (from similar transformer Table A, p 132–133)

2 Magnetic density (Table B, p 134)
3 Current density (Table C, p 134)
4. Relation of iron losses to copper losses.

III. Obtain Iron losses, (curves, Fig 149)

IV Obtain
1 Volume of iron
2 Shape of core

V Calculate the number of turns from

$$N = \frac{10^8 \times E_{eff}}{4.44 \times \phi \times f} \tag{45}$$

VI 1 Calculate wire size.
2 Sketch section of core with wire in place
3 Check sizes of core and copper and iron losses

VII Calculate exciting current. (Method of Fig 154)

Practical Application of Principles. The transformer considered will be a small experimental transformer of 1500 volt-ampere capacity to step down from 110 volts to 55 and 27½, the frequency is to be 60 cycles.

Then from I,
1 Kv-a rating = 1500
2. Primary volts = 110
3 Secondary volts = 55 and 27½
4. Frequency = 60 cycles

II Referring to Table A, it is seen that 95% will be a fair efficiency to assume for a transformer of this size Table B shows that a density of 10,000 lines per sq c.m is a suitable density when the transformer is to be run on a 60-cycle circuit. A density of 1000 C.M per ampere has been assumed for the current density in both primary and secondary coils which is ample for this transformer As the transformer is used for experimental purposes, one-half of the total losses has been allowed in the iron and one-half in the copper

III. Since the output is 1500 watts (100% P.F.) the inp must be 1500 + 95 = 1580 watts, or the losses 80 watts T iron loss is to be one-half and the copper loss one-half or we c say the iron losses will be 40 watts and the copper 40 watts

From curve, Fig 149, for a density of 10,000 lines and a f quency of 60 cycles per second the

Eddy current loss is	.77	watt per 100 cu cm of iron
Hysteresis loss is	3.0	" " " " " " "
So the total losses are	3.77	" " " " " " "

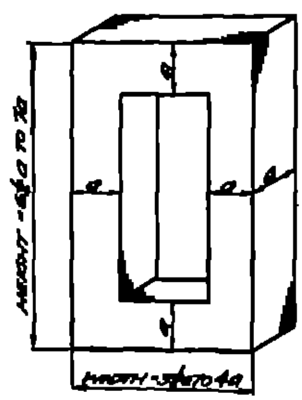


Fig. 151.—Proportions of Transformer Core.

IV From III, since in ev 100 cu cm of iron there are 3 watts lost, there will need to be many hundred cu cm of iron $40 \div 3.77$ or 10.6 hundred, v 1060 cu cm

Figure 151 shows common p portions of a core-type transform The volume of the core in terms dimension "a" for a length (and width 3½a is,

$$V = (6.5a \times a \times a)2 + (1.5a \times a \times a)2 = 13\,a^3 + 3\,a^3 = 16\,a^3$$

$$a = \sqrt[3]{\frac{V}{16}}$$

Substituting $a = \sqrt[3]{\frac{1060}{16}} = \sqrt[3]{66.2} = 4.05$ cm $= 1.6''$ Use 1

V Primary turns. The primary turns will be,

$$N_P = \frac{10^8 \times E_{eff}}{4.44 \times \phi \times f}$$

$$E_{eff} = 110$$

$$\phi = AB = 1.75'' \times 1.75'' \times 6.45 \times 9 \times 10{,}000 = \ldots$$

$\times$ 10,000. (The net area of the iron is assumed to be .9, the gross area.)

$$f = 60$$

$$\text{Substituting, } N = \frac{10^8 \times 110}{4.44 \times 17.8 \times 10{,}000 \times 60} = 232 \text{ turns}$$

The secondary turns will be

$$N_S = \frac{55}{110} \times 232 = 116 \text{ turns}$$

Primary and Secondary Currents. Since the output is to be 1500 watts, the secondary current will be,

$$1500 \div 55 = 27.3 \text{ amperes}$$

and the primary current, neglecting the exciting current, will be,

$$1500 \div 110 = 13.65 \text{ amperes}$$

VI. 1. *Size of Primary and Secondary Wire.* From Table C we see that 1000 circular mils per ampere will be safe for this transformer, so the circular mils required for the primary will be,

13.65 $\times$ 1000 = 13,650 or #9 wire (13,594 C M.)

The circular mils for the secondary will be,

27.3 $\times$ 1000 = 27,300 or #6 wire (26,250 C M.)

The diameter of #9 d c.c. wire is .126″ and its resistance per 1000 ft. is .7908 ohms.

The diameter of #6 d c c. wire is .174″ and its resistance per 1000 ft. is .3944 ohms.

2. If we wind the primary with 47 turns per layer we shall need $\frac{232}{47}$ or 4.94 layers.

The length of primary winding will be 47 $\times$ .126″ = 5 92″. Allow 6″. The depth of primary winding will be 5 $\times$ .126″ = .630″. Allow ¾″.

To keep the secondary about the same length as the primary we can use 34 turns per layer. We shall need 116 $\div$ 34 = 3.4 layers.

The length of secondary will be 34 $\times$ .174″ = 5.92″. Allow 6″.

The depth of secondary winding will be $4 \times .174'' = .6$[cut off]
Allow ¾″

The windings may be placed as in Fig 152

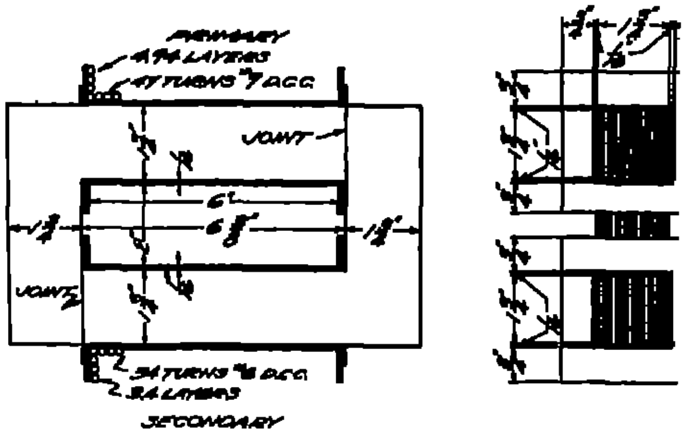


Fig. 152 Section of Small Transformer

3 *Check on Core and Copper Losses* The mean length [cut off] primary turn is, $(1\frac{1}{2}'' + \frac{1}{16}'' + \frac{1}{16}'' + .63'') \times 4 = 10.02''$

Total length primary winding is, $\frac{10.02''}{12} \times 232 = 194$ ft.

Resistance of primary is, $\frac{194}{1000} \times .7908 = .153$ ohms

Primary copper loss is, $13.65^2 \times .153 = 28.5$ watts

Mean length of a secondary turn is,

$$(1\frac{1}{2}'' + \frac{1}{16}'' + \frac{1}{16}'' + .696'') \times 4 = 10.28''$$

Total length secondary winding is, $\frac{10.28''}{12} \times 116 = 99$ ft.

Resistance of secondary is, $\frac{99}{1000} \times .3944 = .039$ ohms

Secondary copper loss is, $27.3^2 \times .039 = 29.1$ watts.

Total copper loss is, $28.5 + 29.1 = 57.6$ watts.

The actual volume of the core will be,

$$(9.875'' \times 1.75'' \times 1.75'')\ .9 \times 2'' + (2'' \times 1.75'' \times 1.75'')\ .9 \times 2$$
$$= 54.4 + 11 \text{ cu. in.}$$
$$= 65.4 \text{ cu. in.}$$
$$= 65.4 \times 2.54^3 = 1071 \text{ cu. cm.}$$

The iron losses will be, $.0377 \times 1071 = 40.4$ watts

Total losses will be,

Copper	57.6
Iron	40.4
Total	98.0

These are somewhat larger than assumed, making the efficiency

$$\frac{1500}{1598} = 94\% \text{ approx.}$$

In order to get efficiency more nearly 95% it will be necessary to try slightly different proportions of copper and iron.

VII. *Calculation of Exciting Current.* The actual length of the path of the flux is approximately,

$$(6\tfrac{3}{8}'' + \tfrac{7}{8}'' + \tfrac{7}{8}'') \times 2 + (2'' + \tfrac{7}{8}'' + \tfrac{7}{8}'') \times 2 = 16\tfrac{1}{4}'' + 7\tfrac{1}{2}'' = 23\tfrac{3}{4}''$$

From curve, Fig. 153, it is seen that 10 ampere-turns are needed per inch of core to magnetise the iron to a density of 10,000 lines per sq. cm., so the core will require,

$$23.75 \times 10 = 237.5 \text{ ampere-turns.}$$

The iron may be cut so that there will be but two joints in the magnetic circuit.

The ampere-turns for each joint are approximately $\frac{.001}{6.45} \times B_{max}$

or for the two joints in series,† $\frac{.001}{6.45} \times 2 \times 10{,}000 = 3.1$

† Still gives $.001 \times B''$ for each joint. See Elements of Electrical Design — Still. McGraw Hill Book Co., Inc.

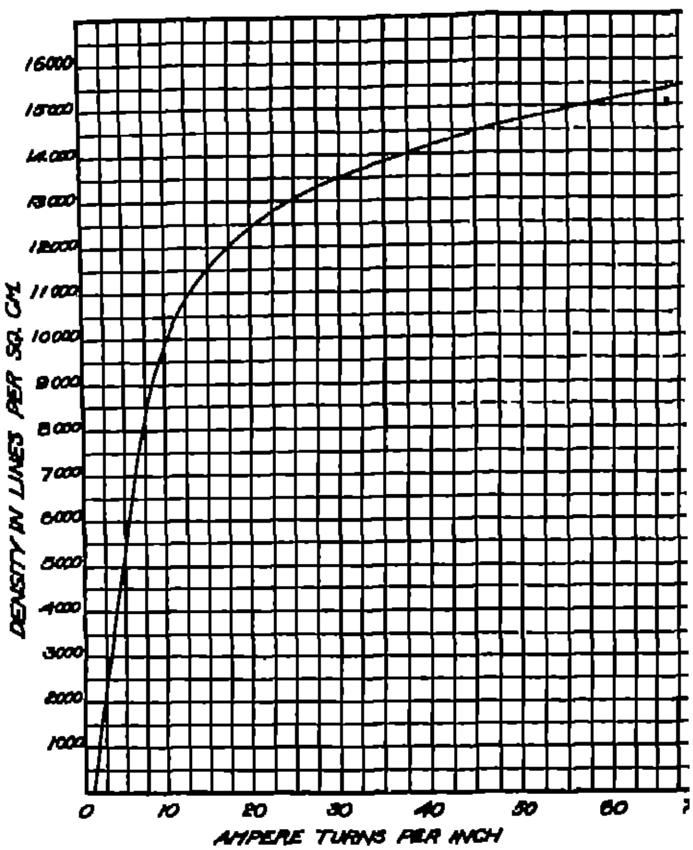


Fig. 153.—Magnetization Curve for Average Quality Transformer I
(Elements of Electrical Design—Still)

The total ampere turns will then be, 237 5 + 3 1 = 240 6

Since there are 232 turns in the primary, the magnetizing cu rent will be I_{max} = 240.6 ÷ 232 = 1 04 amperes. The relations b tween exciting current, magnetizing current, and component exciting current necessary to supply iron losses may be obtain from transformer diagram as in Fig 154

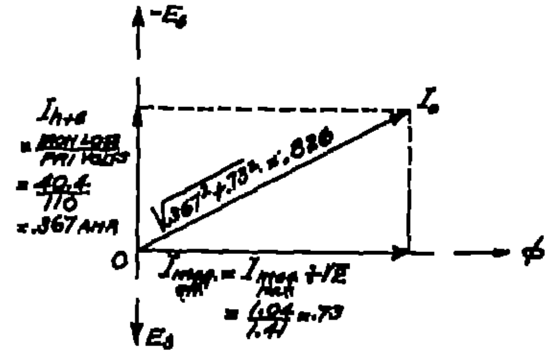


Fig. 154.—Determination of Exciting Current by the Transformer Diagram

Use of Transformer Diagram. Equivalent Resistance and Reactance. The transformer diagram enables one to obtain the operating characteristics of a transformer, such as phase-angle, efficiency regulation, etc. A study of the diagram will show that all the quantities necessary for the construction of the diagram of Fig 147 may be obtained by test except primary and secondary reactance.

Readings of voltage and watts may be taken from one side of the transformer with the other side short-circuited, and from these readings the combined reactance of the primary and secondary, known as Equivalent Reactance, and the combined resistance of the primary and secondary known as Equivalent Resistance may be calculated.

By using the equivalent resistance and reactance as measured above and constructing a modified form of the transformer diagram, the characteristics of the transformer may be obtained The procedure is as follows Short-circuit one side of the transformer through an ammeter and apply enough voltage to the other side of the transformer to cause full load current to flow in the short-circuited coil. Full-load current will flow in the other

coil as well It will be found that the voltage necessary to cr full-load current to flow will be about 5% or 10% of the nor voltage of the transformer, so the core loss is negligible. wattmeter should be placed in the side which is being used ɩ primary and watts as well as volts read As the core loss is nɩ gible the wattmeter reads the copper loss in the windings of transformer, viz., $P = I^2R = IE$. The voltmeter reads the v age necessary to send full load current against the impedanc the windings, viz., $E = IZ$.

The relations of the impedance drop, resistance and reacta drops are as in Fig 155

E_z is read directly, E_R is obtained by dividing the wattm reading by I, and E_x is obtained constructing a right-angle triangle wh E_z and E_R are known

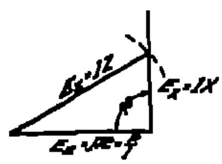


Fig. 155. — Relations of Impedance Drop, Reactance Drop and Resistance Drop in a Transformer

The resistance and reactance of b windings are included in the values tained That is, IX and IR are drops equivalent to combining the ɩ mary and secondary resistance and re tance drops measured separately ɩ added

To apply the results obtained by test to the transformer ɩ gram, consider first that the regular secondary side is the s that has been short-circuited If "a" is the ratio of the prim turns to secondary turns, the following relations exist

$$E_p = E_s a \quad ($$
$$I_p = \frac{I_s}{a} \quad ($$
$$R_p = R_s a^2 \quad ($$
$$X_p = X_s a^2 \quad ($$

Use the diagram of Fig 156 and reduce secondary quanti to primary quantities, thus,

$$E_p = E_s a, \quad I_p = \frac{I_s}{a}, \quad R_p = R_s a^2, \quad X_p = X_s a^2$$

The diagram becomes as Fig. 157.

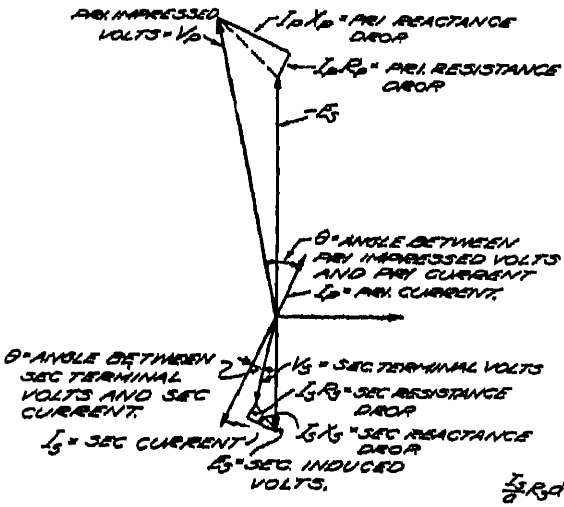


Fig. 156. — Transformer Diagram with Exciting Current Omitted.

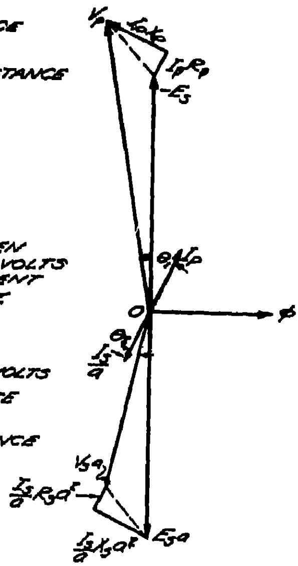


Fig. 157 — Diagram of Fig 156 with Secondary Quantities Reduced to Primary

Turn the lower part of the diagram about O as a center until $\frac{I_s}{a}$ falls upon I_p. $\frac{I_s}{a}R_sa^2$ and $\frac{I_s}{a}X_sa^2$ move to the positions shown by Fig. 158.

The diagram of Fig. 158 may be drawn as in Fig. 159.

Then when R_e = Equivalent resistance

$$R_e = R_p + R_sa^2 = R_p + R_s\left(\frac{N_p}{N_s}\right)^2 \tag{50}$$

when X_e = equivalent reactance

$$X_e = X_p + X_sa^2 = X_p + X_s\left(\frac{N_p}{N_s}\right)^2 \tag{51}$$

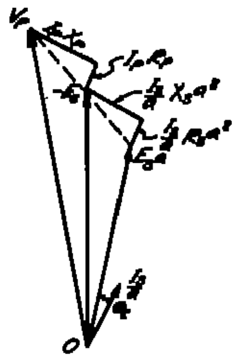


Fig. 158. — Transformer Diagram with Secondary Quantities Reduced to Primary $\frac{I_s}{a}$ Superposed on I_p.

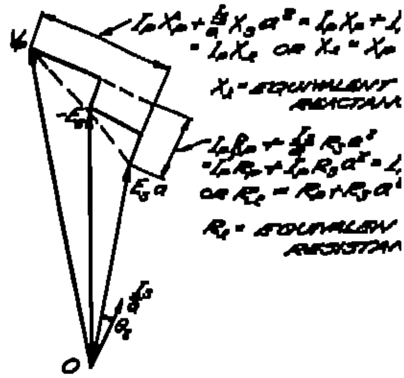


Fig. 159 — Transformer Diag Showing Equivalent Resist and Reactance.

Revolve Fig 159 still more so that OE_sa becomes hori as in Fig 160. Add lines E_san, pq, mp and nq Figure 1

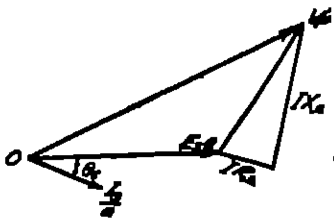


Fig. 160. — Transformer Diagram with Primary and Secondary Resistance Drops Replaced by Equivalent Resistance and Reactance Drops.

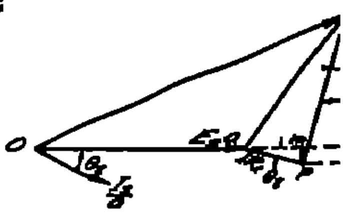


Fig. 161 — Transformer Diag Suitably Drawn for Use Test Readings.

Then

$$E_sam = IR_e \cos \theta_1$$
$$mn = pq = IX_e \sin \theta_1$$
$$pm = IR_e \sin \theta_1$$
$$qV_p = IX_e \cos \theta_1$$
$$nV_p = qV_p - qn$$

$$\text{Regulation} = \frac{OV_p - OE_s a}{OE_s a}$$

If more convenient, the regular secondary side may be used as the primary and the opposite side short-circuited, then,

$$E_s = \frac{E_p}{a} \tag{52}$$

$$I_s = I_p a \tag{53}$$

$$R_s = \frac{R_p}{a^2} \tag{54}$$

$$X_s = \frac{X_p}{a^2} \tag{55}$$

Problem Illustrating Use of Transformer Diagram in Calculating Regulation. The following results of a test on a small transformer will make clear the application of principles.

Rating of transformer = 3 kv-a.
Primary volts = 2200
Secondary volts = 220
Primary resistance at 75° F = 16 65 ohms
Secondary resistance at 75° F = 208 ohms
Ratio a = 10 1
Impedance volts measured from high voltage side with low voltage short-circuited = 70
Primary amperes = 1.36
Impedance watts = 60
Equivalent primary resistance $= R_e = R_p + R_s a^2$

Substituting $R_e = 16.65 + 208 \times 10^2 = 37\ 45$ ohms.

Equivalent resistance drop $= IR_e = 1\ 36 \times 37\ 45 = 51$ volts

Equivalent reactance drop $IX_e = \sqrt{E_z^2 - \left(\frac{P}{I}\right)^2}$

Substituting $IX_e = \sqrt{70^2 - \left(\frac{60}{1\ 36}\right)^2} = 54.4$

Regulation will be computed for two power factors, 80% and 100%. Regulation for 80% power factor

$$
\begin{aligned}
E_s am &= IR_e \cos\theta_2 \\
&= 51 \times .80 = 40.8 \\
mn &= pq = IX_e \sin\theta_2 \\
&= 54.4 \times .60 = 32.6 \\
pm &= IR_e \sin\theta_2 \\
&= 51 \times .60 = 30.6 \\
qV_p &= IX_e \cos\theta_2 \\
&= 54.4 \times .80 = 43.5 \\
OV_p &= \sqrt{(2200 + 40.8 + 32.6)^2 + (43.5 - 30.6)^2} = 2273 \\
\text{Reg} &= \frac{2273 - 2200}{2200} = \frac{73}{2200} = 3.32\%
\end{aligned}
$$

These quantities on the diagram of Fig. 161 are as in Fig. 162.

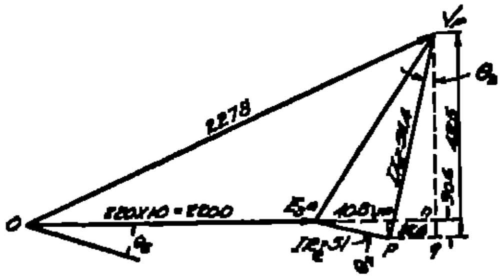


Fig. 162. — Transformer Diagram for Calculating Regulation at 80% P.F.

At 100% P. F., $\theta_2 = 0$

$$
\begin{aligned}
E_s Am &= 51 \times 1 = 51 \\
mn &= 54.4 \times 0 = 0 \\
pm &= 51 \times 0 = 0 \\
OV_p &= \sqrt{(2200 + 51)^2 + 54.4^2} = 2251.5 \\
\text{Reg} &= \frac{2251.5 - 2200}{2200} = \frac{51.5}{2200} \\
&= 2.32\%
\end{aligned}
$$

These quantities on the diagram of Fig. 161 are as in Fig. 163.

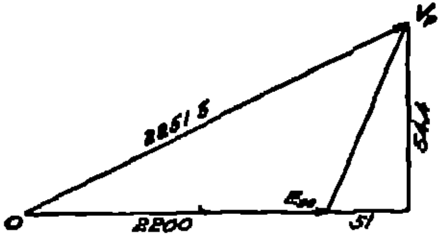


Fig. 163 — Transformer Diagram for Calculating Regulation at 100% P F

Polarity By polarity of a transformer we mean the relative direction of the E. M F induced in the secondary in relation to the primary impressed E M F The standard system of marking

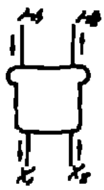


Fig 164 — Marking of Leads for Subtractive Polarity

Fig. 165 — Marking of Leads for Additive Polarity

leads is H_1 and H_2 for primary and X_1 and X_2 for secondary With this system, when the voltage is acting from H_1 to H_2 in the primary, it must act at the same instant from X_1 to X_2 in the secondary

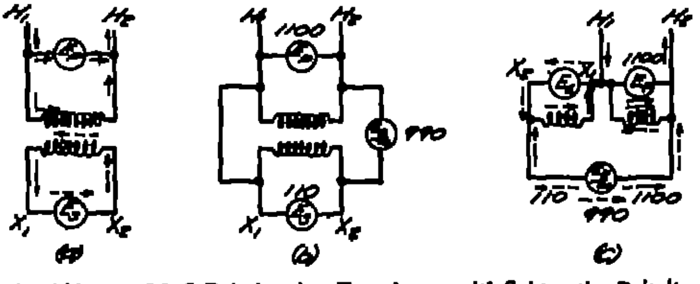


Fig 166. — E. M. F Relations in a Transformer with Subtractive Polarity when used as an Autotransformer

When H_1 and X_1 are adjacent, that is directly across as in Fig 164, the polarity is called "subtractive" When H_1 and X_1 are diagonally across as in Fig 165 the polarity is "additive." At (a), Fig 166, H_1 and H_2 are the primary leads across which a volt age of, say, 1100 is impressed acting from H_1 to H_2. If the ratio is 10 1 there will be 110 volts acting from X_1 to X_2. If H_1 and X_1 be connected together as at (b) the voltage across H_2 X_2 will be $1100 - 110 = 990$

(c) shows the relations graphically, the impressed volts being denoted by the full arrows and induced volts in each coil by the dotted arrows.

The Autotransformer The autotransformer is a single-coil transformer and may be used either for stepping down or stepping up a voltage. When used for stepping down, the coil that is connected across the high voltage line is wound with enough turns to give the proper working density in the core for the given voltage and frequency A tap is taken off the winding at such a distance from the end that the turns between the end and tap are the same proportion of the total turns that the low voltage is of the high voltage The tap forms one side of the low-voltage winding and the end of the coil the other side. Thus in Fig 167 the high-voltage coil has 240 turns and is designed for 120 volts It is desired to step down to 40 volts or one third of 120 volts, so the tap is taken off at one third of 240 or the eightieth turn Neglecting losses, if the load on the transformer of Fig 167 is 30 amps. at 40 volts or 1200 watts, the input will be 1200 watts also and the primary current will be $\frac{1200}{120} = 10$ amperes. At a given instant the 10 amperes will flow in at A and out at G The part of the winding between the points B and E acts like the secondary of a two-coil transformer so that the 30 amperes load-current flows around

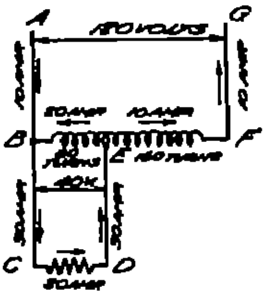


Fig. 167 — Autotransformer for Stepping Down Voltage.

the low-voltage circuit counter-clockwise for the instant shown. The current in BE is therefore 30 − 10 or 20 amperes and flows as shown by arrow.

When used for stepping up, voltage is applied at C and D and a higher voltage taken off at A and G. Figure 168 shows the autotransformer of Fig. 167 when used for stepping up. 40 volts applied at C and D will be stepped up to 120 volts at A and G. If the load is 1200 watts as before, 10 amperes will flow in the high side and 30 amperes in the low side. If, at a given instant, 30 amperes are flowing in at C, 30 amperes will flow out at D. The part of the coil between B and E will act like the secondary of a two-coil transformer and try to send current from E to B. The actual current in BE will be 30 − 10 = 20 amperes as before.

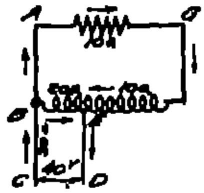


Fig. 168.—Autotransformer Used for Stepping Up Voltage.

The Instrument Potential Transformer. The instrument potential transformer, commonly called a potential transformer, or "pot" transformer, is in general similar to a regular static transformer. It is used to step down the voltage of a high-voltage line to a voltage suitable for ordinary voltmeters, wattmeters, etc. Potential transformers range in size from a 50-volt-ampere rating to as high as a 1000-volt-ampere rating. A 200-volt-ampere size is common. Figure 169 shows a potential transformer for outdoor service, Fig. 170 one for indoor service and Fig. 171 a portable transformer for test use.

Fig. 169.—Potential Transformer for Outdoor Service. (General Electric Co.)

The vector relations shown by the transformer diagram of Fig. 147 hold for the potential transformer. As explained in the discussion of the transformer diagram, there is a drop in the windings due to the load, or burden as it is called, in connection with

instrument transformers Hence, if a transformer has a turn ratio of 10 1 it will not give a voltage of exactly 10 1 where a burden

Fig. 170. — Potential Transformer for Indoor Service. (General Electric Company)

of several instruments is put on the secondary This error is called a ratio error, and can be compensated for at a given burden by winding the transformer with a turn-ratio just enough less than the voltage ratio desired to make up for the drop in the windings. In order to obtain correct readings at other burdens than that for which the transformer is compensated, curves plotted from test readings are necessary

Fig 171. — Portable Potential Transformer (General Electric Co.)

Another error, in a potential transformer, is an error due to phase displacement It was shown by the transformer diagram that the secondary current is not always 180° out of phase with the primary current. The angle by which it differs, depends upon the ratio of the resistance and reactance of the secondary circuit and upon the load While a phase-angle error would not affect the accuracy of a voltmeter reading, it would affect the accuracy of a meter such as a wattmeter or power-factor meter

Figure 172 shows a set of curves made up from test readings that give the ratio and phase-angle corrections to apply to the readings of a certain type 20 1 Westinghouse potential transformer

An example will explain their use Suppose the total burden on

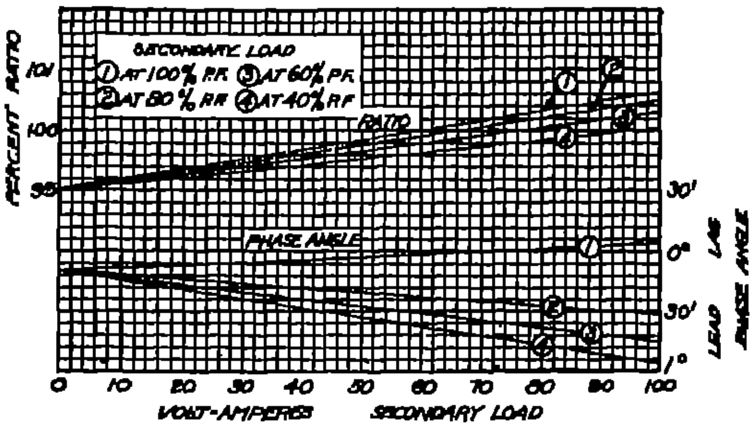


Fig 172. — Ratio and Phase-Angle Curves for Dry Type Voltage Transformers Ratio 20 to 1, 60 cycles. (Westinghouse Electric & Mfg. Co.)

a 20:1 potential transformer, consisting of a voltmeter, wattmeter, watthour meter, frequency meter, and synchroscope, is 50-volt amperes and its power factor is 80% The true line-voltage and the phase-angle correction are required Refer to Fig 172. On the axis of abscissas find the volt-ampere burden of 50, and read vertically to the "Ratio" curve marked (2) which is for a power factor of 80%. Read horizontally to the axis of ordinates

If the voltage read on the voltmeter is 110, the true line voltage will be $110 \times 20 \times 99.75 = 2194.5$ volts Reading similarly to the curve marked "Phase Angle," an angle of lead of 0° 21′ will be found

Operation. Potential transformer secondaries should never be short-circuited while the primary is connected to a live line of the rated voltage of the transformer The impedance of the transformer will be greatly reduced by short-circuiting the secondary and large secondary and primary currents will flow and burn out the windings.

The Instrument Current Transformer The instrument current or series transformer, commonly called a current transformer, is a transformer with separate primary and secondary coils It is

connected in series with the line and is used to step down tl line current to a value suitable for ammeters, wattmeters, wat hour meters, relays, trip coils, etc Having separate primary an secondary coils, it acts as insulation between the instruments c other apparatus connected to the secondary side, and the hig voltage of the line in which it is connected

The operation of a current transformer can be illustrated b means of an ordinary step-down lighting transformer connecte

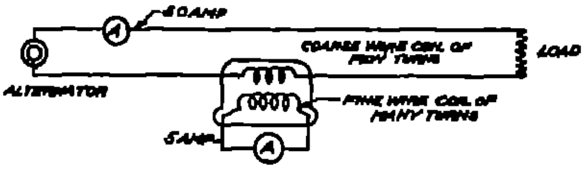


Fig. 173.—Lighting Transformer used to Illustrate the Principle of a Current Transformer

with the low-voltage winding in series with the line and the high voltage winding short-circuited through an ammeter See Fig 173

Assume that the transformer used for illustration is designed sc that the coarse wire winding normally carries 50 amperes and the fine wire winding 5 amperes when used as a regular step-down potential transformer As connected in Fig 173, when the coarse wire winding carries 50 amperes, a flux will be set up in the core

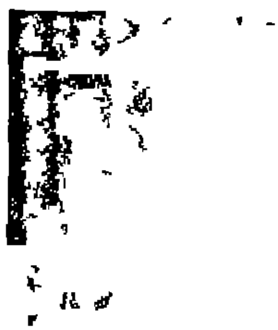


Fig. 174 —4500-Volt Dry Type Current Transformer (General Electric Co.)

which will induce a voltage in the fine wire winding Since the fine wire winding is short-circuited, current of 5 amperes will im

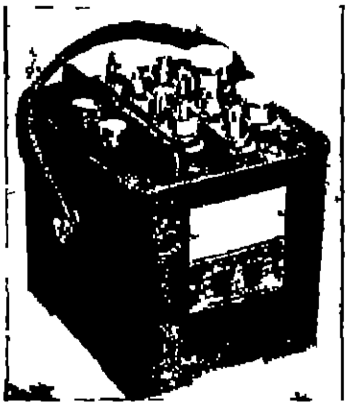


Fig 175 — Type P-3 Portable Current Transformer (General Electric Co.)

mediately flow in the fine wire winding This current will set up a flux that will oppose the main or primary flux, or, expressed another way, the secondary ampere turns will oppose the primary ampere-turns.

It should be noted that in a current transformer used for stepping down current, the primary winding has the small number of turns and the secondary winding the large number of turns. In this respect it is just the opposite of a potential transformer Figures 174, 175 and 176 show different types of commercial current transformers.

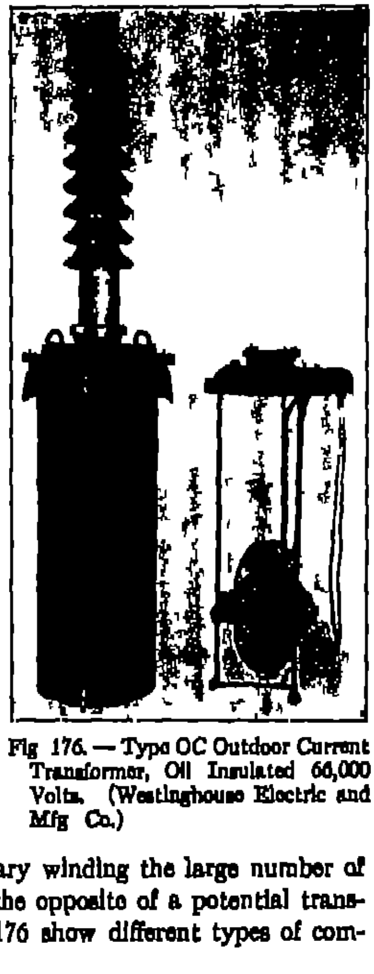


Fig 176. — Type OC Outdoor Current Transformer, Oil Insulated 66,000 Volts. (Westinghouse Electric and Mfg Co.)

Inherent Errors in a Current Transformer The vector relations in the transformer used for illustration are shown graphically by Fig 177 This is the usual transformer diagram. It will

be seen from a study of Fig 177 that, if the exciting current can be made very small, approaching zero, that the primary am pere turns $N_pI'_p$ will approach N_pI_p Figure 178 shows the dia

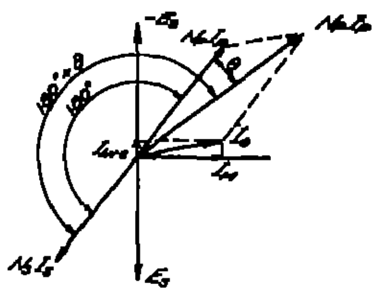


Fig. 177 — Vector Relations in a Current Transformer with a Large Exciting Current.

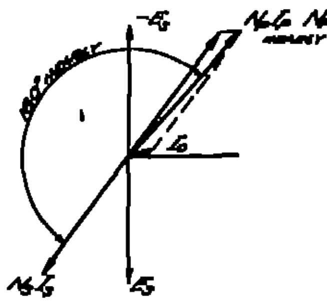


Fig 178. — Vector Relations in a Current Transformer with a Very Small Exciting Current.

gram of Fig 177 redrawn with the exciting current much reduced to illustrate this fact.

Using the transformer of Fig 173 for illustration, the equation $N_pI_p = N_sI_s$ will be true but $N_pI'_p$ will not equal N_sI_s unless I_e is zero Further, the secondary current will not be exactly 180° from the primary current unless the exciting current is zero. This also appears from Fig 178

Effect of Exciting Current on Ratio and Phase Angle. The exciting current of a transformer does not increase at the same ratio as the magnetizing force. This will appear from a study of Fig 179 which is typical of the magnetization curve of the iron used in a transformer core. It will be seen that while the flux or primary current increases from I_1 to I_1', the exciting current increases only from I_0 to I'_0 or less rapidly At points still farther down on the curve the ratio will be still different.

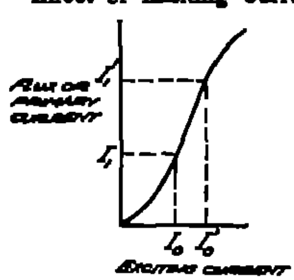


Fig. 179 — Typical Curve of Exciting Current.

If the exciting current increased at the same rate as the primary current the vectors of Fig 177 would all change at the same rate and the phase-angle displacement would be constant Since this condition does not hold, due to the magnetic characteristics of the iron, a correction must be made for different loads on the secondaries of current transformers and also for loads of different power factors.

The proper corrections to make are obtained by tests made by the manufacturers of the transformers and are plotted in curves.

Methods of Correcting — Ratio and Phase Angle Curves. Figure 180 shows a set of ratio and phase angle curves for a typical Westinghouse dry type current transformer at 60 cycles

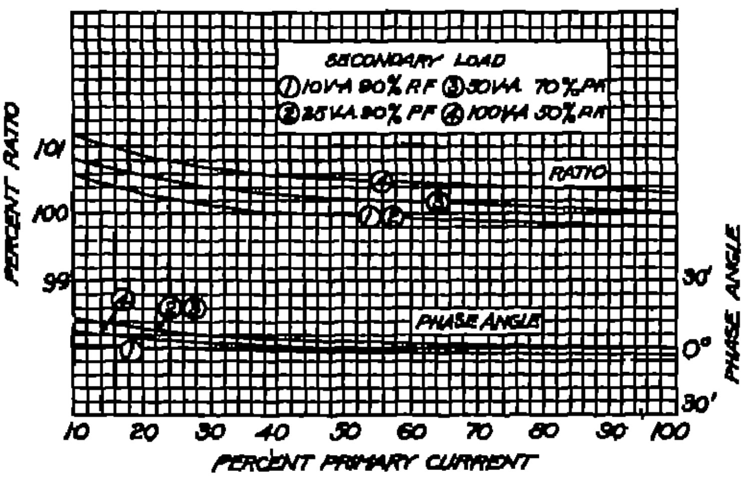


Fig 180. — Ratio and Phase Angle Curves for Types KA, KB, KC, MA, MB MC Current Transformers, 60 Cycles, For Circuits From 6800 to 23000 Volts. (Westinghouse Electric & Mfg. Co.)

The method of using the curves will be shown by an example. Suppose the transformer has a 500 to 5 ratio and is used with a 5-ampere ammeter The full scale will be marked 500 amperes, since the current transformer ratio is 100 1 There are other instruments than the 5-ampere ammeter connected on the transformer making the total burden 7 25 volt-amperes With this

burden, the ammeter reads 425 amperes. What is the true lir current?

From the curves it is seen that 10 volt-amperes and 25 volt amperes give nearly the same performance (curves #1 and #2 Curve #1 will therefore represent very closely the performance a a burden of 7 25 volt-amperes. This curve should be read a $\frac{425}{500} = 85\%$ of primary current. This gives a per cent ratio o 99.85 and the true current will be $425 \times 9985 = 424\ 4$ amperes The curve also shows that the phase angle under these conditions is -3 minutes. The phase angle is of no use in correcting for ampere readings but is useful in making corrections on such instruments as wattmeters and power factor meters.

Operation. The secondary of a current transformer should never be open when the primary is carrying current A dangerously high voltage will build up at the secondary terminals. This may become high enough to kill a person touching the secondary It can become high enough under certain conditions to break down the insulation and actually cause the transformer to short circuit and explode The reason for this high voltage is that when the secondary is open there are no secondary ampere-turns opposing the primary ampere-turns and a large flux builds up from the primary ampere-turns. This flux induces voltage in the secondary and as the turn ratio is usually high, the voltage becomes very high Another reason for not opening the secondary under load is, that the iron becomes highly magnetized and, unless demagnetized afterward, the transformer will not give an accurate ratio

If it is necessary to change a meter under load, be sure that a reliable short-circuiting device is put across the secondary before the meter is removed

Constant Current Transformer The constant current transformer is a piece of apparatus that is used principally to supply current to series arc or incandescent lamps The transformer is of the shell type and has a rather long central core that stands in a vertical position The primary and secondary coils are placed on

this core. One of these coils is fixed and the other is free to move. The movable coil is suspended by cable attached to rocker arms and is counterbalanced by weights. Figure 181 shows a General

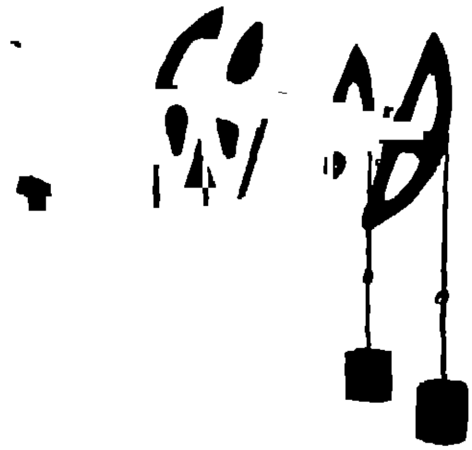


Fig 181 — General Electric RV-Station-Type Constant Current Transformer

Electric Type RV station type constant-current transformer suitable for feeding street lighting circuits. Constant-current transformers are designed to take current from a constant-potential primary circuit and supply constant current to a secondary circuit. Such transformers are designed for primary [illegible] high as 13,200 and in capacities up to 70 kw. [illegible] designed to supply a secondary current of 6.6 [illegible] age of which will depend on the nature of the load.

The operation of the transformer is as [illegible] movable coil, which we assume [illegible] position at the end of the core [illegible]

If the primary is connected to a source of constant potential, it will draw a certain amount of current. If the secondary is open circuited, this current will be exciting current only Suppose, now, that, say, 10 lamps are connected in series with each other and switched on the secondary, and the secondary unlatched so that it is free to move. Whatever voltage that was induced in the secondary by the lines of force from the primary will cause current to flow through the lamps. The primary will now draw the exciting current as before and, in addition to this, more current to balance the load-component of the secondary current. The secondary coil, although unlatched and free to move, will not drop down against the primary because it is carrying current in the opposite direction from the current in the primary and the two currents will repel each other The coil will take some position which will depend on the characteristics of the transformer and secondary circuit If counterweights be removed from the secondary, the secondary will move closer to the primary and be cut by more lines of force This will cause a higher voltage in the secondary and more current will at once flow The secondary will then move away from the primary slightly as this current reacts on the primary current. The counterweights are adjusted at the factory for the normal load that the transformer is to carry If, with this adjustment, more than normal load is put on the transformer, by putting more lamps in series, and therefore more resistance in the secondary circuit, the secondary current immediately drops off and the secondary coil moves nearer to the primary and into a stronger field This stronger field induces more voltage in the secondary coil and brings the current back to its normal value again Similarly, if fewer lamps are connected to the secondary, the resistance will be less than before and the current will rise. The secondary will be repelled from the primary and move into a weaker field. The secondary voltage will then be less and the current will again become normal

Experience in design and manufacture has enabled manufacturers to produce transformers that will regulate to within one per cent of rated current for any loads within their rated capacities

Figure 182 shows a constant-current transformer connected in circuit.

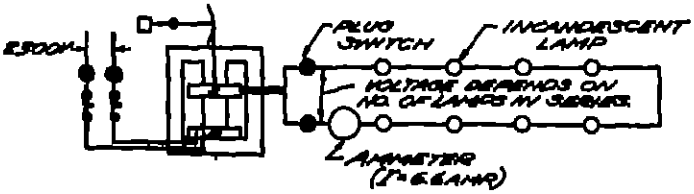


Fig. 182 — Constant-Current Transformer Connected to 2300-Volt Line.

In the simple system described above, all the lamps would go out in case the filament of one lamp became broken. In practice, this trouble is prevented by providing a cut-out in the base of the lamp which operates in case the filament burns out. One form of cut out consists of a film gap in parallel with the filament of the lamp. In case the filament breaks, the voltage breaks down the gap and establishes a circuit through the cut-out so that only the broken lamp is out of service.

Special Forms of Current Transformers. A constant current transformer such as has just been described is suitable for use in a station supplying incandescent lamps. When used with constant current arc lamps, a mercury-arc rectifier similar to that described in Chapter XI must be used with the transformer, because the modern magnetite arc lamps require unidirectional current.

A constant-current transformer operating on the same principle as the one described has been developed for use in lighting sections remote from a station. This type can be mounted on a pole adjacent to a power line and may be switched on and off by means of a time switch. A similar transformer that is entirely waterproof has been developed for subway use.

Another type of transformer that will give a fairly constant current is shown diagrammatically by Fig. 183.

This transformer depends for its action on magnetic leakage the same as the type just described but has no movable parts. If a transformer of the type shown by Fig. 183 be operated with

vanous loads from open circuit to short circuit and the reading plotted, it will be found that the relation between the secondar voltages and secondary currents will be shown by a curve of th general shape of Fig 184. Inspection of this curve will show tha

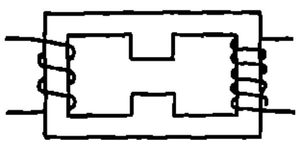


Fig. 183. — Constant-Current Transformer with Stationary Coils.

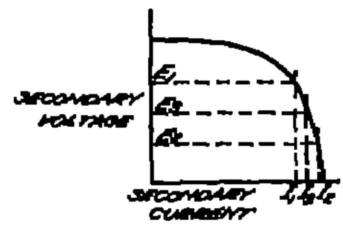


Fig 184. — Voltage-Current Curve for Type of Transformer Shown by Fig 183.

between currents I_1 and I_3 there is a very small change in current for a large change in voltage Such a transformer should be designed so that the normal load is somewhere between I_1 and I_3. If the normal secondary current is I_3, then, when the load is increased, as for instance by connecting more lamps in series, the voltage will rise to E_1 or a large amount shown by the distance E_3 E_1 while the current will remain practically constant, changing only the very small amount I_3 I_1

Fig. 185. — Constant-Current Transformer for lighting Neon Signs. (Chicago-Jefferson Fuse & Electric Co)

Figure 185 shows a transformer of this type made by the Chicago-Jefferson Fuse and Electric Company which is used for

obtaining approximately constant current from an ordinary constant-potential lighting circuit, to light the tubes in a neon sign

Transformers for Welding Transformers for welding purposes are subject to practically short circuits on the secondaries and so have to be designed mechanically and electrically to withstand such severe short-circuit conditions. In the arc-welding process, the short circuit occurs when the operator strikes the arc. In the spot and seam welding processes, the short circuit occurs when the electrodes are brought in contact with the pieces to be welded In butt welding two pieces such as rods or small parts are forced together under considerable pressure and current sent through them The circuit thus formed is of low resistance and the current large

The voltages used are low When actually welding, an arc-welding transformer operates with a secondary voltage of about 18 to 22 volts depending on the length of the arc that the operator is drawing On open circuit, the secondary voltage may go to nearly 100 volts. The current capacities of the secondaries of the smaller sizes run from about 150 amperes to 300 amperes On heavy-duty machines the current runs much higher In spot and seam welding the voltage runs from about 5 volt on real light work up to about 9 volts. For most light work the voltage runs about 1 5 to 4 volts The current may run about 1000 or 3000 amperes. On heavy-duty types the voltage runs somewhat higher and the current may go to 20,000 amperes or higher

The operation of the welding transformer is as follows When the secondary is open-circuited, the flux set up by the primary current cuts the primary turns and generates a counter electromotive force in them that is sufficient to keep the line current down to a small value This current is called the exciting current of the transformer When the secondary is short-circuited, the current in the secondary sets up a strong flux of its own that opposes the main or primary flux. This secondary flux, if kept entirely within the iron core of the transformer, would buck down the primary flux to such an extent that the primary im-

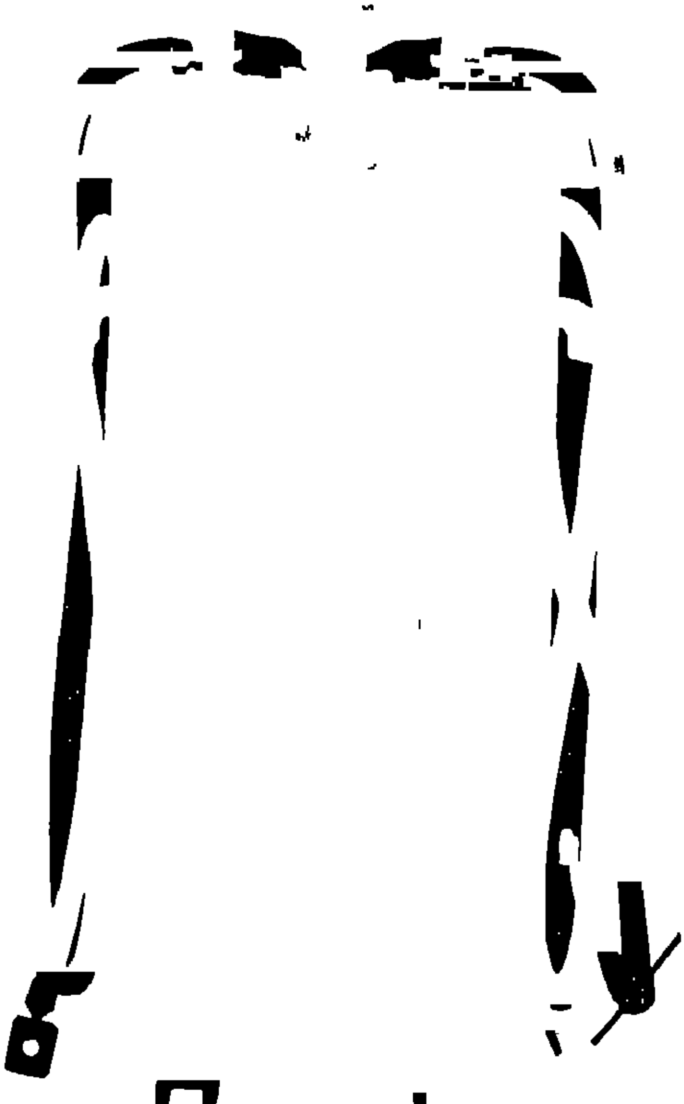


Fig. 186 — "Zeus" Arc Welder (Gibb Welding Machines Co.)

pressed voltage would send enough current through the transformer to amount to practically a short circuit on the line In the welding transformer, means are provided for shunting or

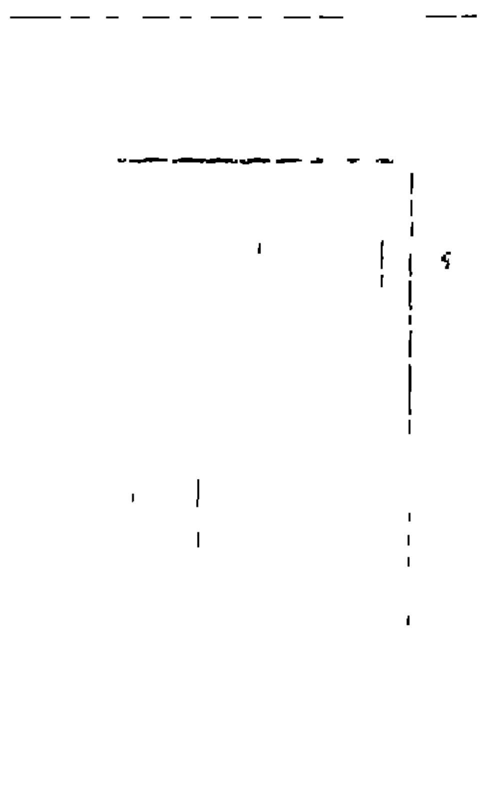


Fig. 187 — Portable Arc Welding Transformer
(Allan Mfg and Welding Co.)

by-passing some of this secondary flux so that it cannot react on the primary flux to an extent that would allow a primary current of very large value flow Various methods are used by different manufacturers to shunt or by-pass this flux set up by the secondary current.

Figure 186 shows a "Zeus" arc welding transformer made b the Gibb Welding Machines Company In this transforme magnetic leakage or shunting of the secondary flux is obtained b placing the primary and secondary coils some distance apar The transformer possesses the added feature of an adjustabl secondary coil The adjustment of the coil is made by means c a handwheel which raises or lowers the secondary coil and thereb

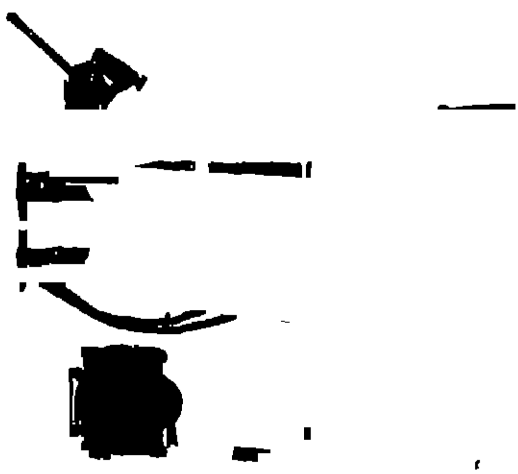


Fig. 188. — Transformer and Welder for Spot Welding (Federal Machine and Welding Co)

secures suitable leakage and regulation for the particular work to be done

Figure 187 shows a portable welding transformer made by the Allan Manufacturing and Welding Company This transformer can be obtained for operation on either single phase or poly phase circuits. The necessary magnetic leakage is obtained by the proper spacing of the primary and secondary coils. Taps are provided by which the secondary voltage may be varied for different kinds of work

Figure 188 shows a large spot welder made by the Federal Ma-

chine and Welder Company At the lower left-hand corner of the figure is shown the transformer which is used with this machine The transformer goes in the base of the machine The secondary voltage of the transformer is varied for different kinds of work by means of an autotransformer regulator The handle of the regulator is shown on the side of the machine

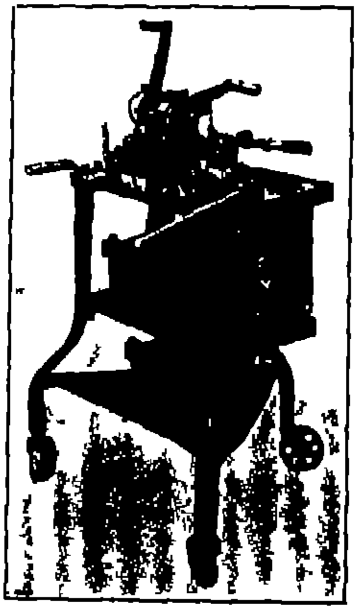


Fig. 189 —Portable Butt Welder
(Federal Machine and Welding Co.)

Figure 189 shows a portable butt welder made by the Federal Machine and Welder Company capable of welding up to [illegible]″ round wire

X-Ray Transformer An X-ray transformer has to supply current to a tube of the thermionic type As explained under the X ray tube, X rays are produced by electrons that move at a high velocity and impinge on a tungsten target in a highly evacuated tube The current supplied to the tube to set up this

flow of electrons must be unidirectional In X ray apparat up to about 60,000 volt capacity, the tube acts as a rectifi Above this voltage rating, a mechanical rectifier or a kenotr is used to rectify the current before it reaches the tube

In radiographic work, the load on the transformer that su plies the tube with current is extremely variable, being on f from about 1/120 of a second up to a maximum time of abo 30 seconds depending on the nature of the work For therај work, the transformer may have to operate continuously for a hour or more at full load

The current rating of the secondary is low, ranging from abo 5 milliamperes in the small machines to as high as 500 millian peres in the large machines. An average value of current fo radiographic work is around 20 milliamperes. For treatmen work, a machine may be called upon to deliver 50 milliampere continuously Some machines for this type of work are designe to deliver a secondary voltage of 200,000 volts or even mor Special equipment can be obtained to deliver 500 milliampere continuously

Transformers for X ray work must have good regulation Th shell type of construction offers a relatively small leakage sinc there is a path through the iron of low reluctance and th primary and secondary coils can be placed close together on th center leg of the core.

One well-known manufacturer places the primary on the cor and then places over this a thin cylinder of micanite Th secondary is wound in sections which are slipped over this cylin der These are separated by glass and paper washers. Th voltage between layers is kept low by this method of construc tion. There is a tendency for coils carrying currents to mov relative to each other, so it is necessary to fasten the coils to prevent slipping under load Movement of layers relative to each other, and movement of coils is prevented by taping the coils and by tying spacers between the washers with heavy ar- mature twine

The transformer is immersed in a tank which contains a high

grade transformer oil. This oil has a breakdown value around 25 to 30 kv, 30 kv per inch through the oil direct, and about 20 kv through the oil along the surface of an insulating medium such as a micanite tube, are average working values for the oil.

The cores of X ray transformers are worked to moderate den

Fig 190. — 230,000 Volt X-Ray Transformer (Kelly-Koett Mfg Co.)

sities On 60-cycle machines the densities run about 65,000 or 70,000 lines per square inch The current densities in the secondaries run about 400 circular mils per ampere, but due to the large number of turns of wire required and the desirability of keeping the coils small, a wire is selected that has the necessary strength for winding rather than one to meet definite current densities A wire that is practical to wind will generally be large enough to carry the small current required.

Figure 190 shows a high grade X ray transformer suitable f delivering 230,000 volts The cut shows the transformer remov from the tank and shows clearly the method of placing the coi on the core and the manner of carrying the high voltage termina through the cover of the tank.

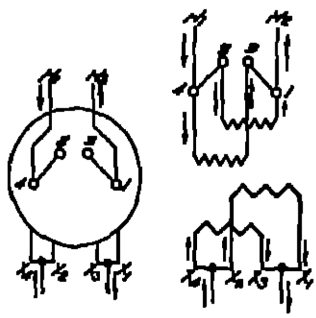


FIG. 191
ADDITIVE POLARITY

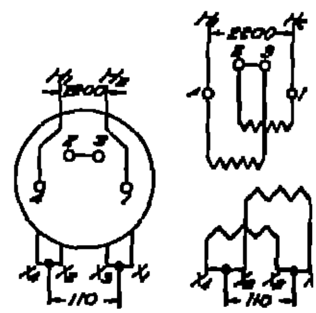


FIG. 193
2200-110

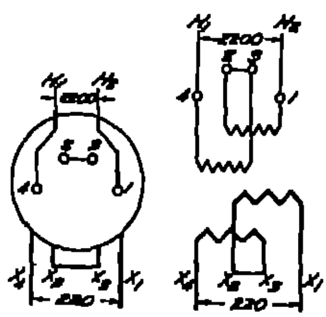


FIG. 192
2200-220

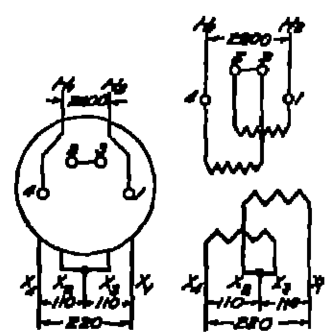


FIG. 194
2200 220 110

Transformer Connections. Figures 191 to Fig 205 show some of the methods of connecting transformers using for illustration a 1100/2200-110/220-volt Westinghouse distribution transformer with additive polarity

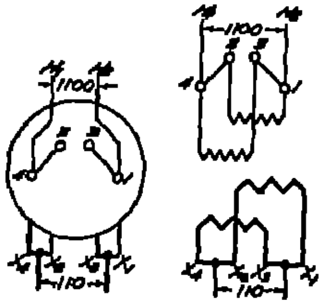


FIG 195
1100-110

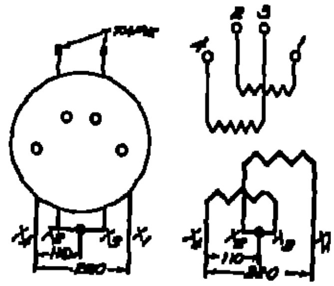


FIG 196
110-220

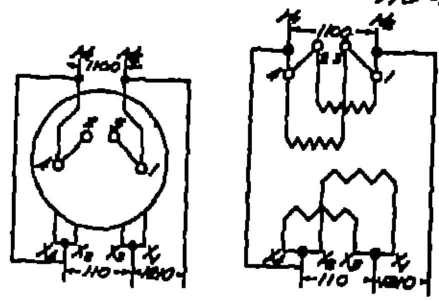


FIG 197
1100-1210

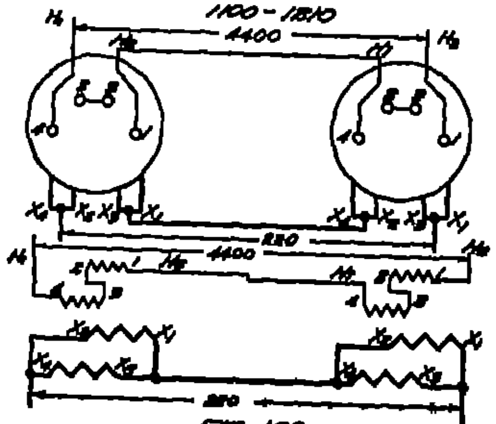


FIG 198
4400-220

Figure 191 shows the method of numbering primary and ondary leads There are two primary coils that are arrang that they may be connected either in parallel or series by m of metal straps These connections are made on a terr board. The secondary coils are brought out of the case separ and may be connected in parallel or series outside the cas soldering or by suitable connectors. Figure 191 shows fu the relative directions of primary and secondary currents v the transformer is of additive polarity

Figure 192 shows how to step down from 2200 volts to volts The primaries are in series and the secondaries ar series.

Figure 193 shows how to step down from 2200 volts to volts The primaries are in series and the secondaries ar parallel

Figure 194 shows how to connect so as to step down f 2200 to 110 and 220, three-wire. This connection is sim to the Edison three-wire circuit for direct current. Lamps r be connected from the middle wire to either outside wire 220-volt lamps or other apparatus connected across the out wires.

Figure 195 shows how to step down from 1100 volts to volts The primaries are in parallel and the secondaries are parallel.

Figure 196 shows how to step up from 110 to 220 volts, or s down from 220 volts to 110 volts. The primaries of the tra former should be taped when used with this connection becau high voltage will be induced in the primaries

Figure 197 shows how to use the transformer to "boost" fr 1100 to 1210 If H_1 were connected to X_1 instead of X_4 the tra former could be used to "buck" from 1100 to 990 volts.

Figure 198 shows how to step down from 4400 volts to 2 volts using two 1100/2200–110/220-volt transformers.

Figure 199 shows how to connect transformers for a three-pha primary line-voltage of 2200 and get a three-phase secondary lin voltage of 110. This is the delta-to-delta connection.

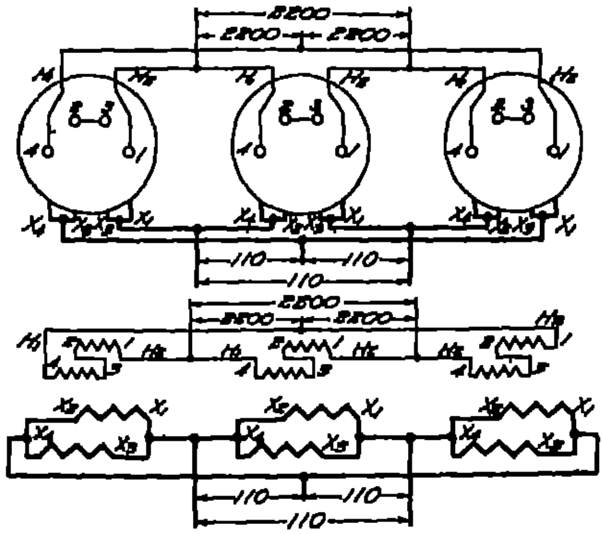


FIG. 199
2200 Δ - 110 Δ

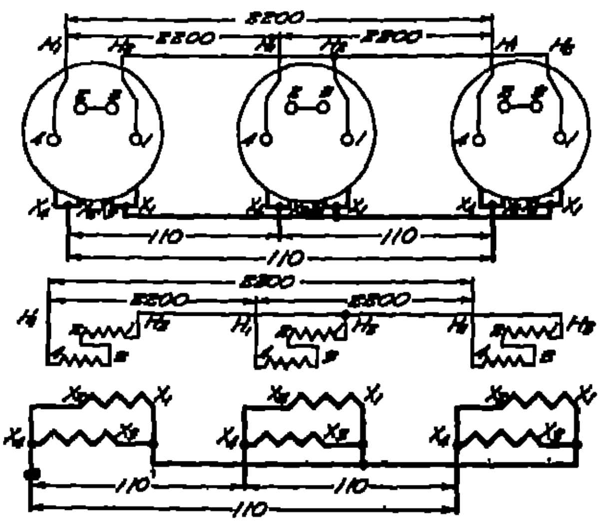


FIG. 200
2200 Y - 110 Y

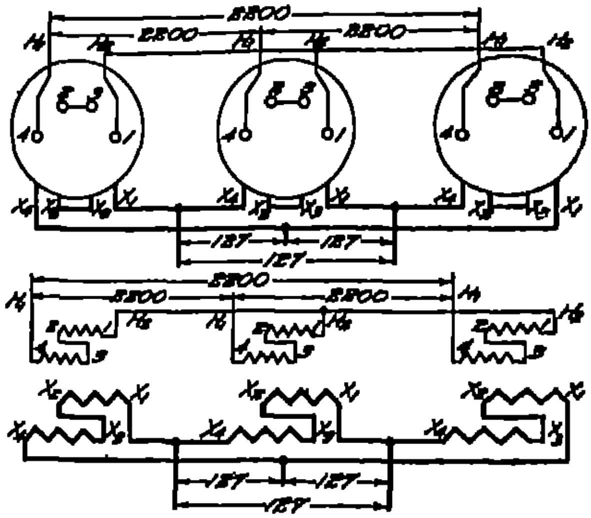


FIG. 201
2200 Y - 127 Δ

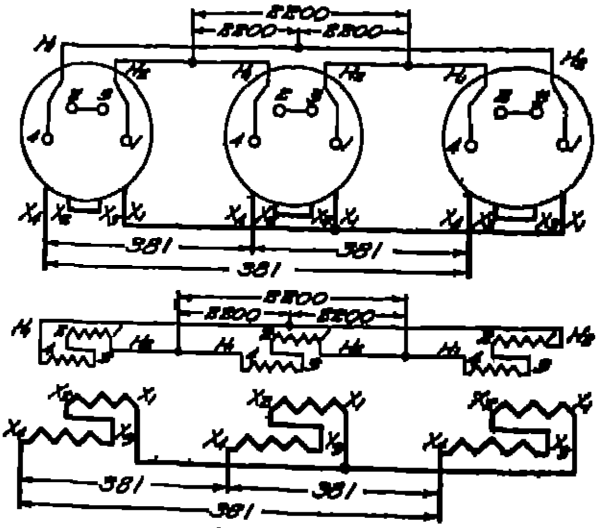


FIG. 201 A
2200 Δ - 381 Y

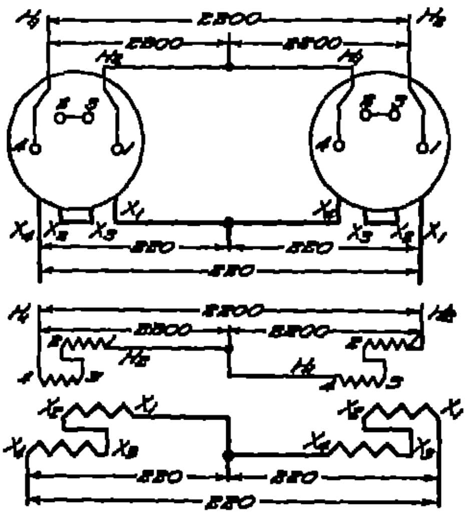


FIG 202
2200 TO 220
(OPEN DELTA OR V)

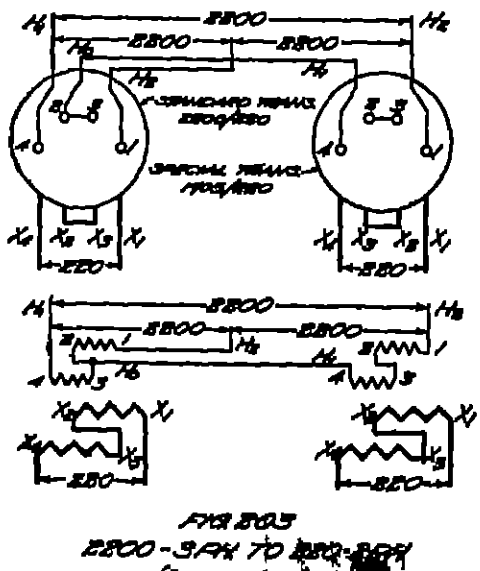


FIG 203
2200-3PH TO 220-2PH
(SCOTT OR T)

Figure 200 shows how to make the star to star connection.

Figure 201 shows the primaries connected star (Y) and th secondaries connected delta with the corresponding line-voltage

Figure 201(a) shows how to connect primaries Δ and second aries Y

Figure 202 gives a method of getting a three-phase secondar voltage from a three-phase primary line by means of two trans formers This is known as the open-delta connection

Figure 203 gives a method of changing from three-phase 220 volts to two-phase 220 volts, or vice versa, by using one trans former with a voltage ratio of 2200/220 and a special transforme with a voltage ratio of 1905 to 220 This connection is know as the Scott or "T" connection The principles on which i operates can be seen from Fig 204(a) AB represents the pri

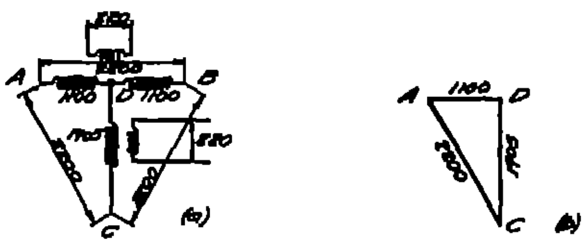


Fig. 204.—Scott or "T" Connection of Transformers for Changing from three-phase to two-phase.

mary of the 2200/220-volt transformer which has primark brought out so that the middle point can be reached One phas of the three-phase voltage is applied to AB A special trans former with ratio 1905 to 220 is connected at D and the othe two phases of the three-phase voltage are applied from A to C an C to B The transformer AB will get 2200 volts and step dow to 220 Examination of sketch (b) will show that CD will g only 1905 volts when 2200 are applied across AC and CB It wi be noticed that the ratio 1905 to 220 is nearly 9 to 1, so in a emergency a 10 1 and a 9 1 transformer may be used to chang from three-phase to two-phase.

Parallel Operation of Transformers. In order that transformers may operate successfully in parallel and deliver to the secondary buses an output equal to the combined output of the individual transformers, several conditions must be met. Among these are the polarities must be correct, the voltages must be the same, the relation of reactance ~~to reactance~~ to resistance should be the same and the regulation must be the same

Two identical transformers will operate successfully in parallel if two similar primary leads are connected to one bus and the other two primary leads connected to the other bus. Two similar secondary leads are connected to one secondary bus and the other two leads to the other bus

In case the transformers are of different makes it is possible that the polarities are different In this case they should be tested out for polarity Connect as shown by the full lines in Fig 205 Apply voltage to the primaries and put a voltmeter across the secondary terminals X_1 and X'_1 If the polarities are alike, the voltmeter will read zero if opposite the voltmeter will read the sum of the two secondary voltages. This will be clear from a study of the arrows which show the instantaneous direction of primary and secondary voltage.

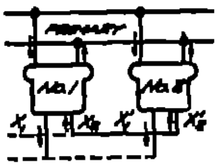


Fig. 205 — Connections for Testing Polarity Previously to Paralleling.

Another short test can be made immediately following this first test by connecting an ammeter or fuse across X_1, X'_1 after it has been determined that the polarites are alike. A reading on the ammeter or blowing of the fuse will indicate that the voltages are not exactly alike, although the difference might not be noticeable on the ordinary high-reading voltmeter If the secondary voltages are found to be alike, the transformers' secondaries may be paralleled by connecting together X_1 and X'_1 as shown dotted

The transformers may, or may not, operate successfully when connected together as described and load taken off at X_1, X'_1 and X_2, X'_2. If the regulation at full load of transformer #1 is 2% and transformer #2 is 2 2%, then if the rated secondary voltage is 100,

#1 will have a full-load voltage if 110 − (110 × .02) = 107 8. #2 will have a full-load voltage if 110 − (110 × .022) = 107.58. The difference in voltage of .22 volts will cause current to circulate in the windings of the two transformers. This current will be fairly large. Hence, current equal to the combined current-rating of the two transformers cannot be taken off the secondary buses without overheating the transformers, due to the current that circulates in the two transformers and heats them without helping supply the secondary load.

Suppose the ratio of reactance to resistance is not the same in the two transformers and we attempt to load them. The condition is shown schematically by Fig. 206 when #1 is shown with a large reactance compared to its resistance and #2 with a small reactance.

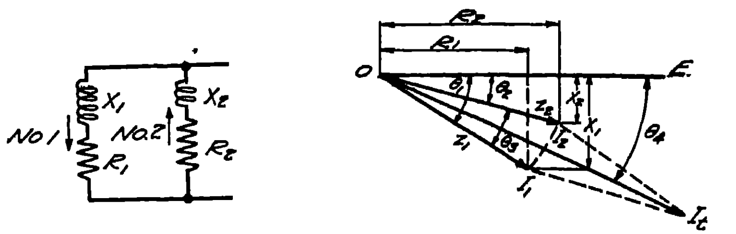


Fig. 206 —Transformers with Unlike Ratios of Reactance and Resistance Connected in Parallel.

The total impedances of the two are alike and the currents will be alike but they will be out of phase with each other by the angle θ_3. The lines Z_1 and Z_2 may represent currents as well as impedances by using a suitable scale. Hence, OI_t is the total or line current. OI_t is less than the actual arithmetical sum of the currents in the two transformers and it lags the voltage by an angle θ_4 which is different from the angles of lag of either of the two transformers.

From the above we see that, for successful operation of two transformers in parallel, their secondaries must be connected to give like polarities, their voltages should be the same, and their regulation and ratio of resistance to reactance should be the same.

If these conditions are not met the transformer will probably overheat when full load is taken from the secondary buses.

All-Day Efficiency of a Transformer. The ratio of the watthours output per day to the watthours input per day, expressed as a per cent, is called the all-day efficiency of a transformer.

Taking a lighting transformer as an illustration, the core loss takes place during the entire 24 hours of the day, because the transformer is on the line continuously, ready to give service. We can consider that the copper loss occurs only when the transformer is loaded because the small copper loss in the primary, due to the exciting current, is negligible.

The all-day efficiency is then,

$$\frac{\text{watthours output per day}}{\text{watthours input per day}} \times 100$$

or

$$\frac{\text{secondary load} \times \text{hours used}}{(\text{secondary load} + \text{total copper loss}) \times \text{hrs. loaded} + \text{core loss} \times 24} \tag{56}$$

As a numerical example, consider a 10 kv-a. lighting transformer with core loss of 77 watts and copper loss of 305 watts. The transformer is run fully loaded 5 hours per day and is without load 19 hours but continuously on the line. Its all-day efficiency is,

$$\frac{10{,}000 \times 5}{(10{,}000 + 305)5 + 77 \times 24} = 93.7\%$$

Transformer Transients. A transient in a circuit may be considered as a passing condition of voltage or current that takes place in a circuit, between two steady conditions in a circuit; for instance, the rush of current when an oil switch is closed until the current builds up to its steady or normal value, or the condition that exists between the time that a lightning arrester discharges until the circuit again becomes normal.

Transformers, when switched on a line, may cause dangerous transients due to a resonant effect they cause. A transmission line has capacity since it consists of conductors separated by a

dielectric. It also has a certain amount of inductance. A transformer with open-circuited secondary has a very high self-inductance and this combined with the line-inductance may be sufficient to establish resonance by satisfying the condition,

$$2\pi fL = \frac{1}{2\pi fC}$$

The line-capacity reactance and the inductive reactance of the line and transformer may be considered as in series with each other, so if a resonant condition should be established by the throwing off of the load of a transformer or the sudden switching in of an unloaded transformer, an enormous current might flow and voltages be induced far beyond the strength of the line insulation.

The following example will illustrate The resistance of a 120-mile #0000 three-phase line is approximately 32 ohms and its inductive reactance at 60 cycles, when the wires are 14 feet apart, is 95 ohms. Such a line, when operated at 100,000 volts between wires, or $\frac{100{,}000}{\sqrt{3}} = 80{,}800$ volts between a line and neutral, has a charging current, due to its capacity, of 42 amperes.

Its capacity reactance $X_0 = \frac{E}{I} = \frac{100{,}000}{42} = 2380$ ohms. Assuming that a transformer which had a normal exciting current of 3.5 amperes suddenly lost its load, its reactance would jump to $X_L = \frac{8080}{3.5} = \cancel{2285}$ 2308 ohms. The total reactance in the circuit would then be the line reactance of 95 ohms and the transformer reactance of ~~2285~~ 2308 ohms or a total of ~~2380~~ 2403. This inductive transformer and line reactance would exactly balance the capacity reactance of 2380 ohms and establish a condition of resonance in the circuit. The 80,800 volts would then try to send a current through the circuit of,

$$I = \frac{80{,}800}{\sqrt{32^2 + (2380 - 2380)^2}} = \frac{80{,}800}{32} = 2525 \text{ amperes}$$

and this current would try to build up voltages across the capacity reactance and inductive reactance

$$2525 \times 2380 = 6,009,500 \text{ volts}$$

which, of course, would destroy the circuit.

In a line, the proportions of capacity and inductive reactance are of such magnitudes that the switching in of an unloaded transformer may introduce a condition of resonance that will wreck the circuit

PROBLEMS

1. What will be the voltage induced in a coil with 100 turns if the flux is 10,000,000 lines and the frequency 60 cycles?

2 What should be the area of a core for a transformer with 200 turns of wire, if the density is 5000 lines per sq cm, the frequency 60 cycles per second, and the voltage 110?

3. What will be the number of turns required for a transformer with a core 2″ × 2″ worked to a density of 5000 lines per sq cm? The voltage is 110 and the frequency is 60

4 What will be the density in a core 3″ × 2″ wound with 200 turns of wire and connected to a 230-volt 60-cycle circuit?

5. What will be the density in the core of Prob 4 if the same voltage is used but the frequency is 25 cycles?

Solve problems 6 to 15 by using the curves of Fig 149

6. Find the eddy-current loss in 100 cubic centimeters of iron, 14 mils thick (063 cm) at 100 cycles The density is 12,000 lines per sq cm. Use K = 1 65.

7. Find the eddy-current loss in 100 cu cm of iron, 14 mils thick at 60 cycles The density is 12,000, K = 1 65

8. Find the eddy-current loss in 100 cu cm of iron, 14 mils thick at 25 cycles. The density is 12,000, K = 1 65

9. What will be the eddy-current loss in Prob 7 if the density is reduced to 10,000 lines per sq cm., other conditions being the same?

10. What will be the eddy-current loss in Prob. 7 if the density and frequency are kept the same but the thickness of the sheets made .05 cm. instead of .036 cm.?

11. What is the effect on the eddy-current loss of,

(a) decreasing the frequency?

(b) decreasing the density?

(c) decreasing the thickness of the plates?

(The density to be kept constant.)

12 What will be the hysteresis loss in the iron of Prob. 6? K = .002

13 What will be the hysteresis loss in the iron of Prob. 7? K = 002

14 What will be the hysteresis loss in the iron of Prob. 8? K = .002

15. What is the effect on the hysteresis loss of,

(*a*) decreasing the frequency?

(*b*) decreasing the density?

Does the thickness of the material make any difference in the hysteresis loss?

16 The resistance of the primary coil of a transformer is 12 ohms and the full-load current in the coil is 78 amperes. What will be the copper loss in the coil at full load?

17 The resistance of the primary coil of a transformer is 4.2 ohms at 25 C and the resistance of the secondary is 0 066 ohms at the same temperature. The full-load current in the primary is 3 4 amperes and the full-load secondary current is 34 amperes What is the total copper loss ?

18. The core loss (sum of hysteresis loss and eddy-current loss) in a transformer was 43 watts The copper loss was 82 watts. If the transformer is a 5 kv-a , what is its efficiency?

19 The core loss in a transformer was 239 watts and the copper loss 520 watts. The transformer is a 50 kv-a. What is its efficiency?

20 In a certain transformer rated 7.5 kv-a , the core loss was 65 watts, the full-load current in the primary 3 4 amperes, the full-load secondary current 34, the primary resistance 4 5 ohms and the secondary resistance 065 ohms What will be its efficiency at full load and at half load?

21 If a 50 kv-a transformer has an efficiency of 98.48% at full load, what are the losses?

CHAPTER IX

ASYNCHRONOUS MOTORS

Principle of the Polyphase Induction Motor. The induction motor depends for its operation upon transformer action between currents in the stationary part called the stator and the revolving part called the rotor. In the polyphase motor the field set up by the stator "revolves" and induces currents in the rotor. These currents react in such a way on the stator field, through the fields that they set up, that the rotor turns in the direction that the field is turning.

Application of Principle to a Two-Phase Motor. The principle of the revolving field may be shown clearly by the study of a two-phase motor which is shown diagrammatically by Fig. 207. N_1 and S_1 are the north and south poles of one phase and N_2 and S_2 the poles of the other phase. For the purpose of showing the operation of the motor, direct current will be used but it will be made to vary in each phase through a cycle by means of variable rheostats and reversing switches so that conditions similar to those in a stator, supplied by two-phase alternating current, will be established.

For example, suppose that the rheostat is adjusted in phase A so that the current has a value of 10 amperes and that the switch is open in phase B so that the current in phase B is zero. The flux will pass directly from N_1 to S_1. This setting of the rheostats and switches will give the condition in a two-phase circuit when phase A is passing through its maximum value and phase B is passing through its zero value, as shown by the curves at the lower part of sketch (a). A quarter of a cycle or 45 degrees later, the rheostats are adjusted so that currents in phases A and B are each 7.07 amps. The field will stand as shown at (b). Another quarter cycle later, phase A will be zero and phase B a

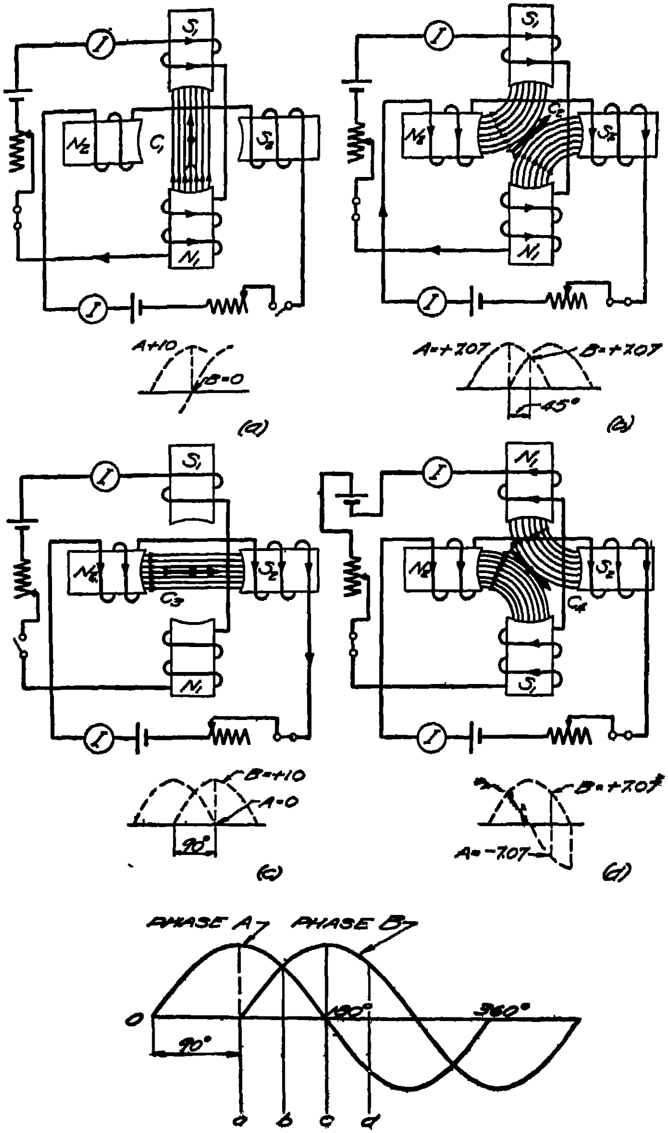


Fig. 207.—Diagram Showing Principle of a Revolving Field.

maximum value of 10 amperes giving a field as shown at (c). Another quarter-cycle later, phase A will be 7.07 amperes in value but reversed in direction but phase B will be 7.07 amperes in the same direction as before. This condition is shown at (d).

A compass needle placed in the polar space will take the positions C_1, C_2, C_3 and C_4. If the currents are carried through a complete cycle of values the needle will make a complete revolution. Thus the magnetic field "revolves" as the field currents pass through their cycle of values.

In an induction motor the needle is replaced by a core on the circumference of which are copper bars placed in slots in the core parallel to the shaft. These bars are short-circuited at the ends making an arrangement of the bars and end rings similar in construction to the wheel of a squirrel cage. The revolving part of the motor is called the rotor and, in this type of machine, a "squirrel-cage" rotor. When the field revolves it sweeps past the bars in the rotor, cuts them and induces currents in them. These currents react on the field in such a way that the rotor turns in the direction that the field is moving. Figure 208 shows how this occurs. Let NS be the direction of the field and let it be moving counter-clockwise and cutting the conductor C. The relative motion is the same as if the field were stationary and the conductor moved clockwise. By applying the three-finger rule for a generator we see that the direction of induced current is up from the paper. Next apply the motor rule (three-finger rule using left hand) and it will be seen that the conductor tends to turn counter-clockwise or in the same direction that the field is turning. The conductor does not move as fast as the field, however, for if it did, there would be no cutting action and no current. The amount it drops behind the field depends upon how much load it has upon it, either in the form of losses or in mechanical load applied to the belt of the motor. The lag in the speed of the

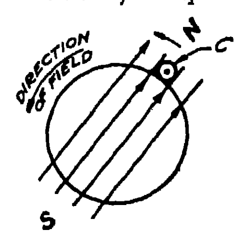


Fig 208. — Diagram Showing that a Conductor on the Rotor Tends to Move in the Same Direction as the Field.

motor behind the speed of the field is known as the "slip" of the motor.

Slip. The amount that the motor lags behind the field in the stator is expressed as a per cent of the speed of the field. The field revolves at what is known as synchronous speed, which is the speed obtained by the generator formula in $V = \frac{60f}{P}$ where V is the revolutions per minute, f the frequency and P the number of pairs of poles. Synchronous speed may be thought of as the speed at which an alternator, with the same number of poles as the motor, would revolve to give the frequency in consideration.

Example; A 4-pole motor runs at 1750 r. p. m. What is its slip? The alternator with 4 poles runs at, $V = \frac{60f}{P} = \frac{60 \times 60}{2} = 1800$ r. p. m. Synchronous speed is 1800. If the motor runs at 1750 its slip is, $\frac{1800 - 1750}{1800} = \frac{50}{1800} = .028 = 2.8\%$

Starting Polyphase Motors. The squirrel-cage motor will draw a very large current at low power factor if thrown directly across the line at starting. This is objectionable because it upsets the

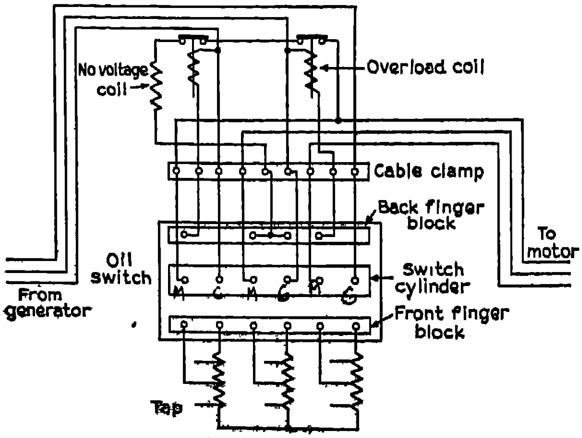


Fig. 209.—Connections of Three Phase Starting Compensator. (General Electric Co.)

system causing fluctuation of voltage and change in power factor. The large current in the motor leads makes it necessary to use extra large wires to the motor or get an excessive voltage drop in the motor line and thus less power in starting

A common method of starting motors of 5 horse power or larger is to use autotransformers on each of the phases and step down the voltage somewhat in starting, then, after the motor has speeded up, throw the motor directly in the line. Such an apparatus with autotransformer is called a starting compensator Figure 209 gives the connections of a General Electric compensator connected to a three-phase motor.

Fig 210. — Type H-1852 Starter is equipped with Bradley units (Graphite compression resistors) which provide stepless acceleration and also prevent severe current inrushes.

Figure 210 shows another kind of starter recommended by the Allen Bradley Company which employs resistance in the motor circuit in starting. The starter is known as a compression-resistance starter and provides variable resistance in each leg of the motor circuit. The resistance units consist of a large number of specially treated graphite discs stacked up in a steel tube with insulated lining. One of these units is shown by Fig. 211. The discs offer a fairly high resistance when loosely stacked in the tube, but when compressed, the resistance of the stack or column of discs is very much reduced. The discs are compressed by the operating handle of the starter.

As the motor speeds up, the resistance is gradually cut out, and after the motor is up to speed, the resistance units are short-circuited.

The advantages claimed for this starter are smooth starting conditions and no opening of the circuit from starting condition to running condition.

Wound Rotors. The discussion thus far has concerned itself with the squirrel-cage type of rotor or that which has a large number of bars imbedded in it close to the circumference and parallel to the shaft. The bars are short-circuited at their ends by heavy rings thus making closed circuits in which currents induced by the alternating-current field can flow. The resistance of this type of rotor is very low, and large currents flow when the rotor is stationary or just beginning to turn. It would seem that the reaction between these large currents and the field would give a large starting torque but it has been found that a rotor with a higher resistance will give a larger starting torque The reason that the ordinary low-resistance, squirrel-cage type of motor does not give high starting torque is that the ratio of its reactance, which is high, to its resistance, which is low, is a large number. That is, $\frac{X}{R}$ is large. This means that the angle of lag of the current is large and the power factor low at starting. Another way of thinking of it is, to consider that currents which should produce a positive torque, lag or do not reach their maximum until the rotor has turned so that they come under a pole that produces a counter or negative torque, and so a less effective torque.

Fig 211. — The Bradley unit consists of an insulated steel tube, filled with specially treated graphite discs

A low-resistance rotor gives a good speed regulation, and a high resistance rotor good starting torque. Where good speed regulation is not essential, high starting torque can be obtained by making the squirrel-cage type of rotor with a higher resistance than would be obtained by ordinary, heavy, short-circuited bars.

From the above it will appear that a motor could give a large

starting torque and good speed regulation if its resistance were large at starting and small at running. This condition can be obtained by winding the rotor like the armature of a revolving-armature type of alternator instead of using the squirrel-cage construction. Such a rotor is known as a "wound rotor" It has slip rings and brushes. Variable resistances are connected between the brushes The resistance is all cut in for starting and all cut out for running, thus giving high starting torque and good speed regulation. Figure 212 shows the electrical connections of a 3-phase Y-connected 4-pole motor with wound rotor.

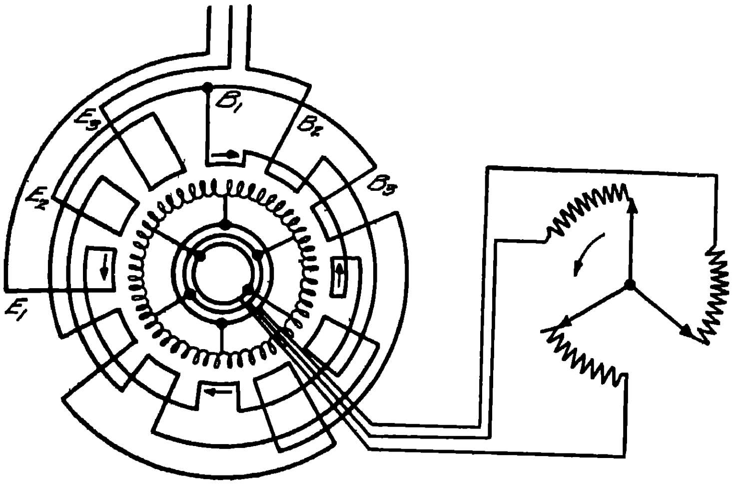


Fig 212. —3-phase Y-connected Motor with Wound Rotor Connected to Starting Resistance.

The Single-Phase Induction Motor. A polyphase motor, if once started, will operate on one phase if the other phases be disconnected. A single-phase motor is constructed with a main winding similar to one of the windings of a two-phase motor, and with a starting winding placed 90 electrical degrees from the main winding. The starting winding may be cut out after the motor has been started. When operating single phase, the single-phase motor depends for its action on an alternating field, and the fact

that the currents in the rotor lag greatly behind the E. M. F.'s that produce them. For the purpose of studying the operation of the motor, the angle of lag may be considered nearly 90 degrees.

While the complete theory of the operation of the single-phase motor is complicated, Figs 213 to 216 will give an idea of the principle of operation

Let Fig 213 be the stator and rotor of an induction motor and,

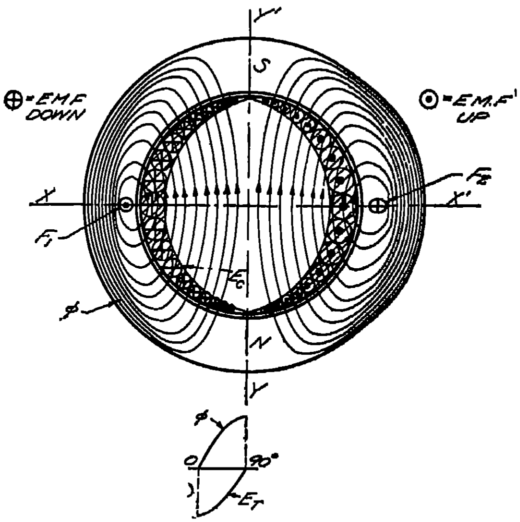


Fig. 213. Rising Flux, Rotor Stationary.

for the sake of simplicity, let the field-winding consist of a single turn of wire, passing down through the rotor at F_2 and up at F_1. If current be made to rise from zero to a maximum in the field-windings in the direction indicated, lines of force will encircle the conductors of the field-winding as shown. These lines will start at the conductor and expand outward as the current rises. Applying the three-finger rule, the E. M. F.'s induced by transformer action in the rotor conductors will be as indicated by E_0 in the shaded portion of the rotor. The shaded portion indicates graphically the magnitude of these E. M. F.'s. If we apply

the three-finger rule for the motor to different parts of the rotor we see there is no tendency to turn. That is, the forces are balanced and the motor may be thought of as a transformer with short-circuited secondary.

If now the motor has been started by some external source and is turning at some speed near synchronism, the conditions are as in Fig. 214. The rotor conductors will have the E. M. F.'s due to the transformer action just described and will have other E M F.'s due to the fact that the conductors move across the field The E. M. F.'s due to the motion of the conductors are called "speed E. M. F 's" These "speed E. M. F 's" will be greatest in the conductors that are near the vertical line YY′. They will be greater in quadrants YX and Y′X′, Fig 214, than in

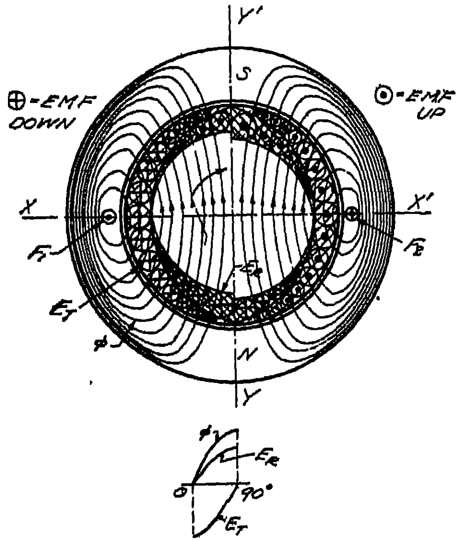


Fig. 214. — Rising Flux, Rotor Turning Clockwise at Near Synchronism.

YX′ and XY′ because the conductors are moving against the lines in one case and with the lines in the other, so the cutting action is greater in one case than the other. The double-shaded portions of the diagram indicate these E. M. F.'s and the sym-

bols their directions. Due to the fact that there is considerable inductance in the rotor and very little resistance, the currents lag about 90 degrees behind the E. M. F.'s so that the currents will not have risen in directions indicated by symbols until 90 degrees later, or until the rotor has turned 90 degrees, as shown by Fig. 215. It will be noticed that the "transformer" and

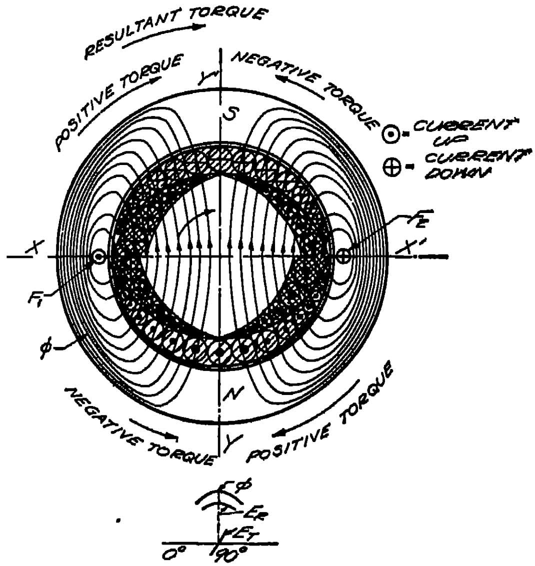


Fig. 215 —Maximum Flux, Rotor 90° Ahead of Position in Fig. 214.

"speed" currents are alike in sign in quadrant XY′ and X′Y and unlike in Y′X′ and YX. That is, we should expect a large torque in XY′ and X′Y and a small torque in Y′X′ and YX. Application of the three-finger motor rule shows further that the large torque is positive or in the direction that the motor is turning and the small torque is negative or in the opposite direction. The positive torque being greater, the rotor continues in turn. The same analysis may be applied to the other three quarters of the cycle showing that the resultant torque is positive.

If, instead of starting the motor clockwise, it had been started

counter-clockwise, it would have continued to turn counter-clockwise, as the analysis just given would show.

Starting Single-Phase Motors. Referring again to the statement that a polyphase motor when once started will continue to operate if one phase be disconnected, it will follow that if a single-phase current could be split up into a two-phase current a regular two-phase motor could be started on the two windings and then run on one if desired.

A single-phase current can be "split" by inserting inductance and resistance in a branch taken from the main circuit as in Fig 216. The current in the branch circuit consisting of extra inductance

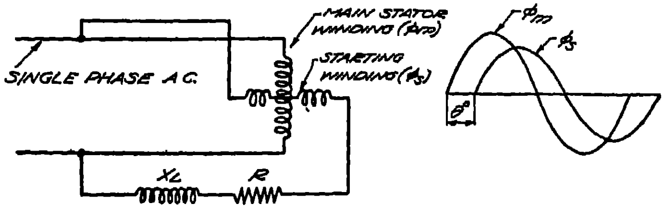


Fig 216 — Method of Splitting One-phase into Two Phases

X_L and the starting winding W_s will lag behind the current in the line and the main winding W_m and produce a condition in the motor similar to that in a two-phase motor, for starting The starting winding may be disconnected after the motor is up to speed.

Motor and Generator Windings are Similar. The windings described for use on the armatures of revolving-field type alternators are the same as those used on the stators of induction motors. Figure 217 is an experimental machine that may be used either as a motor or a generator. The machine was originally a three-phase four-pole squirrel-cage induction motor. In order that various connections could be readily tried, the terminals of each of the 48 coils on the stator were brought to binding posts on the fibre ring shown at the end of the machine As it was desirable that the same machine could be used as a generator as well as a motor, a four-pole revolving field was made and wound with enough turns of wire to give sufficient resistance so

that it could be connected to a 110-volt direct-current circuit. Current is carried to this field through slip rings mounted on the end of a hollow shaft. When the squirrel-cage rotor is removed and the field substituted, the machine becomes a revolving-field-type alternator that is very suitable for experimental work.

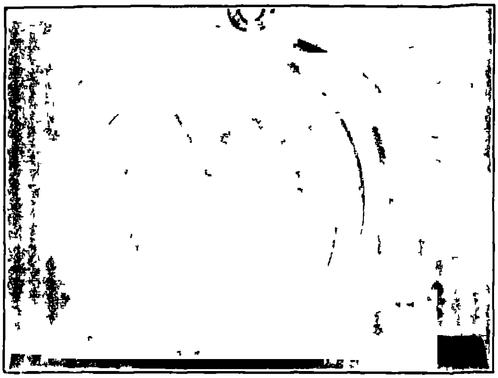


Fig. 217. — Experimental Alternating-Current Machine. (Buffalo Technical High School)

The Circle Diagram for a Polyphase Induction Motor. If tests be made on an induction motor with different loads, it will be found that the motor behaves like a circuit containing a constant inductive reactance and a variable resistance That is, as the load is varied, the locus of the current is a semicircle.

This fact is made use of in what is known as the circle diagram for an induction motor. The circle diagram enables one to determine the complete performance of an induction motor from a few simple readings which can be taken without putting full load on the motor.

To get a general idea of the circle diagram, assume that several readings of volts, amperes, and power factor are taken on an induction motor, first running with no load, then with light, medium and heavy loads. For the purpose of constructing the diagram it is desirable to refer the current readings to the line voltage. The current for each load can be resolved into two

components, a power component and a reactive component. In any A. C. circuit, the power component is in phase with the line voltage and equals I × Cos ϕ = I × P F and the reactive component equals I × Sin ϕ and is 90° out of phase with the line voltage.

In Fig. 218 draw OX as a base line and draw OV at 90° from

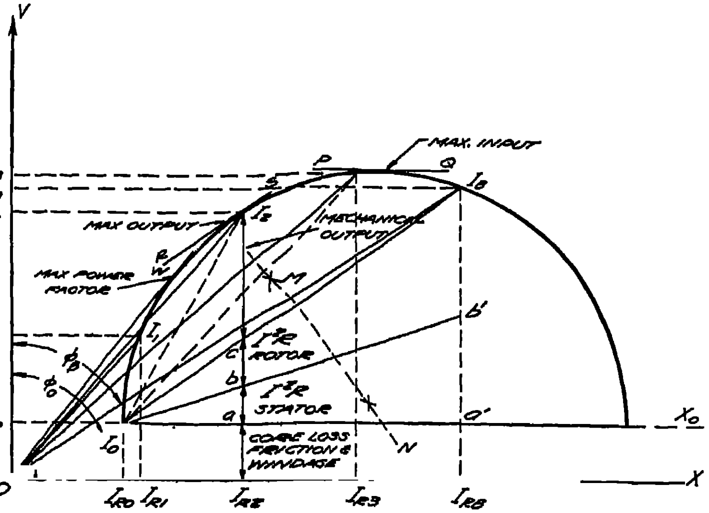


Fig. 218 — Circle Diagram for Polyphase Induction Motor.

OX. Let OV equal the line voltage to scale. OI_0, OI_1, OI_2 and OI_3 are the currents with the motor running, first with no load, then with light, medium and heavy loads. OI_{e0}, OI_{e1}, OI_{e2} and OI_{e3} are the energy components, obtained by multiplying OI_0, etc., by the respective power factors (Cos ϕ) taken during test, and I_{R0}, etc., are the reactive components obtained by multiplying OI_0, etc., by their respective reactive factors (Sin ϕ). If the readings are carefully taken, I_0, I_1, I_2 and I_3 will be found to lie on a curve that is practically an arc of a circle. Hence, if suitable readings be taken so that a semicircle can be drawn to scale, currents for any load within the capacity of the motor can be read off. A few additions to the diagram thus drawn, will give the complete performance of the motor.

I_0 is obtained with the motor running light, and is the starting point of the semicircle. I_0 represents the exciting current and I_{e0} and I_{R0} the energy and reactive components. The diagram is similar to the transformer diagram in this respect, except that I_0 is much larger in a motor than in a transformer. I_{e0} when multiplied by the voltage OV represents the power used up in the motor, since there is no useful mechanical output. This power includes core loss, friction and windage, and a very small copper loss that is practically negligible Since these losses are practically independent of the load, if a horizontal line I_0X_0 be drawn through I_0 parallel to OX, the distance from this line to OX will represent the core loss, friction and windage for any load, if the copper loss be neglected.

If, now, we consider all the different ways in which the power supplied to the motor for a given load as OI_2 is used, we shall find them to be mechanical output, rotor I^2R losses, stator I^2R losses and the friction, windage and core loss previously mentioned So if we could properly divide a line such as $I_{R2}I_2$, and draw lines from these points of division b and c to I_0, the intersection of these lines with any load line as I_B I_{RB} would give the proportional amounts of power used for mechanical output and total copper losses for the load I_B. We draw these lines to I_0 instead of O because we have considered the copper losses negligible up to the point I_0 and the power output to be zero.

In a motor with a wound rotor we can measure both R_S, the stator resistance, and R_R the rotor resistance. R_R must be referred to the stator resistance by multiplying by the ratio of stator to rotor turns In a squirrel-cage motor we cannot measure the rotor resistance directly but if we block the rotor and measure the input of the motor we can calculate it. A wattmeter will give the losses. We already know the losses included between the horizontal line I_0X_0 and OX from the running-light test, so the difference between $I_{RB}I_B$ and $I_{RB}a'$ is the sum of the copper losses in the stator and rotor. Of these, a′b′ represents those in the stator. So the difference between $a'I_B$ and a′b′

equals the rotor loss. The line OI_B represents the line current with the rotor blocked Similarly, OI_2 the line current for a load I_2, etc. If we draw a horizontal line PO tangent to the circle and parallel with OX, the point of tangency will evidently be at the point of maximum input to the motor

Since the mechanical output of the motor is shown by a line such as I_2c, maximum output will occur when I_2c is maximum It will readily be seen by trial, that this point of maximum output can be obtained by drawing a tangent RS to the circle at such a point that it will be parallel to I_0I_B. The maximum factor power will occur when a line such as OI_2 is tangent to the circle or at the point W. The output is I_2c, the input is I_2I_{R2}, the efficiency is $\frac{I_2c}{I_2I_{R2}}$. I_2b is the power given to the rotor since it is the total input less the core loss, friction and windage and stator copper loss. Then, since the output equals $2\pi nT$ where,

$$n = \text{r p.m at synchronism,}$$
$$T = \text{torque in lb. ft.}$$
$$T = \frac{\text{output}}{2\pi n} = \frac{I_2c}{2\pi n}$$
$$\text{The slip} = \frac{\text{rotor loss}}{\text{rotor input}} = \frac{bc}{bI_2}$$

This will be evident from Fig 219 which shows graphically, but not in true proportion, the flow of power through an induction motor. Of the total power that comes into the motor from the line, the watts lost in I^2R in the stator do not reach the rotor. It is assumed in the sketch that one-half of the total core loss is in the stator, so that one-half the total core-loss watts do not reach the rotor. For a given mechanical output, if the total loss in the rotor be increased as shown dotted, the whole input must be increased. This can only occur by the motor slipping more,

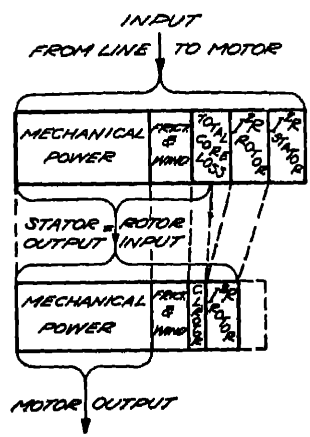


Fig. 219. — Graphical Representation of Flow of Power Through an Induction Motor.

or the slip depends on the rotor loss. That is,

$$\text{Slip} = \frac{\text{rotor loss}}{\text{rotor input}}.$$

A study of the circle diagram will show that, if a perpendicular MN bisecting I_0I_B be drawn to I_0X_0, it will intersect the line I_0X_0 at the center of the semicircle.

The readings for the construction of the circle diagram are:

1. No load volts, amperes and watts per phase at normal voltage and frequency. If the rotor is a wound rotor, its voltage should be measured so that the ratio of rotor to stator may be calculated.

2. Volts, amperes and watts with rotor blocked and current held at about full-load value. Reduced voltage must be used. If the rotor is a wound rotor it should be short-circuited for this part of the test. The motor would draw a very large current and overheat if full voltage were applied with the rotor blocked and short-circuited A reduced voltage sufficient to give about full-load current is used for this part of the test. The current that will flow at full voltage is practically in the same ratio to the current at reduced voltage as the full voltage is to the reduced voltage. The power at full voltage is to the power at reduced voltage practically as the squares of the full voltage and reduced voltage.

3. Resistance of the stator and rotor. Effective resistance should be used When the rotor is of the squirrel-cage type, the wattmeter reading divided by the current squared will give the effective resistance of stator and rotor together.

Series A. C. Motor. Application of the three-finger motor rule to a direct-current series motor will show that it is theoretically capable of running on alternating current. When the field current reverses, the armature current reverses also, so the torque is in one direction. The ordinary series motor, however, behaves badly on an alternating-current circuit, in that it heats excessively, sparks severely, and operates at low power factor. A motor with characteristics similar to a direct-current series motor is desirable in railway and other work where alternating current is

used By modifications in design, such a motor has been developed that operates very successfully.

The modifications to overcome the defects of the ordinary direct-current motor and give a satisfactory A C. motor are briefly as follows: Considerable of the heating that occurs when a direct motor is operated on alternating current takes place in the iron of the magnet poles, cores and yokes. This is due largely to the eddy currents set up in these parts by the reversals of the flux. In an alternating-current series motor, eddy currents are very largely eliminated by making all magnetic parts laminated

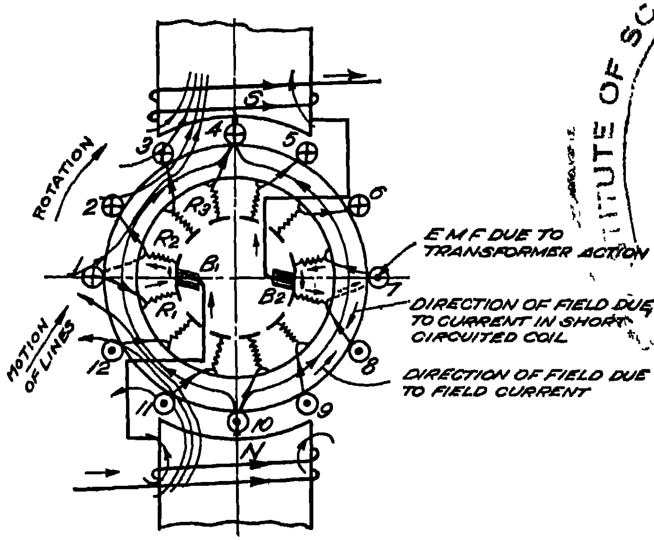


Fig. 220 — Elementary Series A. C. Motor.

Consideration of Fig. 220 will show that while a coil is being commutated it is in an alternating field and therefore has an electromotive force induced in it by transformer action. This electromotive force will cause current to flow in the coil that will heat the coil while under the brush and cause severe sparking as the coil leaves the brush. The generation of this electromotive force and current is as follows: Assume that the field is rising from zero

to a maximum in a direction from N to S. The flux from the lower pole is expanding upward to the right under conductor 1 and upward to the left under conductor 7. An electromotive force will be induced in 1 that acts downward and one in 7 that acts upward. These electromotive forces will cause large currents to flow while the coil is short-circuited by the brush. As the coil leaves the brush, a severe arc will form because a large current is quickly broken in a coil with considerable inductance The currents in the short-circuited coils are considerably reduced in commercial machines by building them with "resistance leads" R_1, R_2, R_3, etc., between the armature coils and the commutator segments. Inspection of the sketch will show that two of these leads are in series to oppose the short-circuit current but the two are in parallel in the load-current circuit. They offer but one-fourth the resistance to the load current that they offer to the short-circuit current

Further study of Fig 220 will show that the flux set up by the current flowing in the coil short-circuited by the brush will set up a flux that will oppose the main flux. This will not affect the sparking but is undesirable because it will cut down the effective flux of the motor and require extra ampere turns on the field.

The reason that the ordinary D. C. series motor operates at low power factor on an alternating-current circuit is that the machine has large reactance. In an A. C. series motor the reactance of the field is made low by using short field poles of ample area and few turns of wire on the fields. The air gap is also made small. Since the ampere-turns on the field are made comparatively small, the armature ampere-turns must be made proportionally large in order to get the necessary torque It would seem that nothing would thus be gained, but it is possible to compensate for the reactance of the armature but not for that of the field, so that the added armature ampere turns, when compensated for, introduce practically no reactance in the motor circuit. Compensation is affected as follows: In Fig 221 the current is assumed to be rising. Current flows through the armature from brush B_1 to B_2. Appli-

cation of the three-finger rule shows that the armature will turn clockwise. The current flowing through the armature will produce a strong cross-magnetization N_1S_1 at right angles to the main field. This flux linking the armature conductors causes a large armature reactance as well as cross-magnetization. If a winding C_1C_2 be placed as shown, this can be made to balance the poles N_1S_1 set up by the armature current and thus compensate for

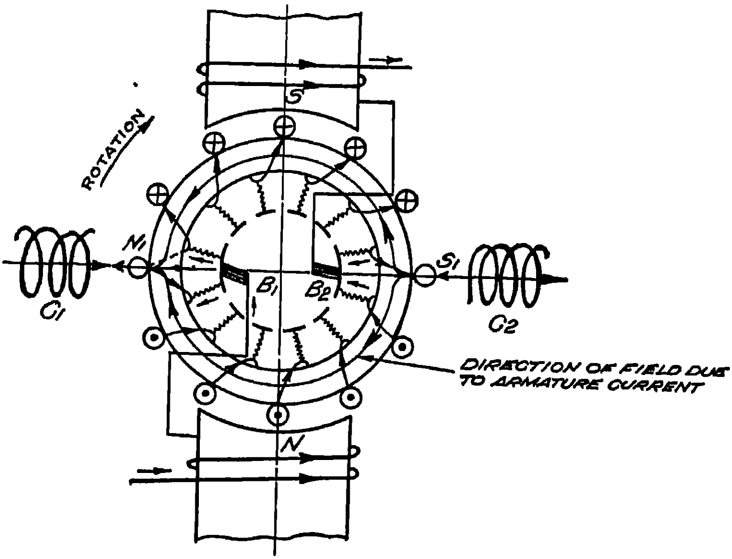


Fig. 221. — Series A. C Motor With Compensating Winding.

the cross-magnetization and the reactance of the armature In one type of motor, known as the conductively compensated motor, this winding is in series with the main winding; in another type, known as the inductively compensated motor, the compensating winding is short-circuited.

In an actual motor the compensating winding is distributed as much as possible by putting it in slots in the poles.

Repulsion Motor. The repulsion motor is a single-phase commutator type of motor that has characteristics similar to a series

motor. It consists of a stator wound with a single-phase distributed winding and a rotor with a winding exactly like that of a direct-current motor armature. There are brushes that stand 180 electrical degrees apart. These brushes are short-circuited and are set at an angle of about 20 electrical degrees with the center line of the poles.

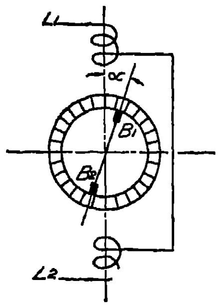


Fig. 222 — Circuits of Simple Repulsion Motor

Figure 222 shows schematically a repulsion motor. Figure 223(a) and (b) show the action of the main field on the armature when the brushes are set 90 electrical degrees from the center line of the poles A ring-winding is shown for simplicity. Considering that the flux is rising and in the direction N_fS_f, it cuts conductors 8, 9, 10, 11, 12 and 1 from left to right as indicated by arrow "a." It cuts conductors 7, 6, 5, 4, 3 and 2 from right to left. The electromotive forces in the two sides of the armature balance and no

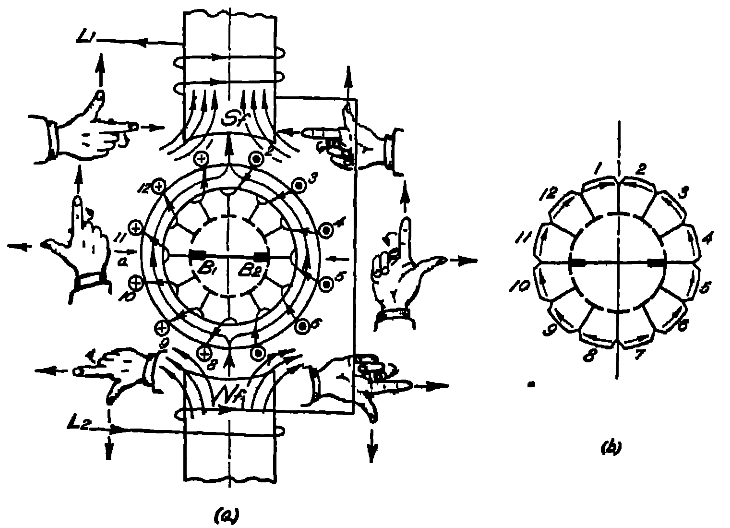


Fig 223 — Repulsion Motor with Brushes 90° From Center Line of Poles.

current flows. This is clearly shown at (b). There is no torque with this position of the brushes.

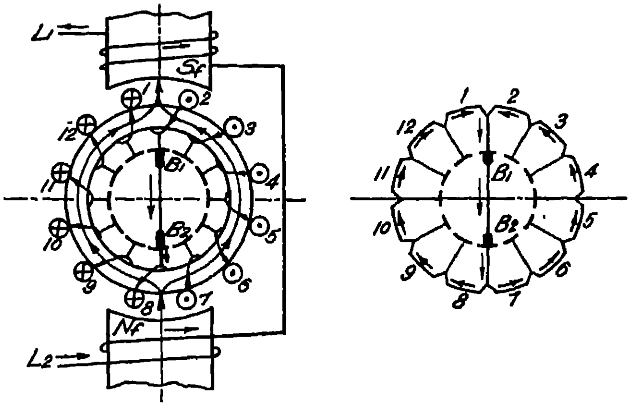


Fig. 224. — Repulsion Motor with Brushes on Center Line of Poles.

Figure 224 shows the brushes set on line with the centers of the poles. With this setting there is no effective torque because the torque developed by conductors 1, 12 and 11 is balanced by the torque in conductors 2, 3 and 4, and the torque in conductors 8, 9 and 10 is balanced by that in 7, 6 and 5. In Fig. 225 the brushes are set at an angle α with the center line of the poles. In this position the electromotive forces in conductors 1, 12, 11, 10 and 9 and conductors 3, 4, 5, 6 and 7 overcome the electromotive forces in 2 and 8 and send current through the short-circuited brushes from B_1 to B_2. If we apply the three-finger motor rule, we see that conductors 2, 1, 12, 11 and 8, 7, 6, 5 produce torque acting clockwise and conductors 3, 4, 9 and 10 torque counter-clockwise. The net effect is to turn the armature clockwise. This may also be seen by marking the poles produced by the armature current at the points N_aS_a. It will be seen that N_a is near N_f and S_a near S_f and since like poles repel, the armature will turn clockwise. Thus the armature currents produce poles on the armature of the same polarity as the main poles to which they are adjacent and repulsion takes place between them. From this fact the motor gets its name.

From the preceding, if the brushes are set on the other side of the center line of the poles, the armature will turn in the opposite direction. The motor will operate best when the angle α is about 20 electrical degrees.

The characteristics of the simple repulsion motor described are similar to those of the direct-current series motor. The machine

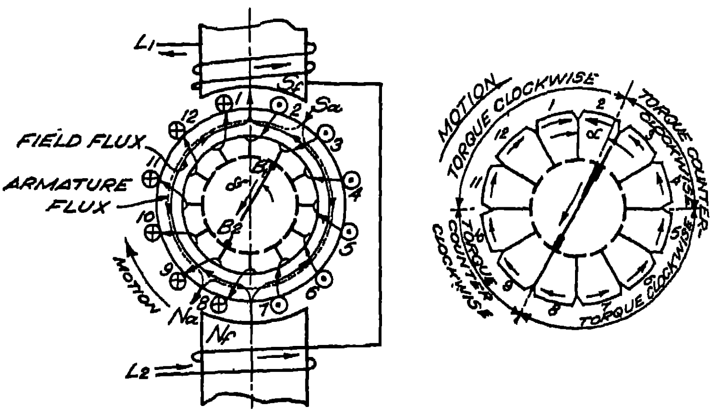


Fig 225 — Repulsion Motor with Brushes at Angle α from Center Line of Poles

has a large starting torque but will race at no load. Several methods of controlling the speed and at the same time improving the power factor have been developed.

Wagner Repulsion-Induction Motor. In this motor advantage is taken of the large starting torque of the straight repulsion motor and the approximately constant speed of the regular induction motor. In construction it is a combination of the two. It will develop in starting, from two to three times full-load torque and draw from two and one-half to three times full-load current if thrown directly on the line If thrown on through a starting resistance, the starting current can be reduced to any value desired but, of course, with a reduction in torque.

The starting connections are exactly like those of the straight repulsion motor shown by Fig. 222. As explained, this motor

gives a large starting torque, similar to the direct-current series motor. In the Wagner motor, when the armature has reached its speed, the brushes are lifted off the commutator by a centrifugal device similar to the fly-ball governer of a steam engine. At the same time the device short-circuits all the commutator bars. It will be evident at once that a short circuit on the whole commutator is equivalent to making the armature a squirrel-cage rotor, so after the centrifugal device has operated the machine operates as a single-phase induction motor, having the same characteristics.

Fig 226 — Wagner Type RA Single-phase Repulsion-induction Motor.

Figure 226 shows the latest form of this motor and Fig. 227 the rotor and governor used on these motors.

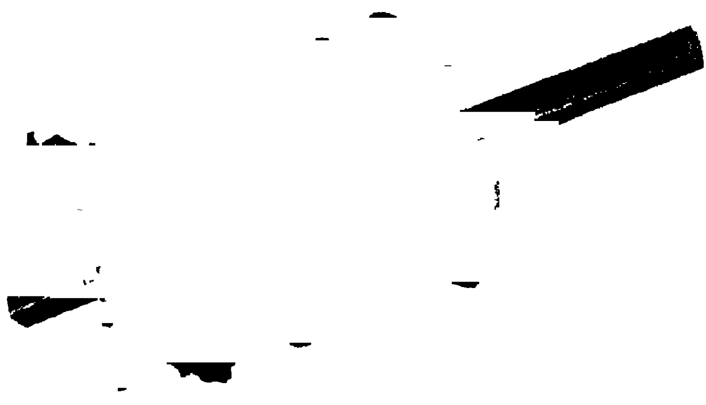


Fig 227 — Rotor of Type RA Single-phase Repulsion-induction Motor (Wagner Electric Corporation)

Single-Phase Commutator Motor — Type SCR. A type of single-phase motor that possesses excellent starting and running

characteristics is made by the General Electric Co. and known as the SCR motor. This motor has a stator similar to that of the compensated repulsion motor and a rotor that has two windings. One of these windings, which is a direct-current or commuted winding, is placed near the outer part of the rotor. This winding is exactly like that of a repulsion motor and is short-circuited by brushes riding on the commutator. The other winding is of the squirrel-cage type and is placed some distance below the commuted winding and separated from it by brass wedges in slits in the rotor. In a general way, the motor has the high starting torque of the repulsion motor and the good speed regulation of the squirrel-cage motor The two rotor windings, working in conjunction, give it certain desirable features peculiar to itself, among them being high full-load power factor and efficiency.

In starting, most of the flux passes through the commuted winding Only a small part enters the squirrel-cage winding and so has little effect. The starting torque is therefore practically that of a repulsion motor which is very high. When the motor is running in the neighborhood of synchronous speed, the currents in the stator and rotor windings set up a revolving field similar to that in a polyphase induction motor. The torque, then, is that due to the repulsion winding and the squirrel-cage winding, or nearly twice that of the squirrel-cage winding alone.

If the load falls off, the commuted or repulsion winding tends to speed up the motor and the squirrel winding produces generator torque This slows the motor down. The motors are built so that the motor torque from the commuted winding and the generator torque from the squirrel-cage winding balance at speeds about 2% above synchronism. At about 10% below synchronism, the motor develops maximum torque and gives an output of about twice full load.

Commutation in the motor is excellent for two reasons, the first being that its normal operation is at about synchronism where the commutation of a repulsion motor is inherently the best, and the second being that when the commutated coils are leaving the brushes, the energy that would otherwise appear in

the form of a spark is transferred magnetically to the squirrel-cage winding and absorbed by it. The squirrel-cage winding has the desirable feature of bringing the line voltage and current more nearly in phase than is the case with the straight repulsion motor, thereby producing a high power factor. It also takes its share of current which results in a division of rotor current between the two rotor windings and high efficiency. Figure 228 shows one of these motors.

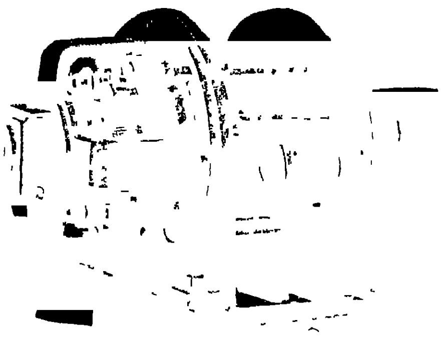


Fig. 228. — Single-Phase Commutator Motor, Type SCR. (General Electric Company)

The Fynn-Weichsel Motor. This motor, which is of the polyphase type, has starting-torque characteristics similar to a slip-ring induction motor and, when running, has speed characteristics similar to a synchronous motor. It operates over its whole working range with a leading current, or a current in phase with its voltage. It can therefore be used to compensate for the lagging current taken by induction motors. If, in a given installation, some of the motors are ordinary induction motors, and several others are of the Fynn-Weichsel type, the whole installation will operate at 100% power factor if the motors have been properly selected.

The stator resembles that of an ordinary induction motor but carries two windings, a main field winding and an auxiliary wind-

ing which is placed 90 electrical degrees from the main winding. The rotor resembles the armature of a rotary converter in that it has slip rings and a commutator. There are, however, two windings on it instead of one. Both of these windings are carried in the same slots One of these is a direct-current winding, or commuted winding, and the other a polyphase winding The commuted winding is at the bottom of the slots and the polyphase winding is at the top of the slots. The commuted winding is connected to the commutator and the polyphase winding is connected to slip rings. Figure 229 shows the various parts of a

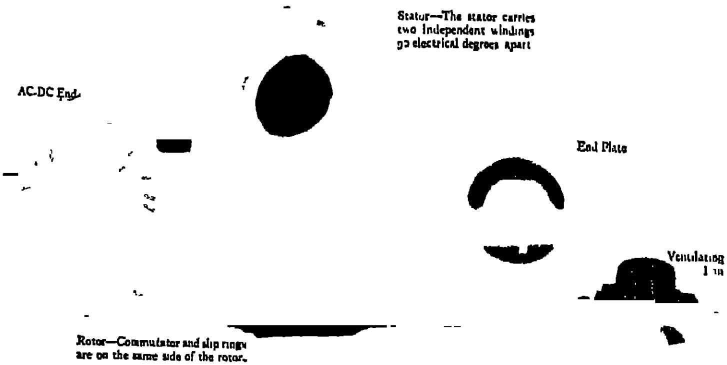


Fig 229 — Fynn-Weichsel Motor. (Wagner Electric Corporation)

Fynn-Weichsel motor and Fig. 230 gives the diagram of connections. Winding F is the main field winding and A is the auxiliary winding. The slip rings are connected to the line. Windings F and A, which are on the stator, act as secondaries of a transformer. Brushes B_1 and B_2 ride on the commutator and connect the commuted winding in series with the field winding F. The auxiliary winding A is short-circuited. The motor is started by connecting resistance in the windings F and A by a starter resembling that used on a slip-ring induction motor. The commuted winding is open-circuited in starting on the large machines. Diagram (b) shows the starting connections used with the large machines.

Diagram (c) shows the arrangement for starting used on the small and medium-sized machines It will be noted that, on these machines, the commuted winding is left in the circuit in starting.

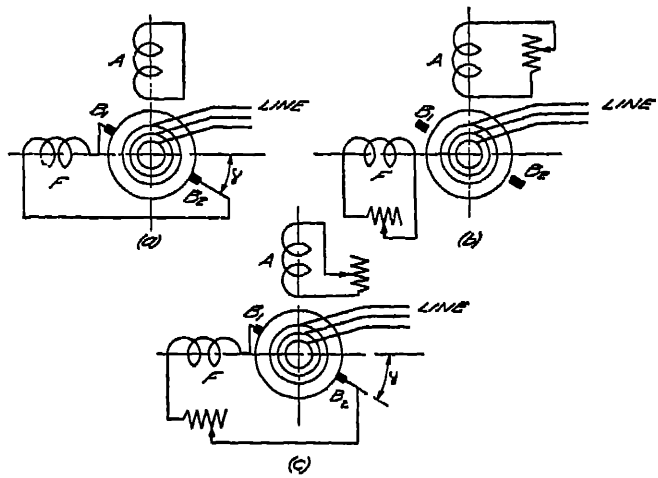


Fig 230 — Circuits of Fynn-Weichsel Motor.

With the connections shown, the motor starts with a large torque and, when up to speed, synchronizes automatically. It will run as a synchronous motor until the load gets about 150% or 200% normal, when it will drop out of step and run as an induction motor. If the load falls off, the motor has the desirable feature of again synchronizing automatically. This is brought about by the action of the coil F. It will be noted from diagram (a) that the brushes stand at an angle α with the axis of the winding F. In this position, the voltage across the brushes is alternating and not direct. It is of the same frequency as that induced by transformer action in the coil F. This voltage acting with that of the coil F gives the motor its ability to synchronize automatically.

Fynn-Weichsel motors are well adapted for machines that require large starting torque, such as compressors, ice machines,

grinding disks, etc. They will carry a considerable overload without drop in speed At severe overloads, they drop their speed slightly but return to synchronism when the overload falls off. A most desirable characteristic of this motor is its ability to correct the power factor of an installation by drawing a leading current which neutralizes the lagging current taken by regular induction motors or other apparatus.

PROBLEMS

1 Explain how a revolving field can be produced by two alternating currents 90 electrical degrees apart

2 What is meant by slip? If the slip of a 4-pole 25-cycle motor is 2%, how fast does it run?

3 Explain the construction of the squirrel-cage rotor and the wound rotor For what kind of work is each adapted?

4 Explain the operation of the single-phase induction motor. How are such motors usually started?

5. Lay out the stator winding for a 4-pole, 24-coil, 3-phase star-connected motor

6 Explain the circle diagram.

7. Construct a circle diagram using the following test data.

Running light. Volts per phase, 133
amperes per phase, 13
watts per phase, 233

Rotor blocked. Volts per phase, 28.6
amperes per phase, 31 5
watts per phase, 520

Resistance per phase of the stator is 21 ohms.

Calculate the energy and reactive components of the running-light current and calculate the power factor running light. Calculate the amperes per phase at normal voltage, which is 133. Calculate also the watts and power factor at normal volts with rotor blocked. Draw the circle diagram and from it find mechanical output, power factor, stator and rotor copper losses, friction and windage loss and efficiency of motor for a current per phase of 50 amperes.

8. If an induction motor gives 10 horse power at 220 volts, what power would you expect to get from it if the voltage drops to 200? Why?

9. Explain the compensated series A_1 C_1 motor.

10 What is a repulsion motor? Explain how rotation is produced by the action of the field on the armature.

11. Explain the Wagner type RA single-phase induction motor. Mention several kinds of work for which this motor is well adapted.

12. Explain the General Electric, type SCR motor. Mention several desirable characteristics of this motor and how they are obtained.

13. Explain the Fynn-Weichsel motor. State wherein it differs from the Wagner type RA motor Mention several desirable characteristics of this motor and state how they are obtained.

CHAPTER X

SYNCHRONOUS MOTOR

Alternator Used as a Motor. Two alternators giving the same voltage and frequency and having approximately the same shaped voltage waves may be run in parallel if they are connected together when in phase If, when running in parallel, the source of mechanical driving power be removed from one machine, it will draw electrical power from the other machine and operate as a motor It will turn at the same number of revolutions as before but slip back a few degrees from the position it held when running as a generator When run thus, it is called a synchronous motor. A synchronous motor is essentially an alternating-current generator operated as a motor. If the motor has the same number of poles as the generator, it will turn at the same speed as the generator. If it has twice as many poles, it will turn half as fast, and if it has one-half as many poles, it will turn twice as fast as the generator The frequency formula on page 6 may be used to determine the speed at which a synchronous motor should operate

In order to get a picture of what happens, consider that two machines, each with four poles, are connected together by a rigid coupling so that the pole pieces of both machines stand in exactly the same relation to the armature coils. This is shown schematically by Fig 231 and represents the condition when the machines are running as generators. If a flexible coupling, such as a coil spring, be substituted for the rigid coupling, machine #2 will continue to turn at the same speed as before but will drop back a few degrees due to the friction and other losses acting as a load or brake. The flexible coupling corresponds to the electrical connection between the two machines when #2 is operated as a motor. Just as the spring will allow #2 to drop back a few

degrees due to the load, so the electrical coupling will allow #2 to fall back a few degrees also as the load comes on. Both machines will run at the same number of revolutions. If #2 be loaded too

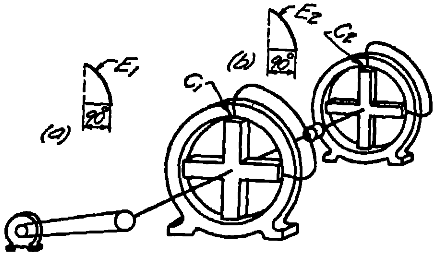


Fig 231 — Two Machines Mechanically Connected to Run in Synchronism.

heavily, the spring will break and the machine will stop A similar condition will exist when the machines are electrically coupled If the load becomes too great, the motor will "fall out of step" and stop

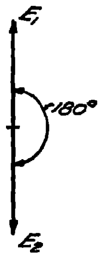


Fig. 232. — Voltage Vectors for Two Similar Machines in Parallel, $E_1 = E_2$

Voltage and Current Relations — Elementary Synchronous-Motor Diagram. Consider first that two machines exactly alike are coupled together and excited to give the same voltage and that they are in phase. The E. M. F.'s of the two armatures will be opposed to each other through the internal circuit of the armatures and leads just as the voltages of two batteries in parallel will be opposed. This condition will be shown by Fig. 232. No current will flow.

Consider next that the machine E_1 gives a voltage higher than E_2. The vectors will be as in Fig. 233. The voltage E_1 being larger will send current from machine #1 to machine #2. Its value will be $I_a = \frac{E_R}{\sqrt{R^2 + X^2}}$ and it will be out of phase with E_R by an angle ϕ whose tangent is $\frac{X}{R}$. X is inductive reactance in the example given, so I_a lags. Suppose now that the rigid

coupling between the two machines be removed. The friction and other losses will tend to make #2 slip back a little, so Fig. 233 becomes Fig. 234. $E_1 = E_2$ but, having a slight phase dis-

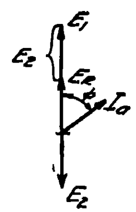


Fig 233 — Voltage Vectors of Two Similar Machines in Synchronism. E_1 is Greater than E_2.

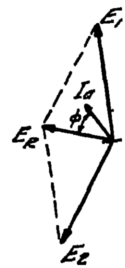


Fig 234 — Elementary Vector Diagram for Synchronous Motor.

placement, there is a resultant E_R which causes current I_a to flow from #1 to #2. This current keeps #2 running as a motor. Figure 234 is an elementary synchronous-motor diagram.

Diagram for Constant Current and Constant Power. Figure 235 shows that for a given current and a given amount of power delivered by the motor, there are two motor E. M. F.'s and therefore two motor excitations at which the motor can be operated. The drawing is a reproduction of the elementary synchronous-motor diagram, drawn in such a position that I_a falls on the horizontal line OX.

Let E_{g1} be the E. M. F. of the generator and let E_{m1} be the E. M. F. of the motor for a given condition of running. In the example given, the resultant E. M. F., E_R, leads the current I_a by an angle of 60°. The generator may be thought of as giving positive power and the motor as giving negative power. Hence $E_{g1}I_a$ cosine α must be + and α must be less than 90° and $E_{m1}I_a$ cosine $(\theta_1 + \phi)$ must be negative or $(\theta_1 + \phi)$ must greater than 90°. Let $OB = E_{m1} \cos (\theta_1 + \phi)$ and be considered negative. OB is thus proportional to E_{m1}. OA is proportional to I_a, since $OA = I_aX$. Hence the area AOBD is proportional to the power received by the motor. If an arc CC_1 be drawn from E_R as a

center with a radius $E_R E_{m1}$, cutting BD extended at the point E_{m2}, E_{m2} will be another position the motor E. M. F. can take because its projection on OX is still OB. E_{g2} will be the corresponding position of the generator E. M. F.

It will be noticed that when the motor E. M. F. is large or equals E_{m1}, that I_a leads the generator voltage E_{g1}, and when the motor E. M. F. is small or equals E_{m2}, that I_a lags behind the generator voltage E_{g2}. This is found to be the case experimen-

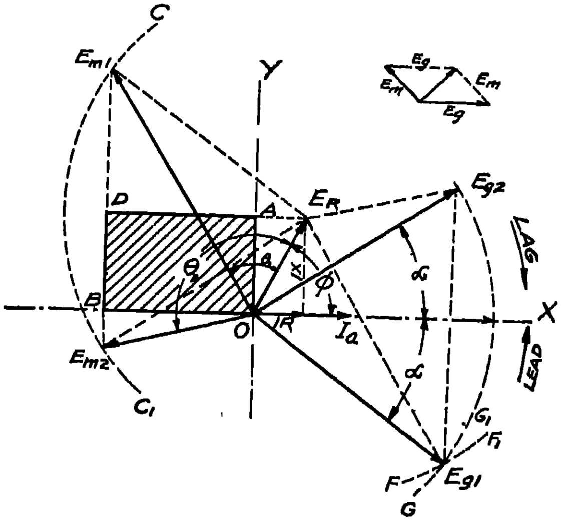


Fig. 235. — Diagram Showing that for a Given Generator Voltage and Motor Current There are Two Motor E. M. F.'s and Phase Positions that Will Give the Same Power.

tally. There are two excitations with corresponding motor E. M. F.'s that will produce the same power with a given current. With the large excitation the current will lead the generator or line voltage and with the small excitation the current will lag behind the generator voltage.

Diagram for Variable Current but Constant Power. Figure 236 shows that for constant motor power there may be more than one motor current and that each motor current may have two

motor E M. F.'s and therefore two excitations. Several rectangles of different shapes but all of the same area have been drawn at the left of the drawing as shown by the shaded portion. The curve HH_1 shown passing through the corners of these rectangles is an hyperbola. The angle between the resultant E. M. F., E_R and I_a is 60° as in the illustration given by Fig. 235. For constant power E_m must always lie on HH_1. Take some motor

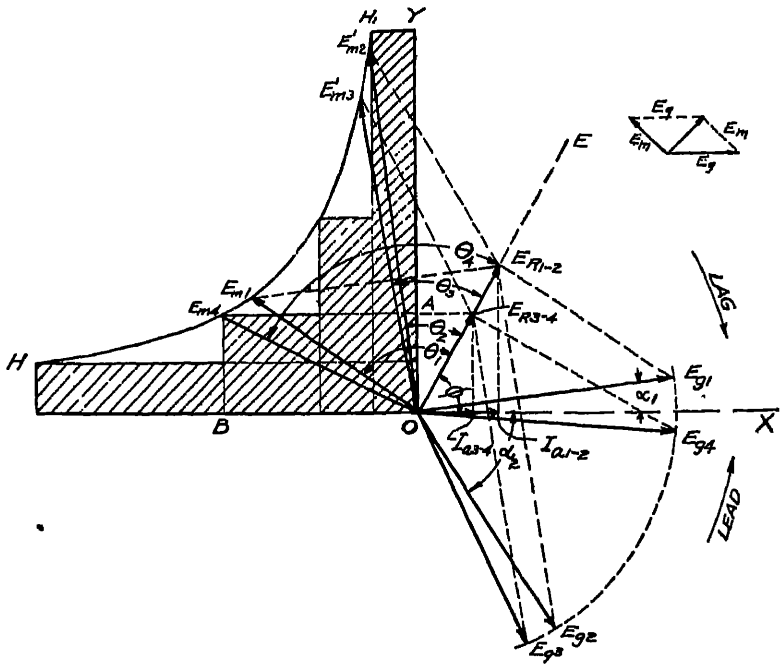


Fig 236.—Diagram Showing that for Constant Motor Power There May Be More than One Motor Current and that Each Current May Have Two E. M F 's and Excitations

E. M. F. as OE_{m1} and from E_{m1} as a center swing an arc with OE_{g1} as a radius cutting OE. E_{R1-2} will be the resultant voltage for this phase position of E_{m1}, and E_{g1} will be the position of the generator voltage. The current I_{a1-2} will lag behind E_{g1} by an angle α_1. Since there are two points on HH_1, equidistant from

E_{R1-2}, OE'_{m2} will be another position of the motor E. M. F. for which the motor will take the same current and give the same power. This will be apparent, since for any points on the curve HH_1, an abscissa as OB, times an ordinate as OA will be a constant quantity. OE_{g2} will be the corresponding position of the generator voltage. I_a will lead the generator voltage E_{g2} by an angle α_2. This is to be expected, since the motor is excited to give an E. M. F., E'_{m2} which is much greater than E_{m1}. (See also Fig. 235.)

Minimum Current. Figure 237 is the diagram of Fig. 236 redrawn. The line E_mE_R is equal to OE_g. Its position is deter-

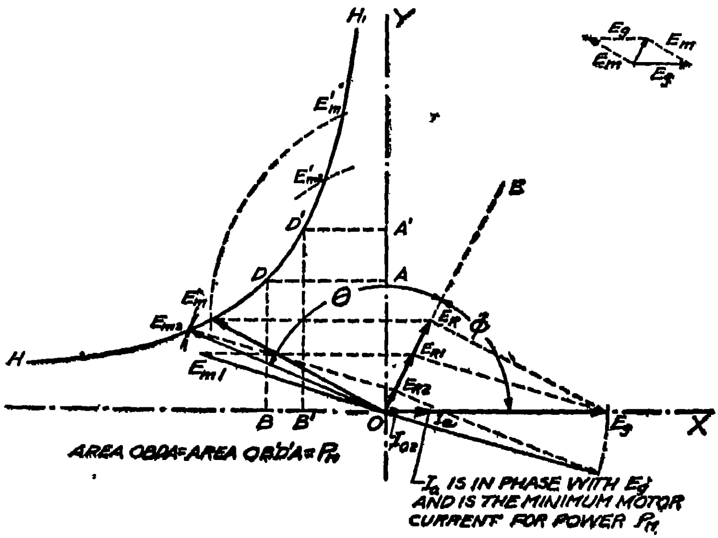


Fig. 237. — Diagram Showing Position of Generator and Motor E. M. F.'s For Minimum Current and Constant Power.

mined by moving the end E_R along OE keeping the line E_mE_R parallel with OX. When the end E_m cuts the curve HH_1, if a parallelogram $E_mE_RE_gO$ be constructed, it will be found that E_g and I_a are in phase. I_a is the smallest current that will carry the

load. If, for instance, $E_{R1}E_{m1}$ be tried, it will be found that E_{m1} does not reach the constant-power curve HH_1, and so this position will not give enough power. If $E_{R2}E_{m2}$ be drawn, E_{m2} will be on the curve HH_1, and the current will be I_{a2}. E_{m2} is about $1.1 \times E_m$ and I_{a2} about $\frac{1}{4} \times I_a$, from the drawing. Also I_{a2} is out of phase with E_{m2} about 150°, or $\cos(\theta + \phi) = \cos 150° = .87$, so the power is only $1.1 \times E_m \frac{I_a}{4} \times .87 = .24\ E_m I_a$ or about one-fourth that needed.

There is, of course, another position for E_m at E'_m and E_{m2} at E'_{m2} and corresponding positions of E_g as explained under constant current and constant power. The constructions for these positions are omitted to add clearness to the diagram.

Synchronizing. It was stated at the beginning of the chapter that an alternating-current generator would run in parallel with another generator of the same frequency, voltage, and wave form, if switched on with it when the two machines are in phase or in synchronism as it is usually called. One method of determining when the machines are ready to be thrown together is by means of lamps. When the machines are 110-volt, the lamps may be connected directly to the armature leads of the two machines, but when the machines are of higher voltage, transformers must be used to step down the machine voltage to a value suitable for the lamps. Another method of synchronizing is by means of an instrument known as a synchronism indicator or synchroscope. The method of synchronizing by means of lamps will be described first

Synchronizing with Lamps. In Fig. 238 machine 1 is connected to the busses and is running at normal frequency and voltage Machine #2 is to be synchronized with #1 and thrown on the busses. Two lamps, L_1 and L_2 in series, are connected from b_1 to b_2 on the machine side of the switches S_1 and S_2. A connection is made from a_1 to a_2. With S_2 open, there is a circuit a_1, b_1, L_1, L_2, b_2, a_2 back to a_1. Suppose that at the instant under consideration, that lead b_1 of generator #1 is + and lead b_2 of #2 is + also. Machine #1 tries to send current through

the lamps and machine #2 tries to do so also. If the machines are in phase, the two waves will appear as one, as at (a) Fig. 239. Then no matter at what instant we consider b_1 plus, b_2 will be exactly equal to it, and there will be no voltage across the lamps,

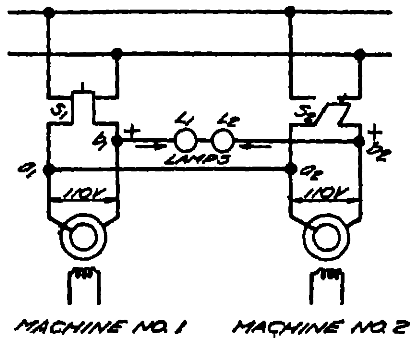


Fig. 238. — Connections for Synchronizing with Lamps Dark

so they will be dark. This will be the proper time to close the switch S_2. If the machines are slightly out of phase as shown at (b) and (c) Fig. 239, then there will be a difference between the E. M. F.'s. E_1 and E_2, at all points in the cycle and current will flow through the lamps. The farther out of phase E_1 and E_2 are up to 180°, the greater will be the difference between E_1 and

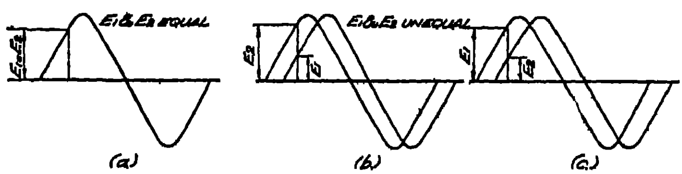


Fig. 239.—Diagram Showing Three of Many Possible Conditions when Machines Are Being Synchronized.

E_2. At 180° phase difference, the voltage across the lamps will be twice the machine voltage. Such a condition may occur when an operator is synchronizing, so two lamps are put in series to prevent burning out the one lamp.

When two machines are synchronized, the operator should have a method of controlling the speed of the incoming machine.

When the machines are considerably out of phase, the lamps light and go out very rapidly. As the incoming machine is properly brought up towards synchronism, these periods of light and darkness gradually lengthen. When the periods of darkness are of at least 2 or 3 seconds duration, the operator should close the switch just as the lights are going out.

Consideration of the above method of synchronizing indicates that it is not perfect. The machines are thrown together when the lamps are dark. Since the lamps require considerable voltage to make their filaments glow, it is possible to throw the machines together when slightly out of phase Further, the method depends on using two lamps in series, or one lamp of double the machine voltage, so that the lamps are worked at much less than full brilliancy under normal conditions of synchronizing. There is a possibility also of a filament burning out, and the operator throwing the machines together when much out of phase.

If the switch is closed when the machines are not in phase, current will flow from one machine to the other. If they are but slightly out of phase, this current will pull them together. If considerably out of phase, the current will be so large as to trip the breakers or blow the fuses

Another method of synchronizing with lamps consists of connecting the leads a_1 and b_2 together through a lamp, and b_1 and a_2 together through another lamp. With this connection, the switch S_2 should be closed when the lamps are bright One lamp must be connected between a_1 and b_2 and another between b_1 and a_2. Otherwise there will be a short circuit when the switch S_2 is closed.

Two- and three-phase machines are synchronized the same as single-phase machines When first connected up they must be "phased out" with lamps on each phase to get all phases to come in together. After this only lamps shown by Fig. 238 are necessary

A more satisfactory method of synchronizing than by means of

lamps is by the use of a synchroscope. This instrument is described in Chap XI.

Hunting of Synchronous Motors. Two machines operating parallel would have their E. M F.'s in a position about as shown by the heavy lines in Fig. 240. E_g is the generator E M. F. and E_m the motor E. M. F. If a sudden load should slow down the motor, E_m would move to a position E_{m1}, as the armature would slip back a little due to the load This would produce a new resultant E M F., E_{R1} which would be larger than E_R, and a corresponding current I_{a1}, larger than I_a.

This current I_{a1} would tend to make E_{m1} swing back to E_m, and if the armature were fairly heavy it might swing as far as E_{m2} producing a resultant E_{R2} in the opposite direction from E_{R1}.

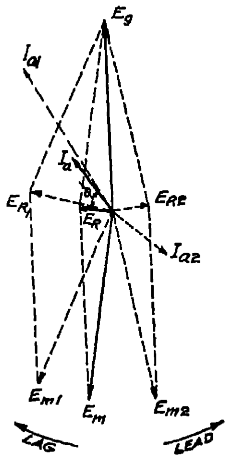


Fig 240. — Diagram Illustrating Hunting

In any case the currents will tend to pull the machines in step, but the design of the machines may be such that they overswing with sudden change of load Some machines have characteristics such that they swing back and forth, in relation to each other, to a considerable extent. Such oscillation about synchronous speed is called "hunting."

Use of a Synchronous Condenser. It was stated under capacity that a condenser might be used to compensate for the lagging current caused by induction motors or other apparatus which drew a lagging current.

Synchronous motors or rotary convertors may also be used to compensate for lagging current, if they have their fields over-excited. The effect of an over-excited field on either a synchronous motor or rotary convertor is to cause a leading current When a synchronous motor is used without a mechanical load, simply for power factor correction, it is called a synchronous condenser.

The following example will make clear the operation of a synchronous condenser in controlling power factor.

Suppose that a line supplying several induction motors is operating at 80% P F with, of course, lagging current, and that the volt-amperes going over the line are 100 kv-a. The true power is 80 × 100 = 80 kw and the reactive kilovolt-amperes are 60. The total kilovolt-amperes, the reactive volt-amperes and the true kilowatts may be represented by a triangle BAC, Fig 241. If in Fig. 241 the power factor is to be made unity (100%), a syn-

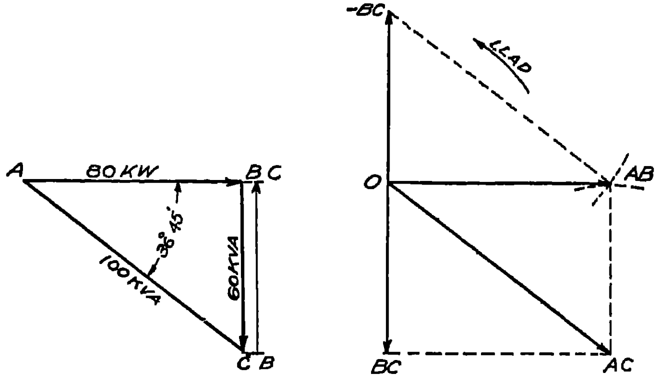


Fig 241 — Diagrams Illustrating Total Kv-a, Reactive Kv-a and True Kw.

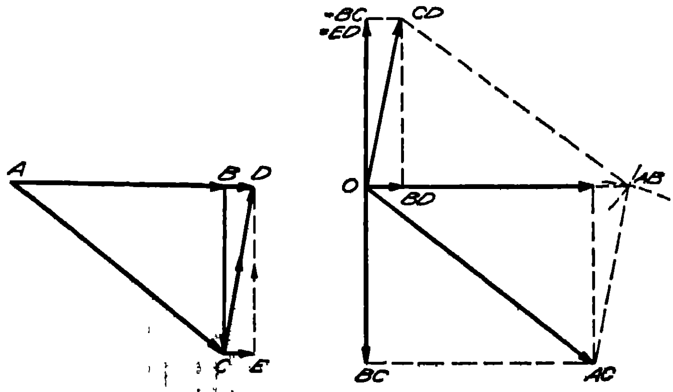


Fig. 242 — Diagrams Illustrating Total Kv-a, Reactive Kv-a. and True Kw. when Losses in Motor are Considered.

chronous motor, over-excited to give reactive kilovolt-amperes 180° from BC = – BC and equal to 60 kv-a., will balance BC and bring the point C to B making angle between AB and AC zero, thus making the power factor 100%

In an actual installation the synchronous machine will have some losses and power must be supplied from the line to overcome these losses. This power is in watts and can be represented on the diagram along the line AB. When losses are considered, the diagram of Fig. 241 becomes that of Fig. 242. Since BD represents the true power lost in the machine and ED the reactive kv-a, the line CD represents the rating of the machine.

In case it were desired to have a synchronous machine carry a mechanical load as well as correct power factor, the diagram can

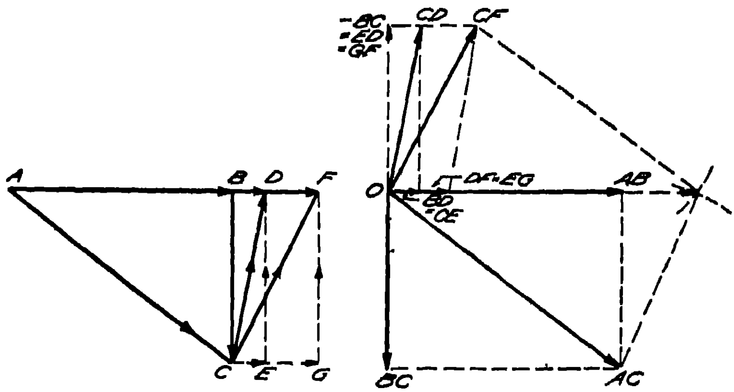


Fig. 243.—Diagrams Illustrating Total Kv-a., Reactive Kv-a. and True Kw. when Motor Carries a Mechanical Load

be modified still further, as in Fig. 243, letting BD equal the losses as before but now adding DF to equal the load the machine is to carry. The effect on the power factor will be seen at once by comparison with Fig. 242. The rating of the machine must now be large enough to take in enough kv-a. to supply the losses BD, the useful power DF and the reactive kv-a. GE, or the rating is now CF.

In case we wished to change the power factor from say 80%

to 90% with a commercial machine using the same kv-a. input with no load on it we must supply reactive kv-a. of such an amount that $\frac{AB'}{AC'}$ shall equal .9 instead of .8. Hence, we must supply in the problem given $60 - 44 = 16$ kv-a. This will be clear from a study of Fig. 244

In this diagram, it is assumed that the losses vary as the load, which is not strictly true.

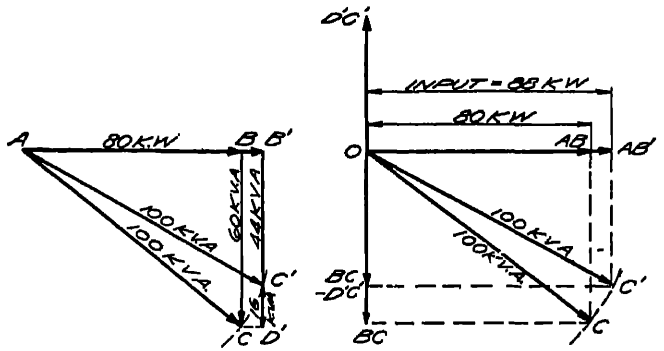


Fig. 244. — Diagrams Illustrating How Power Factor Can be Changed, for a Given Kv-a Input, by Changing the Reactive Kv-a.

The Rotary Converter. The rotary or synchronous converter is a machine for converting alternating current into direct current or for converting direct current into alternating current. In appearance it resembles a direct-current generator or motor, in that it has stationary field magnets and a rotating armature with a commutator. It resembles an alternating-current generator, or synchronous motor, in that it has slip rings. The commutator is usually at one end of the armature and the slip rings at the other. The field magnets receive their current from brushes that ride on the commutator. The magnets may have either a shunt or a compound winding.

The converter is used principally where considerable amounts of power are to be converted from alternating current into direct current or vice versa. Where the converter is operated to convert

from alternating current into direct current, it is said to be operated direct: when it is operated so as to convert from direct current into alternating current, it is said to be operated inverted Converters require no source of outside mechanical power to operate them. They take sufficient electrical power from that passing through them to cause their armatures to rotate. Converters have characteristics similar to direct-current and alternating-current motors. They are started by methods similar to those used for such motors.

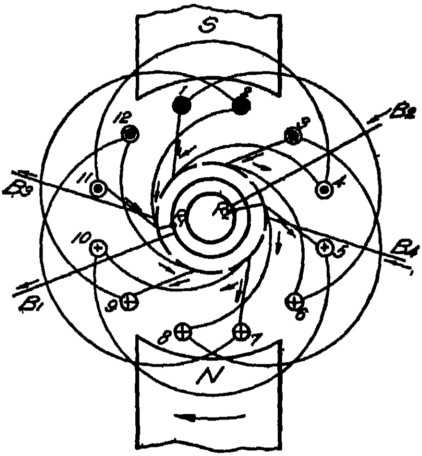


Fig 245. — Drum Winding Tapped for Single-Phase Alternating Current

Construction and Operation of the Armature. The armature is wound like the ordinary parallel or lap winding of a direct-current machine. One of these is shown by Fig. 245. The commutator has brushes at B_3 and B_4 similar to those on a direct-current motor or generator. The alternating-current slip rings R_1 and R_2 are connected to the armature by taps. These taps may be connected directly to the commutator bars if desired as shown by the sketch. Brushes B_1 and B_2 carry the alternating current.

For the purpose of studying the operation of the rotary converter, the ring type of winding will be used. Figure 246 shows

one of these windings. It should be noted that there is but one winding on the armature. The current in the armature is alternating in character, since the conductors alternately cut fields of north and south polarity. If the machine were driven by an outside source of mechanical power, it could be used to supply both direct and alternating current. When so used the machine is called a double-current generator. Part of the armature current is then taken from the slip rings as alternating current and the remainder from the commutator as direct current. When the machine is used as a rotary converter, the current in the armature conductors is the difference between the alternating and direct current, taking into account the instantaneous values The current relations in the separate conductors are complicated but will be explained graphically later on.

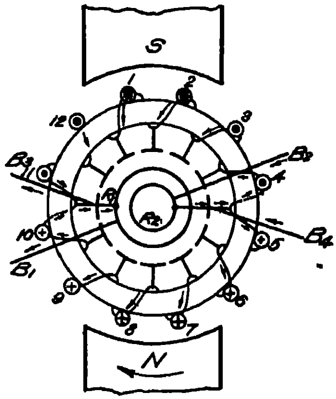


Fig 246 —Ring Winding Tapped for Single-Phase Alternating Current.

Single-Phase and Polyphase Rotary Converters. Rotary converters may be bipolar or multipolar and may be tapped to be single-phase or polyphase. The elementary rotary shown by Fig. 246 is a bipolar single-phase machine. This machine may be made two-phase by tapping the armature at two more points midway between the taps shown by Fig. 246 and adding two more slip rings. Such a machine is shown by Fig. 247.

The same winding may be made three-phase by tapping at three equidistant points as shown by Fig 248.

In case the machine is four-pole, it must have four taps to make it single-phase; eight to make it two-phase, and six to make it three-phase. This will be clear from a study of Fig. 249 which shows a four-pole three-phase machine. On a three-phase machine the taps must be 120 electrical degrees apart and, since

the poles are but one-half as many mechanical degrees apart on a four-pole machine as on a two-pole machine, one set of taps

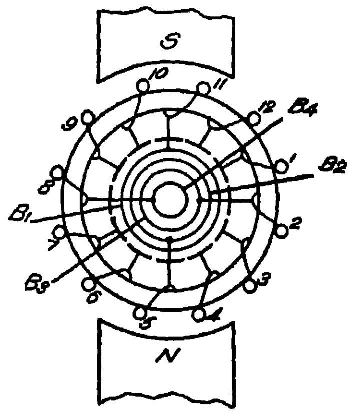


Fig. 247. — Bipolar Two-Phase Rotary Converter.

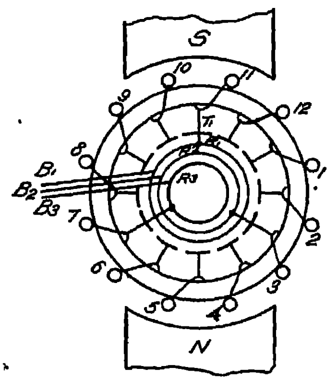


Fig. 248. — Bipolar Three-Phase Rotary Converter.

will be but 60 mechanical degrees apart, such as taps T_1, T_2 and T_3. In order to utilize the part of the windings under the other poles, three more taps T_4, T_5 and T_6 are required.

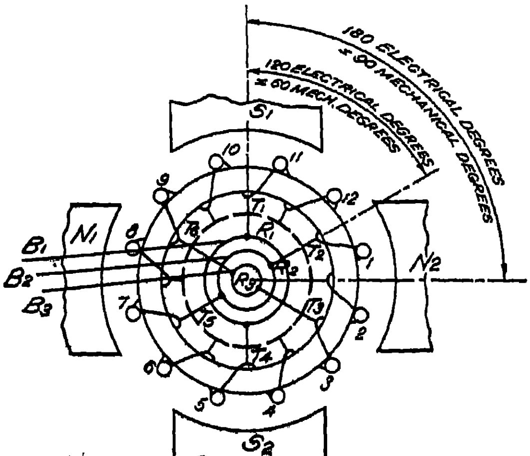


Fig. 249. — Four-Pole Three-Phase Rotary Converter.

In tapping an armature, the principle to keep in mind is that each set of poles must have its own taps. These must be spaced 180 electrical degrees apart in a single-phase machine, 90 electrical degrees apart in a two-phase machine, 120 electrical degrees apart in a three-phase machine, etc.

Relation of Alternating E.M.F. to Position of Taps. In order to understand the relations between the alternating and direct voltages and currents in rotaries, assume a single-phase bipolar machine and consider the alternating-current part first. Figure 250 shows the armature in such a position that the taps are midway between the poles. Conductors 11, 12, 1, 2, 3, and 4 have E. M. F.'s induced in them acting towards ring R_1. Conductors 5, 6, 7,

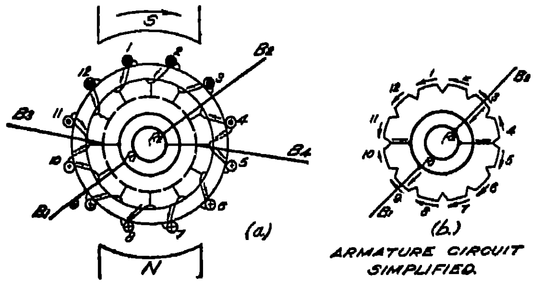


Fig 250 — Armature of Rotary Converter with Taps Midway Between Poles

8, 9 and 10 also have E. M. F.'s induced in them acting towards R_1. The total E. M. F. generated is therefore equal to that generated in one-half of the armature conductors in series. The two halves of the armature feed into ring R_1 and brush B_1. Brush B_2 and ring R_2 form the other side of the circuit. Sketch (b) shows the armature winding simplified. There are no opposing E. M. F.'s in the two halves of the armature so the alternating E. M. F. is maximum for the position of the armature shown.

Figure 251 shows the armature turned 60° from the position shown by Fig. 250. In one-half of the armature, conductors 11, 12, 1 and 2 have E. M. F.'s acting toward R_1 and conductors 3

and 4 have E. M. F 's acting away from R_1. In the other half of the armature, conductors 6, 7 and 8 have E. M. F.'s acting towards R_1 and conductors 9 and 10 have E. M. F 's acting away

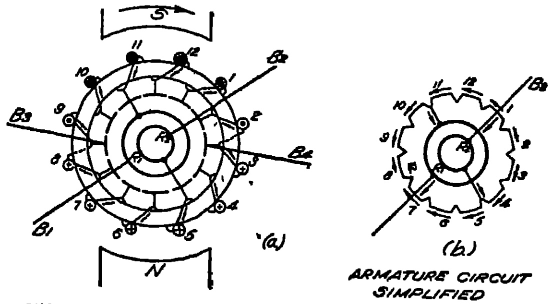


Fig 251. — Armature of Rotary Converter with Taps 60° from Center Line of Poles

from R_1. The E. M. F. across R_1R_2 is therefore less than that in the position shown by Fig. 250

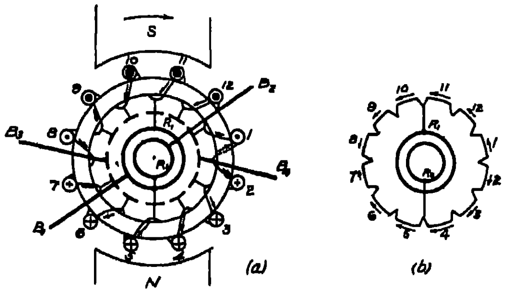


Fig. 252. — Armature of Rotary Converter with taps on Center Line of Poles.

Figure 252 shows the armature turned 90° from the position shown by Fig. 250, or to a position where the taps are on the center line of the poles. In one-half of the armature E M. F.'s act towards R_1 in conductors 11, 12 and 1, and away from R_1 in conductors 2, 3 and 4. In the other half of the armature, E. M. F.'s act towards R_1 in conductors 5, 6 and 7, and away from R_1 in conductors 8, 9 and 10. Inspection of sketch (b)

shows that the E. M. F.'s in the two halves of the armature balance for this position of the armature; that is, the alternating E M. F. is zero.

Thus, when a bipolar armature is tapped at diametrically opposite points, the alternating E M. F. is maximum when the taps are midway between the poles, and zero when the taps are on the center line of the poles. Between these two extreme positions, the E. M. F. varies with the position of the taps; being greatest, of course, when the taps are in positions nearly midway between the poles.

Currents in Individual Conductors. To get an idea of the current values in the various conductors, let (a) Fig 253 represent

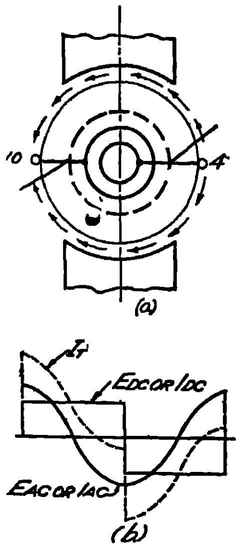


Fig. 253. — Waves of E M. F and Current for a Conductor Connected to a Tap (100% P F)

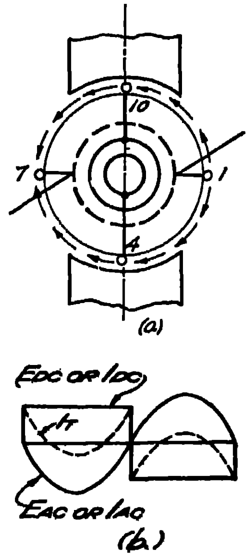


Fig. 254. — Waves of E. M. F. and Current for a Conductor Midway between Taps (100% P F.)

diagrammatically the armature of Fig. 250, and let conductor 10, which is a conductor connected to a tap, stand in position midway between the poles. Conductor 4 will also stand midway between

the poles. In the position shown, there are no opposing E. M. F.'s in either the upper or lower halves of the armature, so, as previously explained, the alternating E. M. F between slip rings is maximum. At this time the direct E. M. F. in 10 is reversing, so the alternating and direct E. M. F.'s will be as shown by (b) Fig. 253. If the power factor is 100%, the current will be in phase with the voltage so the waves will represent current as well as voltage. The direct current is shown as a rectangular wave which reverses, because in a particular conductor as 10, which is passing the neutral plane, the current actually reverses. The total current in the conductor will be the sum of the two waves or the wave I_t, which is obtained by adding the ordinates of I_{dc} and I_{ac}.

Next take the other extreme case of conductor 1 which is midway between taps and let conductor 1 be in position midway between the poles as in Fig. 254. The opposing E. M. F.'s in the right- and left-hand sides of the armature make the E. M. F. across the slip rings zero so that when the E. M. F. is reversing in 1, the alternating E. M. F. across the slip rings is zero. The two waves of E. M. F. and current for 100% P.F. will then be as in Fig. 254(b). The A. C. and D. C. waves are shown in opposite phase because the alternating-current wave represents motor action and the direct-current wave generator action. For a double-current generator, the two waves would have been drawn in phase.

The total current in a conductor midway between taps will be curve I_t. The heating is proportional to the waves I_t and will evidently be greatest for the wave of Fig 253 so we conclude the heating as greatest for a conductor at a tap and least for a conductor midway between taps. In the conductors between those investigated, the heating will vary with the position of the conductor.

Effect of Number of Rings on the Capacity of a Rotary Converter. The capacity of a converter depends on the number of rings it has. The following table gives the capacity of a converter compared with the machine when used as a direct-current generator.

Direct-current generator	1 00
Single-phase converter, 2-ring.	.85
Three-phase converter, 3-ring.	1.33
Two-phase converter, 4-ring	1 63
Six-phase converter, 6-ring	1 93
Twelve-phase converter, 12-ring . . .	2.44

Voltage Relations with Different Numbers of Rings. Referring to Fig. 251 the direct-current E. M. F is the average E. M. F. induced by the conductors of one-half of the armature cutting the lines of force from the field. All of these conductors are in series. At any instant, those that are nearest the centers of the poles have the largest E. M. F 's induced in them, while those near the neutral plane have practically no E. M. F.'s induced in them. All conductors, of course, in their passage under a given point, as for instance a pole center, have the same E. M. F. when in this particular position, but all do not have this E M. F. at the same time. The relation may be expressed by means of

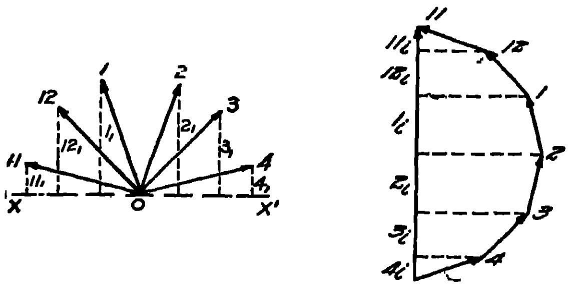


Fig. 255. — Vectors Representing E. M F 's in Conductors of Armature of Fig 250.

vectors. In Fig. 255 the vectors are numbered the same as the conductors. The length of the vertical from the end of the vector to the line XX′ represents the instantaneous value of the E. M. F. in each conductor. For the purpose of studying the converter, the vectors can best be drawn by the topographic method or as shown at the right by Fig. 255.

The resultant of the vectors represents the direct-current E. M. F. since it is the same as the sum of the instantaneous E. M. F.'s 11_1, 12_1, 1_1, 2_1, 3_1 and 4_1. As the other half of the

armature is in parallel with this, its E. M. F is the same and the whole vector diagram becomes as shown by Fig. 256.

From Fig 256 the direct-current voltage must equal the maximum single-phase alternating-current voltage or,

$$E_{dc} = E_{ac\ max}$$

and

$$E_{eff} = E_{dc} \times .707$$

Since the effective value is 707 of the maximum value, if the diagram of Fig 256 be redrawn to scale representing effective values and be made a circle, actual A. C. voltages between any points on the armature can be read off directly from the diagram. For instance, the diameter AB of the circle equals the voltage between slip rings of a single-phase machine. The chords AC, CD, DA equal the voltage between the slip rings of a three-phase machine, the chords AE, EB, BF, FA, the voltages between slip rings on a quarter-phase machine, and AG, GC, etc., the voltages between slip rings on a six-phase machine.

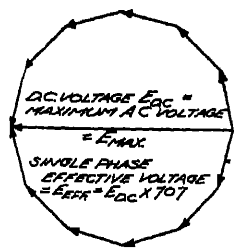


Fig. 256. — Voltage Vectors for Single-Phase Converter.

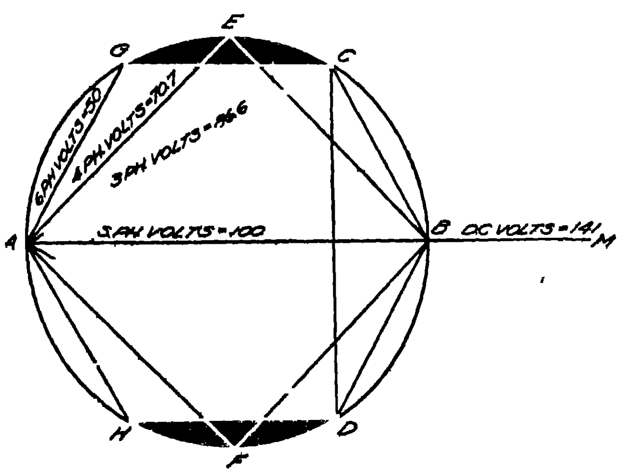


Fig. 257. — Graphical Method of Finding Voltages Between Slip Rings.

With the diagram drawn as above the D.C. voltage will be $\frac{AB}{.707} = 1.41AB$ or will be shown by the line AM.

Connection to a Three-Phase Line — Double-Delta Method. Of the many methods of connecting rotary converters to a three-phase line, only two will be described; the double-delta and the diametrical method. Both of these methods make it possible to convert from three-phase to direct current by using six-phase converters, thereby gaining the advantage of a six-ring converter over a three-ring converter. The table on page 232 shows that the capacity of a converter with six rings is greater than that of one with three rings by the ratio of $\frac{1.93}{1.33}$.

The principle of operation will be clear from the diagram of Fig. 258 which shows a partial double-delta connection. The

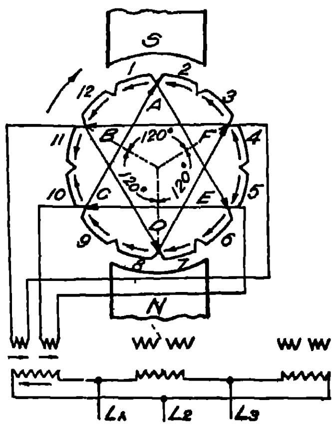


Fig 258 — Partial Double-Delta Connection

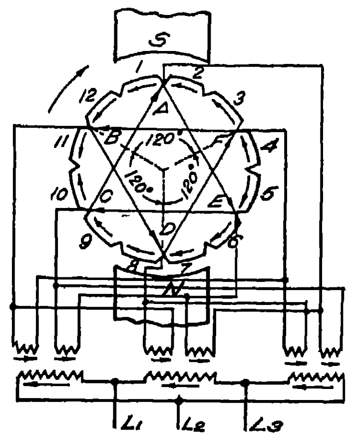


Fig 259 — Complete Double Delta Connection

power to be converted into direct current comes in over the lines L_1, L_2 and L_3 and is stepped down to a value suitable for the converter by three transformers having double secondaries, and therefore twelve secondary terminals. These terminals are con-

nected to the converter slip rings in such a manner that two deltas are formed in the armature. The sides of the deltas are parallel with each other. Considering that the armature is in motion, the sum of the electromotive forces in conductors 3, 2, 1, 12 is FB. At the same instant conductors 6, 7, 8, 9 generate the electromotive force EC. As the armature revolves, BD and AE and later CA and DF take the positions held by FB and EC. FB and EC are parallel and therefore in phase Similarly, DF is parallel and in phase with CA, and BD parallel and in phase with AE. If, now, two points as B and F be connected as shown to one secondary of transformer 1, and C and E to the other secondary, the two electromotive forces will combine to induce an electromotive force in the primary of the transformer. Similarly, DF and CA, which are 120° from FB and EC, may be connected to transformer 2, and BC and AE which are 120° farther around the armature may be connected to transformer 3. The electromotive force, thus generated and impressed on each transformer, may be thought of as the counter-electromotive force of a direct-current motor. The high-voltage line must supply an electromotive force to balance the counter-electromotive force Figure 259 shows the complete connections of the double-delta method of converting from three-phase to direct current.

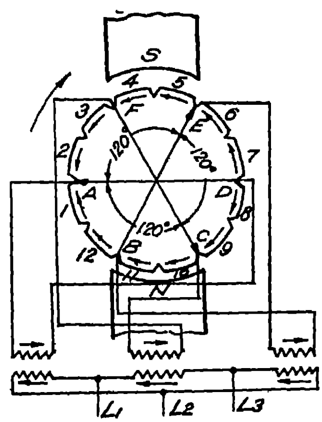


Fig 260. — Diametrical Connection

Diametrical Connection. The diametrical connection is illustrated by Fig. 260. In this method of connecting, rings connected to the armature at diametrically opposite points are connected to the single-coil secondaries of the transformers. In the position shown,

the sum of E. M. F.'s in 7, 6, 5, 4, 3 and 2 is DA

120° later the sum of E. M. F.'s in 3, 2, 1, 12, 11, 10 is FC = DA

120° later the sum of E. M. F.'s in 11, 10, 9, 8, 7, 6 is BE = FC = DA

Hence the taps must be connected so that these electromotive forces act through the transformers 1, 2 and 3 in the same directions, respectively, as the three sections of the armature reach any given position, as for instance, that when DA was midway between poles, Fig. 260.

Methods of Starting Rotary Converters. A converter may be started from the direct-current side, when direct current is available. The procedure is similar to that in starting a direct-current motor. When the converter is up to speed it must be synchronized, because while the alternating voltage might equal the voltage of the line, the machine would not necessarily come in with the alternating voltage in phase with the line voltage. The procedure in synchronizing would be exactly like that in synchronizing a synchronous motor.

Polyphase converters may be started from the alternating-current side. If alternating current be fed into the armature of a polyphase rotary converter through the slip rings, it will form poles on the armature similar to the poles formed on the stator of an induction motor. The poles thus formed will move around the armature producing a revolving field. As these poles move on the converter armature, their lines of force will cut the pole faces, poles, and copper of the field windings and produce a torque, just as the stator field in an induction motor cuts the squirrel-cage rotor and produces torque. The result is that the motor gradually comes up to nearly synchronous speed. Reduced voltage is used for starting. This is obtained by having a double-throw switch by which the converter is thrown on taps of the main transformers. When nearly up to speed, the switch is thrown over to the full-voltage position. When the armature is standing still, or just starting, the transformer action between armature and field windings is sufficient to generate a very high voltage in the field coils which are in series with each other. To guard against this high voltage breaking down the insulation, the individual field coils are disconnected from each other by means of a switch known as a field break-up switch.

When the machine is nearly up to speed the transformer action between armature and field is very small because the armature is turning nearly as fast as the field. The direct-current field may then be put on the machine The armature will fall into step but will not necessarily fall in step with the armature in the correct position with relation to the poles to give the proper direct-current polarity. If the direct-current voltmeter indicates wrong polarity, the field and main switches are opened and the machine allowed to drop back one pole. When the machine has dropped back one pole and the switches are again closed the meter will indicate correct polarity.

Power Factor Control — Loss of Output with Low Power Factor. Since the converter has characteristics similar to a synchronous motor, the power factor can be controlled by varying the strength of the field. As in synchronous motors, a strong field will cause a leading current and a weak field will cause a lagging current.

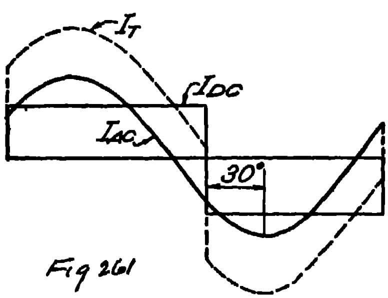


Fig. 261. — Current Relations in a Conductor at a Tap. (Lag of A C Current 30°)

The output of a converter drops off considerably with a decrease in power factor. This is caused by large increase in conductor current. If waves of direct current and alternating current similar to those of Fig. 253(b) and 254(b) be drawn, but several degrees out of phase, and a resultant wave plotted, the resultant wave will have a larger average value than when the waves are drawn for 100% power factor with current in phase with voltage.

The heating will be greater Figure 261 and 262 show the two waves 30° out of phase and the resultant current wave dotted.

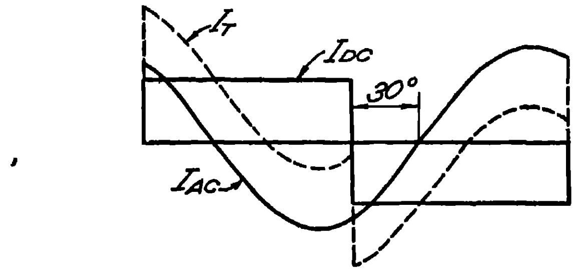


Fig 262. — Current Relations in a Conductor Midway Between Taps. (Lag of A C Current 30°)

Special Apparatus. The relation between the alternating voltage and the direct voltage cannot be changed to any extent by changing the field on the converter Change in the field simply changes the power factor. When it is desired to boost up or buck down the incoming voltage, an alternator with the same number of poles as the converter is coupled to the shaft of the converter and its armature connected to the incoming line. The fields are excited from the same source as the fields of the converter. When the field of this generator, which is called a booster, is raised, the booster voltage adds itself to the converter voltage, when the field is lowered, the booster voltage subtracts itself.

The booster field may be reversed if desired and the machine used to buck the line voltage.

The induction regulator described in Chapter XI may be used to vary the alternating voltage of a converter and thereby vary the direct voltage.

Split-Pole Converter. The split pole converter, in appearance, resembles a machine with commutating poles. The auxiliary poles on a split-pole converter serve a different purpose, however. They are used to change the shape of the alternating wave of electromotive force that is generated when the armature revolves. Assuming that a machine gave a wave similar to wave a in Fig.

263 without the auxiliary poles, if the field be changed by superimposing on it a field from small auxiliary poles it might have a shape somewhat like waves b or c. This change in wave-shape changes the ratio between the effective alternating voltage and the direct-current voltage, and since the alternating voltage remains constant, the direct voltage may be raised or lowered by changing the excitation of the auxiliary poles.

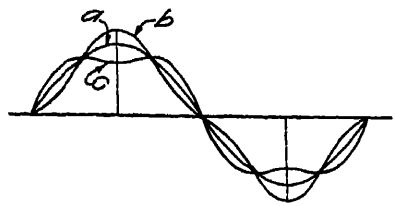


Fig. 263. — Waves Showing Effect of Auxiliary Poles.

PROBLEMS

1. What electrical conditions are necessary in order that two alternators may be run in parallel?
2. Show by a diagram that a synchronous motor may have two excitations that will give the same output.
3 Explain one method of synchronizing two single-phase machines
4. What is meant by "hunting"?
5 What is a rotary converter?
6. What changes would be necessary in a direct-current generator to make it into a rotary converter?
7. Explain why the conductors near the taps on a rotary converter heat more than those at some distances from the taps.
8. Why are rotary converters for power work usually six-phase?
9. Explain by a diagram how you would find the voltage between slip rings if you tapped a 110-volt armature at three points equidistant from each other.
10. Explain two methods of starting rotary converters.
11. What is a split-pole converter?

CHAPTER XI

OTHER ALTERNATING-CURRENT APPARATUS

Types of A. C. Meters. Several types of alternating current meters that are in common use will be described, covering, ammeters, voltmeters, wattmeters, watthour meters and recording or graphic instruments An understanding of the principles of operation of these instruments described and illustrated will give a good working knowledge of well-known standard types and a background for analyzing other types that may come within the reader's observation.

The Electrodynamometer Principle. Two coils with their magnetic axes at right angles to each other will tend to turn so that their magnetic axes point in the same direction if current is

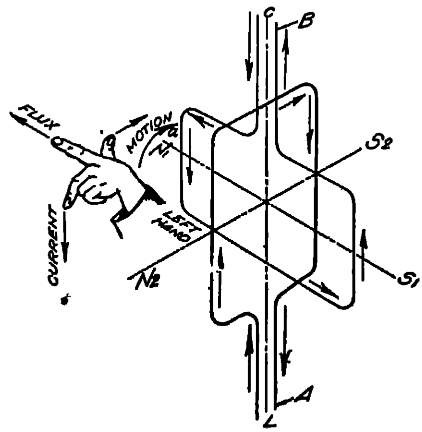


Fig 264. Diagram Illustrating the Dynamometer Principle.

sent through the coils. Figure 264 shows a fixed coil A and a movable coil B, standing at right angles to each other. The movable coil is free to turn on the center line CL. If current be

sent through coil A clockwise as shown by the arrows, this current will produce a field acting in the direction S_1N_1. Current sent through coil B counter-clockwise as shown, will react on the field set up by coil A, so that the coil B will turn clockwise as viewed from the top or as shown by arrow "a." This fact will readily appear by application of the three-finger motor rule as shown at the left of the sketch, or from the fact that the two coils produce resultant fields S_1N_1 and S_2N_2, that may be thought of as bar magnets. Such magnets will tend to arrange themselves parallel.

If the currents be reversed in both coils the movable coil will

Fig 265. Weston Dynamometer-Type Wattmeter.

tend to turn in the same direction as before, since both the field and current will be reversed and the three-finger rule will show that motion will be in the same direction.

Application to Meters. The dynamometer principle is applied in the construction of ammeters and wattmeters. If coil A which is fixed be connected in series with coil B and the tendency of coil B to turn be opposed by a spring, then a pointer attached to coil B will be deflected in proportion to the current. The apparatus when properly calibrated becomes an ammeter.

If coil A be connected in series with the line and coil B be

connected across the line through a high resistance, then the current in coil A is the line current and the current in coil B is proportional to the voltage. The apparatus when properly calibrated becomes a wattmeter.

The Weston Dynamometer-Type Wattmeter. Figure 265 shows a high-grade dynamometer type of wattmeter made by the Weston Electric Instrument Corporation. S is the stationary field coil and M the movable or potential coil.

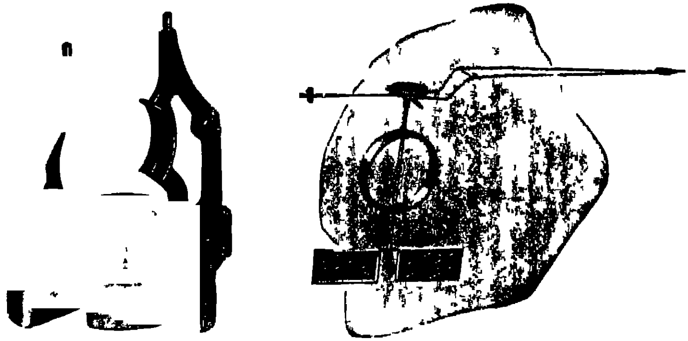


Fig 266 —Left, Clamping Device for Stationary Coil. Right, Movable Elements.

Figure 266 shows the clamping device for the field coil and the movable coil, pointer and damping device. The damping device consists of a very thin but rigid vane which fits into the sector-shaped chambers shown at the bottom of the clamping device for the field coil. The vane moves with a very small clearance in the chambers which have covers on when the instrument is assembled Damping takes place by the compression of the air as the vanes swing across the chambers. Polyphase meters have two field coils, one over the other, clamped in a device similar to Fig. 266 but longer. The movable element has two pressure coils, one above the other, on a shaft about twice as long as that shown by Fig. 266.

The Weston Dynamometer-Type of Ammeter. The Weston dynamometer-type ammeter has its movable parts constructed and arranged, in general, the same as the wattmeter. There are

two field coils which consist of a relatively small number of turns of medium-size conductor The coils can be connected either in series or parallel by suitable links on the instrument, thus making it two-range When used for a low-range meter, the field coils are connected in series with each other and across a resistance called a shunt as in (a) Fig. 267. When connected for high range,

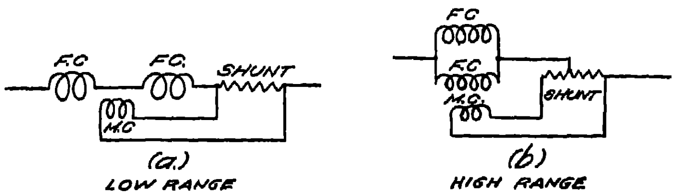


Fig. 267. — Internal Connections of a Weston Two-Range Dynamometer-Type Ammeter

the coils are connected in parallel and across half the shunt as at (b). The movable coil is always connected across the entire shunt The field coils thus draw current proportional to the current through the shunt and the movable coil, current proportional to the drop across the shunt. The drop in the shunt is proportional to the current. The torque of the instrument is thus proportional to the square of the current or effective value.

The Movable-Iron or Electromagnetic Instrument. If two pieces of iron be placed in a coil as shown at AB and CD, Fig. 268 and direct current be sent through the coil as shown, ends A

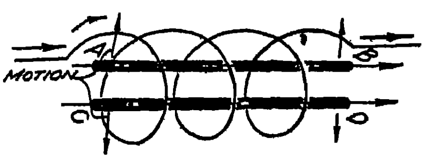


Fig. 268. — Principle of Electromagnetic Instrument.

and C will become south poles and ends B and D north poles. The two pieces, being magnetized with like poles at the same ends, will be repelled. If alternating current be used instead of direct current, repulsion will take place because the poles of both pieces

reverse at the same time. The principle is utilized in the electromagnetic type of instrument by making one piece of iron fixed in position and attaching the other to a pointer which swings over

Fig 269 — Weston Electromagnetic-Type Ammeter.

a suitable calibrated scale Figure 269 shows a Weston ammeter of this type

Inclined-Coil Instrument. The inclined-coil type of instrument is shown schematically by Fig 270. C is a stationary coil which

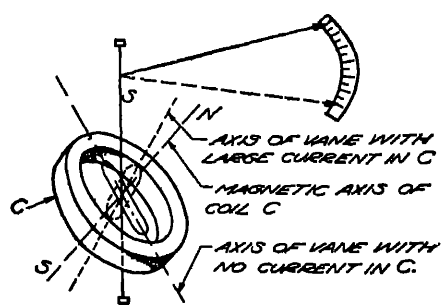


Fig 270 Inclined-Coil Instrument.

is set at a fairly large angle with the shaft S The field of the coil will be along the line SN. In one form of instrument, the shaft S carries an iron vane also set at an angle with the shaft.

When the coil C is energized, the vane tends to align itself with the magnetic axis of the coil C. In another form of instrument, the shaft carries a coil instead of an iron vane The coil is set at an angle with the shaft and, when energized, tends to align its field with that of the coil C. This instrument is really an electrodynamometer with the axes of the two coils at a considerable angle instead of coincident as in that shown by Fig. 264.

The Electrostatic Voltmeter. This instrument depends for its action on the attraction of oppositely charged bodies. In Fig. 271, if sectors D_1 and D_2, which are fixed in position, be connected to one side of the line, and D_3 and D_4, which are attached to a pointer, be connected to the other side, the unlike charges on the two sets of sectors will attract each other and turn the pointer P. This pointer will register volts on a properly calibrated scale.

Fig. 271.—Electrostatic Voltmeter (General Electric Company).

This instrument shown by Fig. 271 is a General Electric Type EL electrostatic voltmeter suitable for voltages from 3000 to 10,000

The Induction Watthour Meter. The induction watthour meter which is an alternating-current meter only, is essentially an induction motor operating on the split-phase principle. Its load consists of a train of gears to which the pointers that register the watthours are attached, and a magnetic damper or brake which is an aluminum disk that rotates between the poles of a permanent magnet. The magnet induces eddy currents in the disk, and these in turn react on the magnet to retard the motion of the disk.

Figure 272 is a schematic diagram of an induction watthour meter connected in an alternating-current circuit to record the watthours consumed by the load L. S_1 and S_2 are two field coils connected in series with the line. P_c is a potential or pressure coil connected across the line. S_1 and S_2 carry the load-current

and P_c carries current proportional to the voltage. The fluxes set up by the two currents react on the disk D proportional to the power in the circuit. The pointers P_1P_4 will register on their dials an amount proportional to the number of revolutions of the disk, or, with proper gearing and damping, the watthours consumed by the load. The relations between line voltage E_L, the line cur-

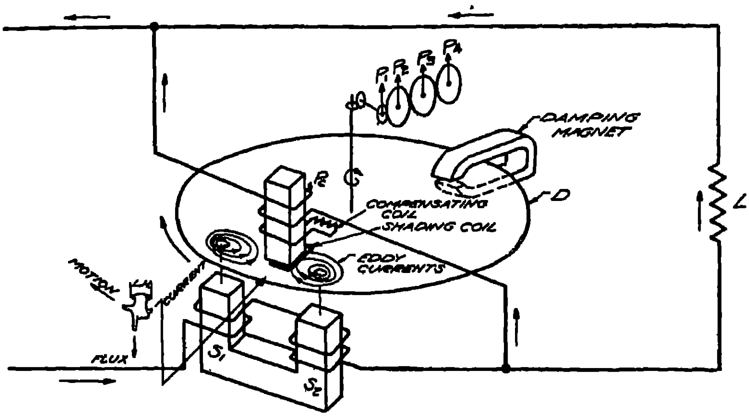


Fig 272 —Schematic Diagram of Induction Watthour Meter.

rent I_s, and the flux ϕ_s set up by the series coils are shown by Fig. 273 All three are in phase. The flux due to the current in the pressure coil P_c is made to lag behind the flux ϕ_s in the series coil by an angle of 90°. With the two fluxes 90° apart, the condition is exactly like that in the elementary induction motor described on page 184 and a revolving field is produced that sweeps over the disk D. The disk follows the field, due to the reaction of the eddy currents produced in the disk It may be seen that the disk will turn by application of the three-finger motor rule. The sketch at the left in Fig. 272 shows the direction of motion for the particular directions of field coil, flux and pressure-coil flux illustrated in the diagram.

Since it is not possible to make the current in the coil P_c lag exactly 90°, due to the fact that the coil has some resistance, the flux in the disk due to this coil is made practically 90° from the

flux in coils S_1 and S_2 by placing a small coil in the path of the flux through the disk and connecting this coil through a re-

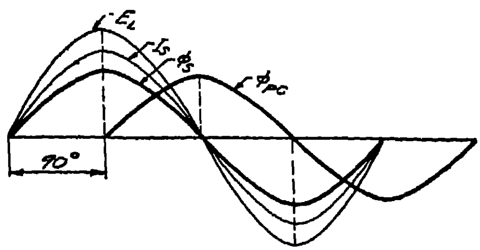


Fig 273—Relations of Voltage, Current and Flux in Induction Watthour Meter

sistance. The actual meter has a small short-circuited coil or ring near the end of the pole P_c that can be moved slightly to right or left for adjustment. This coil is for the purpose of providing enough torque to overcome the friction of the meter A coil placed near one side of a pole, as mentioned, crowds the flux to one side of the pole and produces a splitting of the flux, or what is known as a "shaded pole" Such an arrangement produces torque just as splitting a phase produces torque due to the revolving-field principle.

Oscillograph. The oscillograph is an instrument by which waves of E. M. F. or current can be observed or photographed. In principle, it is very simple. The construction where a photograph of the wave is desired will be described first.

In Fig. 274, N and S are the poles of a powerful magnet L is a loop of very fine wire to which a very small mirror is fastened in a vertical position. The loop and mirror are suspended by a delicate filament F and form the moving element of the instrument. A is an arc lamp which is enclosed in a box having a hole on the side next the mirror so that the light can be thrown only along the line R. A point of light will then appear in the mirror and, if the mirror oscillates about a vertical axis, this point will trace a line on the surface of a cylinder C. The current whose wave is to be traced is carried through the loop of

wire L and causes it to oscillate. The moving element, being extremely light and sensitively suspended, is able to follow the variations of the current. Thus far, only a line will appear on the cylinder. If, now, the cylinder be turned at a uniform rate, a wave will be traced on the surface of the cylinder which will faithfully record the manner in which the current in the loop L varies. The zero line can be obtained by allowing the cylinder to turn with the current shut off from the loop A photographic film on the cylinder C will give a permanent picture of the wave.

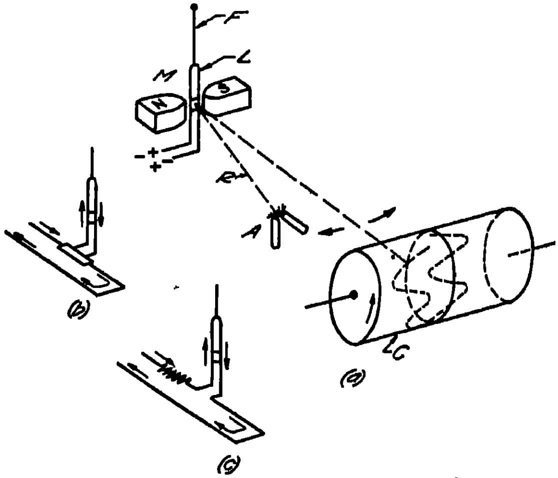


Fig. 274 —Elements of an Oscillograph.

The actual apparatus has necessary light shields, shutters, and motor to turn the cylinder, and has other mechanical details worked out so that the elementary principles of operation described can be carried out with a high degree of precision.

In order to photograph current, a shunt is used across L as at (b). To photograph voltage, a high resistance is put in series with L as at (c).

It is sometimes desirable to observe the wave and not photograph it. In this case the cylinder C is replaced by a prism, usually with 6 faces, each face being a mirror. The prism is

turned by a synchronous motor. The ray from the mirror of the moving element L is thus reflected again and this final reflection is thrown upward on a piece of ground glass. Since the prism of mirrors turns in synchronism with the current through the loop, a picture of the wave will appear on the ground glass.

Oscillographs are made both in laboratory form and in portable form suitable for carrying on a job

Synchroscope. One form of synchroscope that is widely used is that made by the Westinghouse company known as the Type SI. It is essentially a small synchronous motor. The stator is connected to the machine that is running and the rotor is connected to the incoming machine. In order to do away with movable electrical connections, an iron vane is used in place of the usual rotating winding This vane receives its magnetism from a stationary coil which surrounds it. The details of this vane and the method by which it is magnetized will appear from the sketch and following description.

Figure 275 shows the front view of an SI single-phase synchro-

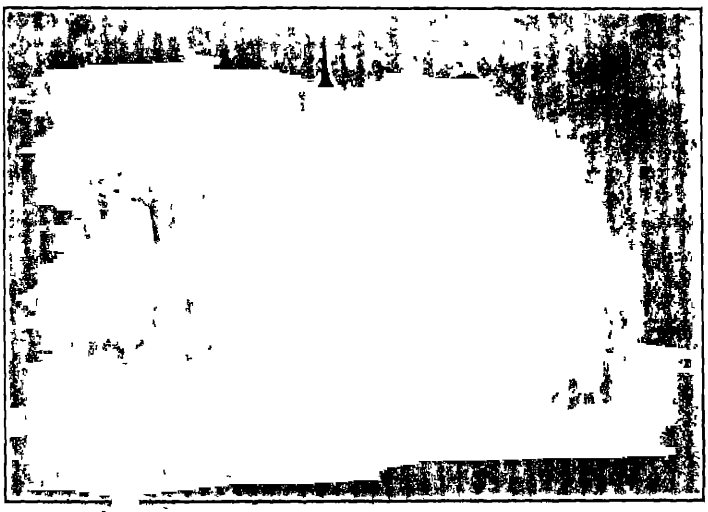


Fig. 275.—Synchronoscope with Case and Cover Removed (Westinghouse Electric & Mfg. Co.).

scope removed from the case, and Fig 276 shows schematically the arrangement of the coils and moving element.

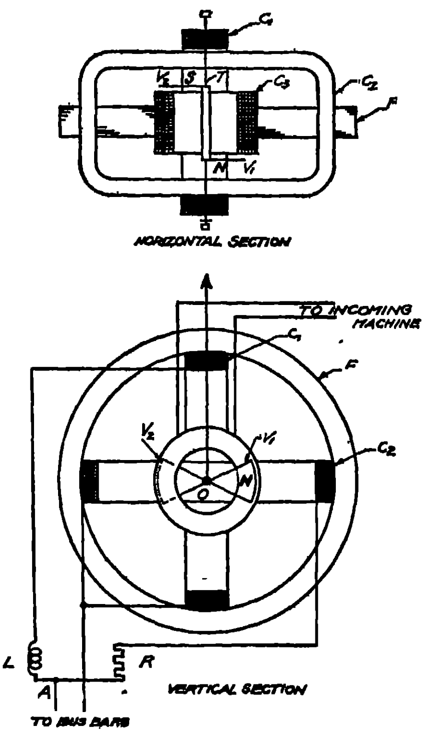


Fig 276 — Circuits of sI Synchroscope. (Westinghouse Electric and Mfg. Co.)

C_1 and C_2 are two stationary coils at 90 degrees from each other like the windings of a two-coil two-phase stator of a synchronous motor F is the laminated stator. One end of coil C_1 is carried to an inductance L, and the end of the other coil C_2 is carried to a resistance R. The resistance and inductance are connected to one side of the line at A. The other ends of the coils are connected together and form the other side of the line. The inductance and coil C_1 are thus connected in parallel with the resistance and coil C_2. The arrangement is really a split-phase motor stator. Current lags greatly in coil C_1, and as explained under induction motors, a revolving field is produced. A conductor such as a disk pivoted within this stator would revolve just as the rotor would revolve in a motor. The arrangement is slightly different in this instrument, however, as two vanes V_1 and V_2, rigidly fastened to a shaft T, constitute the rotor

Surrounding the shaft T and between the vanes there is a coil C_3 rigidly mounted. The center line of this coil coincides with the center of the shaft. Inspection of the drawing will show that

if coil C_3 is made to carry current the vanes will be magnetized with poles at N and S, or vice versa, depending on the direction of the current in coil C_3. If alternating current be supplied to C_3, the polarity of the vane rapidly reverses. The arrangement is analagous to a wound rotor fed with alternating current from an outside source.

Coils C_1 and C_2 are connected through the resistance and reactance previously described to the machine that is running, and coil C_3 is connected to the incoming machine. The rotating field may be thought of as an axis of magnetic lines that rotates about O as a center. What really happens is, that at one instant when the current in C_1 is maximum, the current in C_2 is practically zero, and since the inductance makes the current lay nearly 90°, the magnetic axis is thus horizontal A quarter of a cycle later, the current in C_2 is maximum and in C_1 is zero, so the axis is shifted to a vertical position. Similarly, when the waves are each at the 45° phase, the axis shift to 45° with C_1 and C_2. Current supplied to C_3 will magnetize the moving element V_1TV_2, and, if the axis of its magnetic field is in phase with the magnetic axis set up by C_1 and C_2, the vanes V_1 and V_2 will line up with this axis. For example, suppose current in C_2 is maximum and in C_1, zero, the field will be vertical at this instant. If the field of the vanes is maximum at this instant, the vanes will stand vertical, but if at this instant the current lags a quarter of a cycle it will not reach its maximum until a quarter of a cycle after the current in C_2 reaches its maximum, so the vanes will stand at 45°. The vanes are connected to a needle which moves over a circular scale. In the synchroscope, C_3 is connected to the incoming machine. Hence when the incoming machine is in step with the running machine the needle will remain stationary.

There being no sliding contacts, this instrument is very sensitive and indicates clearly when the machines are in step. Due to the synchronous-motor principle, it also indicates whether the incoming machine is going too fast or too slow by the direction the needle turns.

Power-Factor Meter. The principle used in the synchroscope, just described, is applicable to a power factor meter as well. In a power-factor meter one coil, as C_1, is connected in series with a non-inductive resistor, and the other coil C_2 in series with an inductive resistor. The two sets are connected in parallel across the line of the circuit to be measured. Current, then, in one coil C_1 is in phase with the E. M. F. and in the other coil C_2 it lags the E M F, thus giving a revolving field exactly as in the split-phase motor. The coil C_3 carries current in phase with the line current. The deflection of the vane and needle will depend on the angle between the fields from the coils C_1 and C_2 and the coil C_3, and so the meter when properly calibrated will indicate power factor. Modern power-factor meters, however, are made on the moving-coil dynamometer principle. The type of power-factor meter above described has been superseded by the dynamometer type known as type SY.

Mechanical Rectifier. Figure 277 shows a mechanical rectifier made by the Kelly-Koett Mfg. Co for rectifying high-voltage current. The particular apparatus shown is for use with an X-Ray tube. It is designed to rectify a wave of 230,000 volts peak value The tank which forms the base of the apparatus contains an oil-insulated transformer which steps up the voltage from 220-volt commercial circuit to 230,000 for use on the tube. The motor shown mounted at the center of the apparatus is of the synchronous type and runs in phase with the current that is fed to the transformer. The disks at the ends of the double extended motor-shaft are of bakelite. Each disk is provided with two segments of contact metal fastened to the rim of the disk. The segments are opposite each other and each covers one-fourth of the circumference of the disk. Brass collectors or shoes are provided at diametrically opposite points to collect the current that is commutated by the disk. The arrangement is similar to the two-part commutator generally used in explaining the elementary direct-current machine, the difference being that the collectors, which correspond to the brushes, do not ride on the commutator but have a clearance of about $\frac{1}{16}''$. The voltage being high, cur-

rent readily jumps the small gap between collectors and segments
One disk would rectify the current but, by using two disks, the voltage across the disks can be cut in half.

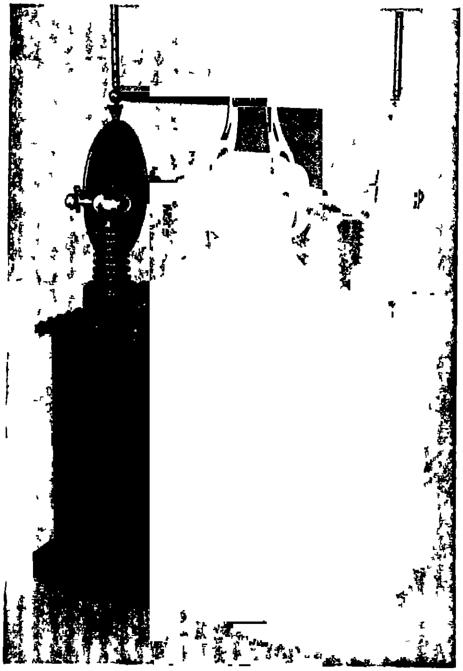


Fig. 277. — 230,000 Volt Transformer and Rectifying Unit (Kelly-Koett Mfg Co)

The schematic diagram of Fig. 278 will make clear the operation of the X-Ray tube and the double-disk rectifier. The X-Ray tube has a filament F which is heated by current from a small transformer. Opposite the filament there is a tungsten electrode T called the target. This target has its surface at 45° with the axis of the tube. The tube is exhausted to a very high vacuum. When the filament is heated it will give off electrons. If a high D. C. voltage is impressed across the filament and target, the electrons will be thrown off the filament and strike the target at a

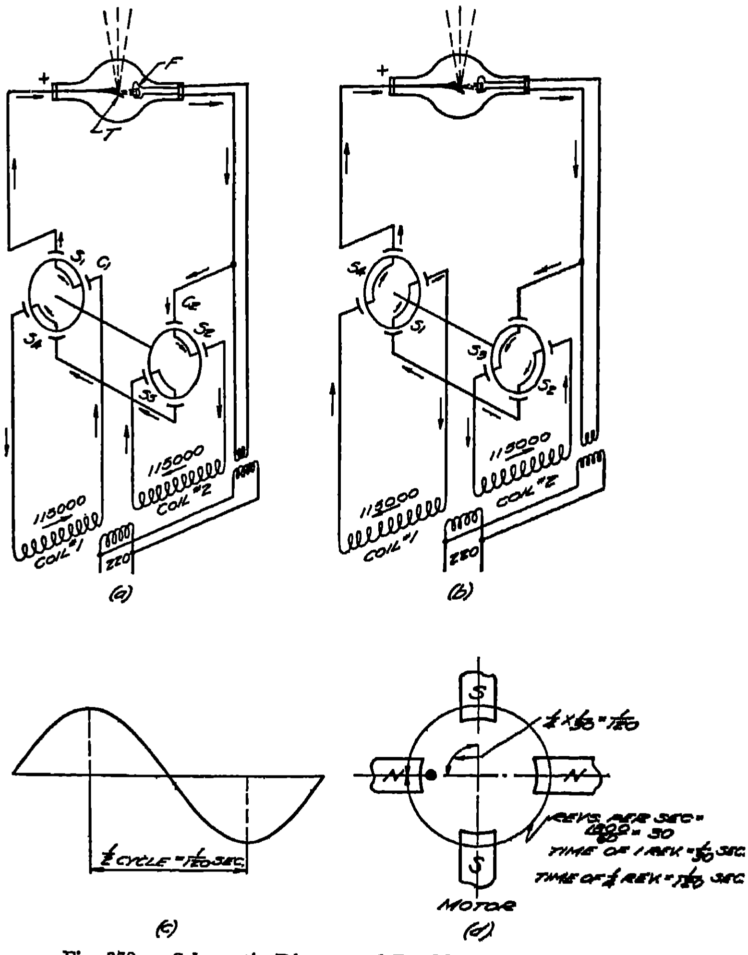


Fig. 278 — Schematic Diagram of Double Disk Rectifier, Transformers and X-Ray Tube. (Kelly-Koett Mfg Co.)

high rate of speed These electrons, in striking the target, produce vibrations which are thrown off from the target as the rays.

The direct current is provided as follows. The transformer has two secondaries each capable of delivering ½ of 230,000 volts or

115,000 volts Assuming the segments are in position shown by (a) at S_1 and S_2, S_3 and S_4 current flows from transformer coil #1 to collector C_1, to segment S_1, to tube, to collector C_2, to segment S_2, to transformer coil #2, to segment S_3, to segment S_4 and back to coil #1.

Assuming the apparatus is 60 cycles, the E. M F. will have reached a maximum value in a direction opposite to that shown by sketch (a) $\frac{1}{120}$ second later. During this time the motor will have turned the commutator to the position shown at (b) which is $\frac{1}{4}$ of a revolution farther on.

This will be apparent from study of sketches (c) and (d) The path of the current will then be as indicated by the arrows of sketch (b) or the current through the tube is in the same direction as before.

The Kenotron. The kenotron shown by Fig. 279 is a form of

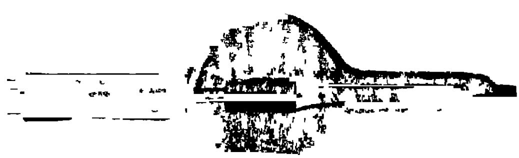


Fig. 279. — High Voltage Kenotron.
(General Electric Co)

rectifier that is used for high-voltage rectification. It consists of a highly evacuated tube in which two electrodes are sealed. One of these consists of a small coil that may be heated by means of low-voltage current. The other electrode consists of a cylinder surrounding the heating coil but not touching it. The connections are shown by Fig. 280. P_1P_2 and S_1S_2 are the primary and secondary coils of the transformer that heats the filament F, and P_3P_4 and S_3S_4 are the primary and secondary coils of the main transformer. The high-voltage current in S_3S_4 is rectified by the kenotron as follows; When the filament is heated and the filament and cylinder are subjected to an electrostatic field or such a field as exists between charged bodies, the filament will throw

off electrons. Electrons are negative in character and will be attracted to a body that is positively charged. Since the filament and cylinder are connected across the transformer coils S_3S_4, the cylinder becomes positively charged during every other half-cycle. During the half-cycle that the cylinder is positive, electrons flow to it. When it becomes negatively charged during the other half-cycle, electrons cannot flow from the cylinder to the filament, because a heated condition of an electrode is necessary, in order that it may throw off electrons. Hence the electron flow is from filament to cylinder only. The kenotron, as shown, rectifies only one half of the wave. The other half is suppressed. The action is similar to a check-valve in a pipe. Water may flow in one direction but, when it reverses, the valve closes Just as pressure will build up at the check-valve, so electric potential will build up at the terminals of the kenotron tube.

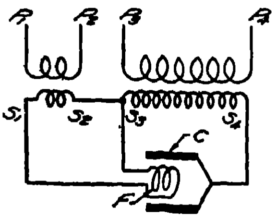


Fig. 280. — Circuits of Kenotron Rectifier

Current flow takes place entirely by electrons in the kenotron. The electron flow as described, is directly opposite in direction to "current flow" as we commonly understand it.

The Tungar Rectifier. The tungar rectifier resembles the kenotron in some respects, and while it depends for its action on the throwing off of electrons from a hot filament, its operation is somewhat different from that of the kenotron. The tungar rectifier is used for low-voltage rectification. It is used to a large extent in battery charging The construction of the bulb is essentially like that of the kenotron, there being a filament and plate, the terminals of which are sealed into the walls of the bulb. The bulb, instead of being exhausted to a high vacuum, is filled with an inert gas under a low pressure. When a gas is under low pressure, its molecules are relatively far apart. Electrons, in moving at a high rate of speed from the hot filament to the plate, knock loose some of the electrons that normally are attached to the molecules and ionize the gas, as it is called. An ionized gas is

a conductor of electricity. The phenomenon may be pictured as electrons tearing away some of the electrons from the molecules and thus breaking up the molecules of the gas into two parts or units. One of these is negative and consists of the electrons themselves, the other is a unit, positive in character. These positive and negative units are called ions. They move towards bodies of polarity opposite to their own.

In the tungar bulb, the negative ions are attracted to the plate which is positive, and the positive ions to the filament which is negative. Due to the fact that the plate cannot give off electrons, since as explained in the case of the kenotron, a heated conductor is necessary for the emission of electrons, electron-flow takes place only from the filament.

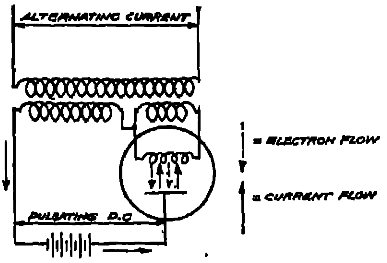


Fig. 281.—Circuits of Tungar Rectifier.

Figure 281 shows the circuits when a tungar bulb is connected to a line through a transformer. The apparatus as shown will rectify but one-half of the wave. Two tubes can be connected so that both halves of the wave will be rectified.

The Three-Element Vacuum Tube. The three-element tube is a vacuum tube containing a filament and plate and a third element known as a grid. The grid is a sieve-like structure placed between the filament and plate. The purpose of the grid is to control the flow of electrons from the filament to the plate or, in other words, control the plate current. It does this with the expenditure of very little energy. The grid is analagous to the gate in a gate-valve. A small amount of energy expended in raising or lowering the valve will control a very large amount of energy, flowing through the valve as, for instance, in the case of steam or gas.

Figure 282 shows a 3-element vacuum tube. The filament F is heated by means of the battery A. In relation to the plate P,

the filament is kept negative, so that electrons flow from filament to plate. If the grid, which is between the two, is made positive it will increase the flow of electrons, since making the grid positive is equivalent to making the plate more strongly positive If the grid is made negative, it will decrease the flow of electrons, since the like charges will repel, and the electrons will be forced back to the filament.

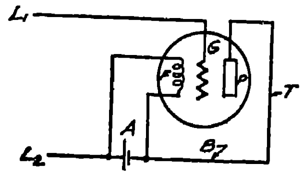


Fig. 282 — Circuits of Three-Element Vacuum Tube.

A very slight change in the potential of the grid will influence the flow of electrons to a very considerable extent.

If a telephone receiver be placed at T and a second battery at B, very slight changes in potential across L_1L_2 will cause loud noises in the receiver. This principle is used in the radio detector.

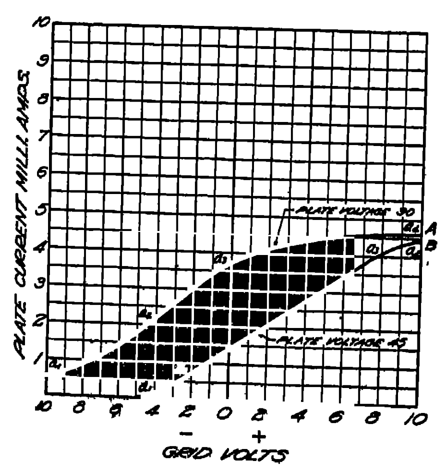


Fig 283. — Characteristic Curve for a Three Element Tube.

The behavior of a tube when subjected to varying grid voltages both + and − can best be understood by means of curves. In

Fig. 283 curve A is plotted for a voltage between filament and plate of 90. The abscissas represent readings of grid voltage, and the ordinates the values of plate current. Curve B is a similar curve for a plate voltage of 45 instead of 90. Several important characteristics appear from the curve. First: The change in plate current is slow at first, increasing its rate from point a_1 until a point a_2 is reached. From a_2 to a_3 the change is very rapid, but practically uniform. From a_3 to a_4 the change is variable again but less rapid than from a_1 to a_2. Second: The curve shows that, at near zero grid voltage, a very slight change in grid voltage causes a large change in plate current. Third: The plate current can be changed by changing either the plate voltage or the grid voltage.

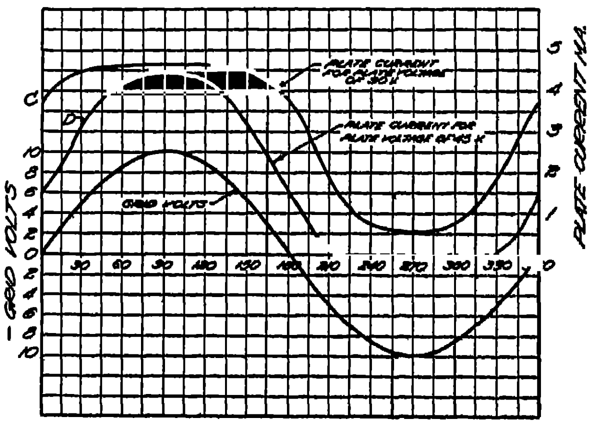


Fig. 283-a. — Variation of Plate Current when an Alternating E. M. F. is Impressed on Grid.

Figure 283a shows how the plate current in the tube, whose characteristics are given by the curves of Fig. 283, will vary when an alternating E. M. F. is impressed on the grid.

Assuming that a sine wave of E. M. F. with a maximum value of 10 volts is impressed on the grid, the plate current will vary as

curve C if the plate voltage be held at 90 and as curve D if the plate voltage be held at 45.

In plotting curves C and D values of grid voltage are taken from the curve marked "grid volts," Fig. 283a, and the values of plate current are taken from the curves of Fig. 283. Curve C is obtained from curve A and curve D from curve B.

The Three-Element Vacuum Tube Used as an Oscillator. It was shown in Chapter V that the current in a series circuit becomes equal to $\frac{E}{R}$ when $2\pi fL = \frac{1}{2\pi fC}$. The circuit is said to be in resonance when the frequency is such as to satisfy the above equation for given values of L and C. Further, when an impulse of E. M. F. is set up in such a circuit, current will surge back and forth or oscillate, as it is called, at the frequency f until it gradually dies out due to the various losses in the circuit. The frequency of the current will be that obtained by solving the equation, $2\pi fL = \frac{1}{2\pi fC}$ or $f = \frac{1}{2\pi\sqrt{LC}}$.

If, with such a circuit, we impart properly-timed impulses of E. M. F., current will continue to oscillate as long as the properly-timed impulses are kept up. A vacuum tube may be used to set up oscillations by using some of the energy of the plate circuit to feed back into the grid. When L and C are properly adjusted in the circuit to which the tube is connected, we can obtain very high frequencies by this means.

The apparatus is analagous to the pendulum of a clock. The pendulum, when once started swinging, uses some of its energy to release one tooth of the escapement wheel at each swing. At these instants, the pendulum receives properly-timed pushes from the clock spring, through the medium of the escapement wheel, to keep it swinging. If the length of the pendulum be changed, the number of swings per minute will change. Similarly, if the product of L and C in an electric circuit be changed, the natural frequency of the circuit will be changed and it will oscillate at a different rate.

Figure 284 shows a three-element tube used as an oscillator.

When electrons start to flow from the filament F to plate P, current flows in the inductance coil L_1 as shown by the full ar-

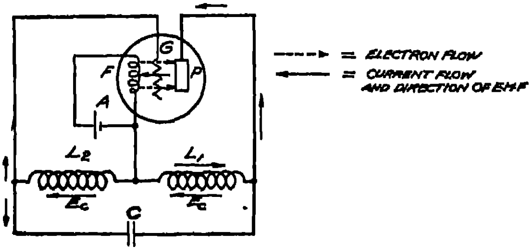


Fig. 284. — Three-Element Tube Used as Oscillator.

row. This current magnetizes the core which is common to L_1 and L_2 and induces a counter electromotive force E_a which sends current to the condenser C and also the grid G The grid is thus made more strongly positive and draws more electrons from the filament. Thus the current increases.

The increase in current will continue until the saturation point for the tube is reached when the current becomes steady for an instant. The current then starts to fall because the voltage E_a in L_1L_2, which is an induced voltage, becomes zero when the current becomes steady. This has been explained under transformers.

The current will fall when the E. M F from L_1L_2, which assists electron flow, becomes zero. As it falls, a counter-electromotive force is set up in the reverse direction which makes the current fall rapidly towards zero.

The relations between plate current and grid voltage will be as in Fig. 285. The reason that the grid voltage begins to drop off at the point "a" is that the plate current increases less rapidly after the point "a" is reached. This will appear from the curve for the tube, which has been shown at the left as consisting of three parts, two curved and one straight. When the current drops off less rapidly, the counter E. M. F. becomes less and consequently the grid voltage as well.

At the point "b," the positive grid voltage that was induced in

L_1L_2 and boosted up the plate current becomes zero, and so the plate current begins to fall. As it falls, a counter E. M. F. is generated in the reverse direction in L_1 and L_2 and this aids in reducing the plate current still more. By the time the plate current is zero, the induced voltage E_c is zero and a cycle of grid voltage has been completed and also an oscillation of plate current. Thus the current delivered to the circuit L_1L_2C is in the form of oscillations of a frequency such that,

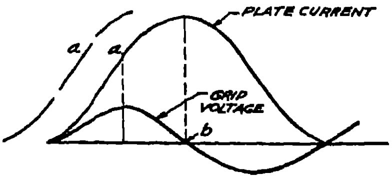


Fig 285 — Relations of Plate Current and Grid Voltage in a Tube Used as an Oscillator

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{35}$$

Mercury-Arc Rectifier. The mercury rectifier depends for its action on the fact that vapor from mercury, contained in a highly-evacuated bulb or tube, possesses the property of allowing current to flow through it in one direction only. If an electrode be sealed in the bulb above the mercury and a second electrode in the bulb below the surface of the mercury, current can be made to flow from the electrode above the mercury to the mercury and thence to the second electrode but not from the second electrode to the first electrode because the mercury vapor acts as a valve and prevents this opposite flow. In order to start the current, it is necessary to tip the bulb so that there is an actual mercury path from one electrode to the other This is necessary because the mercury vapor offers a very high resistance to the flow of current until it is once broken down and the current started.

A form of rectifier common for battery charging is shown diagrammatically by Fig. 286. G is a glass bulb exhausted to a very high vacuum. $A_1A_2A_3$ and C are electrodes sealed into the walls of the bulb. M is the mercury which fills the bulb to a point just below A_3. T_1T_2 is a transformer which steps down the voltage to a value suitable for the bulb. A_1 and A_2 are the regu-

lar operating terminals or anodes and A_3 is a terminal for use in starting the current. C is the cathode or terminal from which the rectified current flows from the bulb to the battery to be charged. To start the current flowing, it is necessary to tip the bulb until mercury flows over to A_3 and forms an actual metallic path. Current will then flow from the transformer through the mercury and out at C. The resistance B limits the current to a safe value in starting.

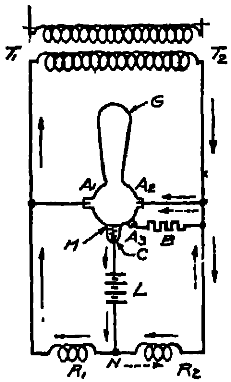


Fig 286 — Mercury-Arc Rectifier

Assuming that the arc has been started and that T_2 is positive, current will flow into the bulb at A_2 and out at C. During the next half-cycle, T_1 is positive and current flows into the bulb at A_1 and out at C. The valve action of the mercury vapor prevents the current from flowing from C to A_1 or from C to A_2.

Without the reactance coils R_1 and R_2, the arc would break when the current passed through zero at the end of a half-cycle The effect of these coils is to sustain the current from one electrode until the current from the other electrode begins to flow, so that the current does not at any time actually fall to zero and the arc break. The rectified wave is of the shape shown by Fig. 287.

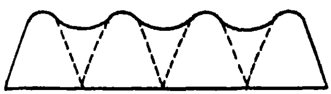


Fig 287. — Rectified Current from Mercury-Arc Rectifier

The coil R_1 and R_2 operate as an autotransformer as follows: Assume that T_2 is positive and that current is flowing from T_2 to A_2 through the bulb and out at C. Part of the current from T_2 flows through the reactance coil R_1 from [illegible] to R_1 as shown by the heavy arrow. This current magnetizes the core of the coils R_1 and R_2 and induces an electromotive force acting from N to R_2 as shown by the dotted arrow. This electromotive force causes current to flow in the circuit NR_2A_2CL according to the principle of the autotransformer.

Horn-Gap Lightning Arrester. The horn-gap lightning arrester consists of an air gap formed by two horn-shaped pieces of metal H_1 and H_2 shown by Fig. 288. The gap is connected from the line end of a choke coil C to ground. The coil C consists of a few turns of large wire and has an air core. The coil C, having low resistance and few turns of wire, offers but little impedance to the power current whose frequency is low.

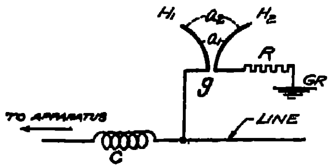


Fig 288 —Horn-Gap Arrester with Choke Coil

Present studies indicate that a lightning discharge is of the nature of an enormous discharge of current that rises with such a steep wave front that it may reach its maximum in 3 or 4 micro-seconds. This charge in distributing itself along a line may be reflected back and forth at a high frequency. An analogy is the sudden throwing of a pailful of water into a tank. The water will surge back and forth until it finally settles down.

A choke coil, if it contained no capacity, would offer a high impedance to a high frequency surge. This can be seen by inspection of the formula $Z = \sqrt{R^2 + 2\pi fL^2}$. When "f" is large, Z becomes large also. In the choke coil, R is so small that it may be neglected and the formula becomes $Z = \sqrt{2\pi fL^2} = 2\pi fL = X_L$ or the coil is practically all reactance.

The choke coil is intended to act as a buffer or stop for the high-voltage, high-frequency surge that tries to find its way to ground by breaking down the insulation of the apparatus. The air gap "g" is to shunt the current to ground. A resistance R limits the current to a safe value

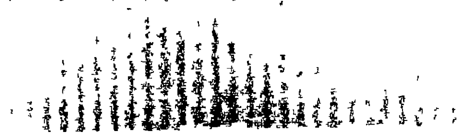
As soon as an arc is formed at "g" the power current tries to flow to ground through the conducting path thus formed by the arc. The arc, however, rises and as it does so, increases in length due to the spreading apart of the horns. At some position "a_2" it becomes so long that the power voltage is not sufficient to maintain it, and it breaks thus causing the power current to cease flowing.

Aluminum-Cell Lightning Arrester. A type of arrester very common a few years ago is known as the electrolytic or aluminum-cell arrester. Figure 289 shows such an arrester. It depends for

Fig. 289. — Type AK Electrolytic Lightning Arrester for 18,500 to 24,600 Volt Service (Westinghouse Electric and Mfg Co)

its action upon a chemical solution of aluminum and the metal aluminum.

If several aluminum cones or trays are stacked up, each containing a solution of aluminum salts as shown by Fig 290 and the upper tray connected to one side of a line and the lower tray to the other side, and voltage applied to the stack, a small current will flow. This current "charges" the stack, that is, forms a film of aluminum hydroxide on the surfaces of the trays. The film has the property of suddenly breaking down after a critical voltage of about 300 volts per tray is reached, and allowing a large current to flow from one end of the stack to the other.



When the voltage drops below the critical value, the film forms again immediately and shuts off the further flow of current. This characteristic is made use of in the electrolytic arrester.

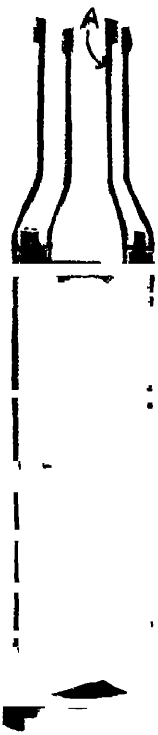


Fig 290 —Tray Structure for Electrolytic Lightning Arrester. (Westinghouse Electric and Mfg Co)

In Fig. 290 one side of line comes in at "A" and the other side comes to the case. The case is grounded in an actual installation. The cones are filled with electrolyte and the whole stack immersed in oil which serves to keep the electrolyte from splashing out and also serves to carry away heat. The charging current, if allowed to flow continuously, would use up considerable energy. It has been found that if the arrester is charged once in 24 hours it will operate satisfactorily. So the arrester is not connected to the line directly except when being charged. It is connected to a horn gap set just above the line voltage to ground. The voltage induced by the lighting disturbance will jump this horn gap, and causes the arrester to operate When it is desired to charge the condenser, the gap is closed through a high resistance. This resistance is turned by means of a handle so as to close the gap.

The Autovalve Arrester. The autovalve lightning arrester made by the Westinghouse Company depends for its action on a glow discharge across small air gaps in series. The gaps have the property of allowing the current to flow when a certain critical voltage is reached and stopping the current as soon as the voltage falls below the critical value. It has been found that the spark-over voltage across an air gap between two flat electrodes is a minimum of about 350 volts, when the spacing of the electrodes is .0003". If the spacing is greater or less than this, the spark-over voltage will be greater than 350 volts. Figure 291

shows a curve of spark-over voltages with different air gaps. If the material of the electrodes is a good conductor, the current will concentrate at some point on the surface of the electrodes and vaporize the material. An arc will form in such a case and the voltage drop to between about 100 and 50 volts. If, however, the electrodes are made of a material with a high resistivity, the current will not concentrate and form an arc, but will continue to pass, as a glow discharge. Figure 292 shows the glow-discharge curve and arc-discharge curve for a spacing of electrodes as described. Disks of high resistivity are used in the autovalve arrester. While

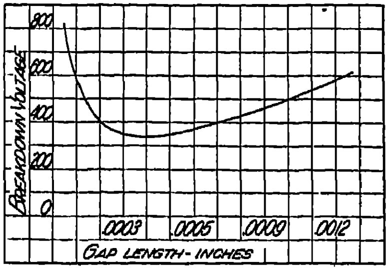


Fig. 291. — Breakdown Voltage of Small Gaps in Air.
(Westinghouse Electric and Mfg. Co.)

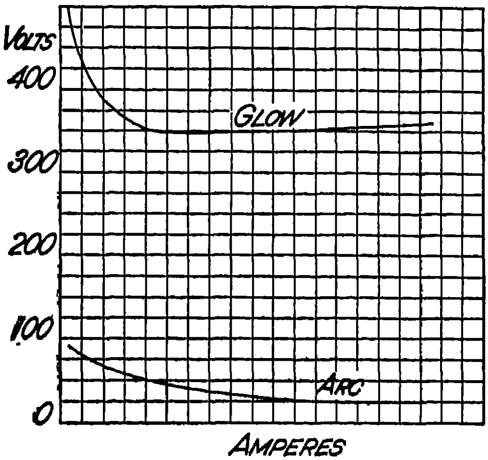


Fig. 292. —Glow and Arc Discharge Curves.
(Westinghouse Electric and Mfg. Co.)

it would seem impractical to make a commercial lightning arrester with gaps as small as .0003″ it has been found that by spacing

the electrodes by as much as .003″ or .005″ by means of mica washers, that the discharge will start at the edge of the washer as shown by Fig. 293 and that the discharge characteristics of

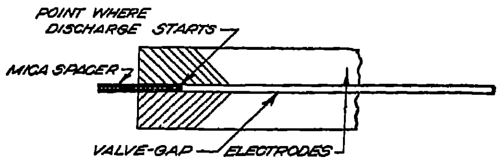


Fig 293 — Enlarged View of Autovalve Electrodes Spacer and Valve Gap.
(Westinghouse Electric and Mfg Co.)

a gap of air alone will be maintained, thus making such an arrester practical to construct.

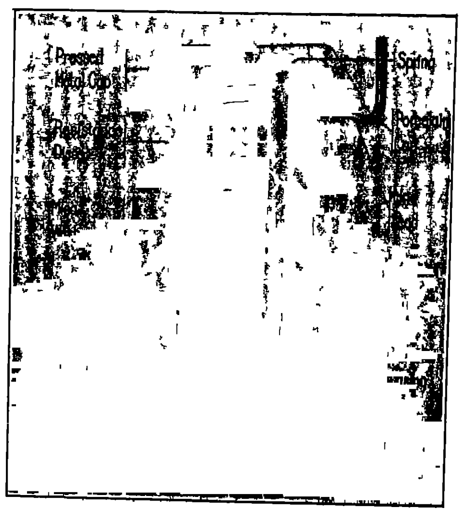


Fig 294. — Sectional View of Autovalve Arrester.
(Westinghouse Electric and Mfg. Co.)

By properly designing the electrodes which are in the form of disks of a composition somewhat resembling porcelain, the discharge current can be controlled for the particular service re-

quired. The arrester consists of a stack of such disks or units connected between line and ground. Enough disks are usually put in series so that the arrester will begin to operate at about twice the normal line voltage The autovalve arrester has the desirable characteristics of maintaining a fairly constant voltage

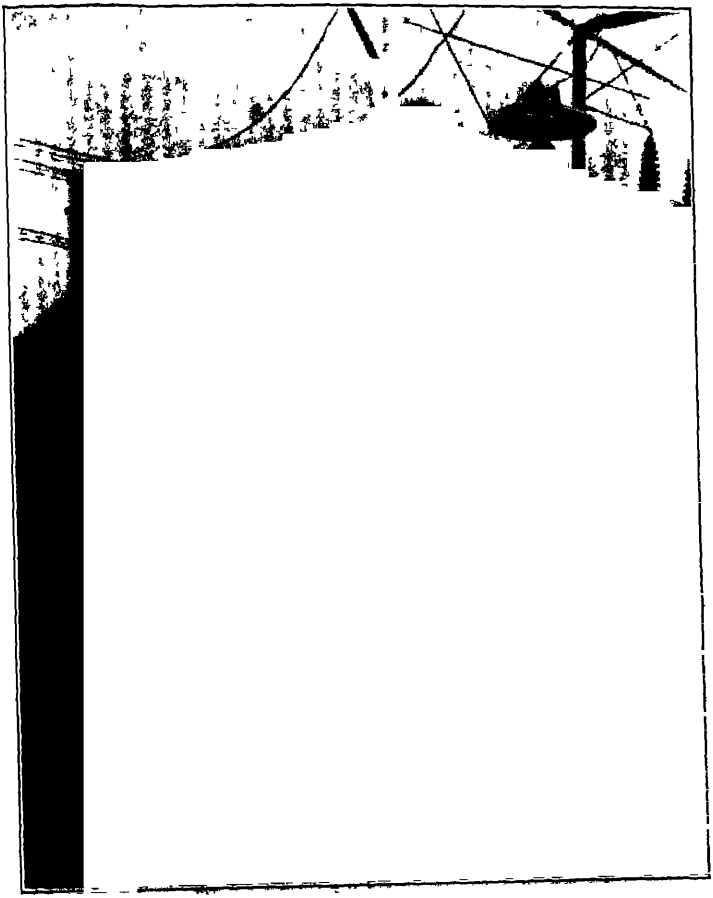


Fig. 295. — Three Phase 73,000 Volt Type S. V. Arrester. (Westinghouse Electric and Mfg. Co.)

of 350 across each gap over a wide range of current-discharge values. That is, the breakdown voltage and discharge voltage are practically the same.

Fig. 296. — Phase Leg of 220 KV. Open Gap Station-Type Arrester.
(Westinghouse Electric and Mfg. Co.)

A sphere gap is placed in series with the stack of disks. This is set for correct operation at an altitude of 1000 feet. For altitudes above 2000 feet, the gap setting should be increased in accordance with settings [illegible] curves furnished by the Westinghouse Company.

Autovalve arresters [illegible] two types. Type S.V. or Station Type up to [illegible] Type L.V. for distribution

and secondary circuits. Figure 294 shows a sectional view of a distribution-type arrester. Figure 295 shows a three-phase 73,000-volt type S.V arrester. Figure 296 shows one phase leg of a 220,000-volt S V arrester. Arresters of the L.V. type for distribution circuits may be had from a voltage of 750 volts to 50,000 volts. Type L.V. arresters for secondary circuits range from 110 volts to 750 volts. Figure 297 shows such an arrester.

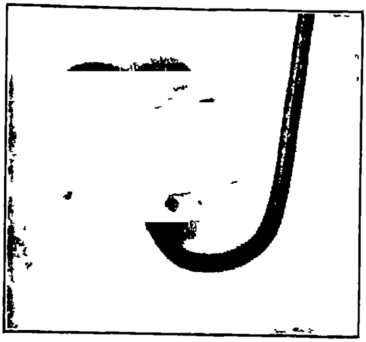


Fig 297. — Type LV Arrester for Circuits up to 750 Volts (Westinghouse Electric and Mfg Co)

Oxide-Film Arrester. The oxide-film arrester is made by the General Electric Company and is suitable for both indoor and outdoor service It is designed for use on A.C circuits from 300 volts to 220,000 volts. In construction, it consists of a stack of disks known as cells, each cell being suitable for about 300 volts. An arrester for a line voltage of 3000 would have 10 cells A spark gap is connected between the stack of cells and the line.

Figure 298 shows an assembled cell and Fig. 299 the parts before assembly. R is a porcelain ring about $7\frac{1}{2}''$ in diameter and $\frac{5}{8}''$ thick. B is a brass plate of which there are two per cell O is some of the powder that is used in the cell. The plates are crimped to the edge of the porcelain ring and form a container for the powder which is lead peroxide. The inside surface of the brass plate is coated with a special varnish.

Fig. 298 — Oxide Film Arrester Cell. (General Electric Co.)

The operation of the arrester is as follows: When the lightning voltage sparks over the gaps, it breaks down the insulating varnish on the metal of the cells. This breakdown is in the form of minute punctures of the varnish. As soon as the breakdown occurs, the current discharges through the cells to the ground and

Fig 299 — Oxide Film Arrester Cell Before Assembly (General Electric Co)

relieves the pressure. The current that flows through the cells immediately causes a chemical change in the powder at the point of puncture. The peroxide is reduced to red lead and litharge. Both of these substances have a high resistance and shut off the generator current that would otherwise follow the lightning discharge. Should another surge come on, the varnish would break down at some other point. The varnish at the point

Fig. 300. — Type of Form BO Oxide Film Arrester for Outdoor Service — Three-Phase 20,000 to 25,000 Volts — Shields of Middle Leg Removed for Inspection. (General Electric Co.)

of puncture is immediately replaced by the oxide or litharge. After the arrester has been in service for a time, the original varnish becomes a honey-comb structure that, in some respects, is better than the original film. The arrester is therefore good for many years of service

Figure 300 shows an outdoor type of arrester suitable for circuits from 20,000 to 25,000 volts. This construction is typical for outdoor service from 15,000 to 37,000 volts maximum. Figure 301 shows an indoor arrester also suitable for circuits from 20,000 to 25,000 volts. This construction is typical for indoor service from 15,000 to 37,000 volts.

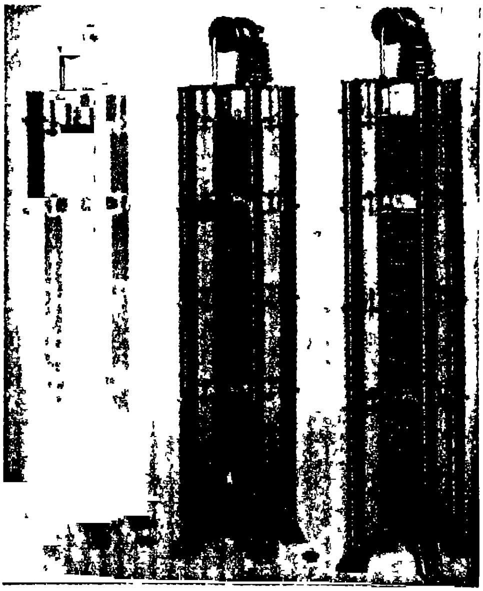


Fig. 301. — Type of Form B Oxide Film Arrester for Indoor Service Three-Phase 20,000 to 25,000 Volts. (General Electric Co.)

Current-Limiting Reactors.—A current-limiting reactor is a reactance coil with a non-magnetic core. It is connected in a circuit to limit the current that will flow in case of a short circuit.

The reactor has a core of concrete and a winding of bare copper cable One form of reactor is shown by Fig. 302. The concrete

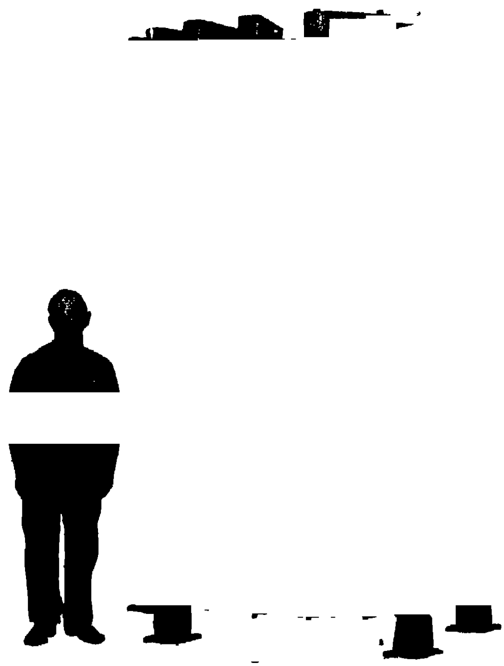


Fig 302 — 80,000 Kv-a. 8000 Volt 10,000 Ampere Current Limiting Reactor.
(General Electric Company.)

serves only as a frame or support. The cable itself is held by wooden pieces bolted to the core. The whole apparatus is supported on legs of insulating material. The necessary reactance is obtained by using a fairly large number of turns in the winding.

Since there is no iron in the circuit, there are no losses due to eddy currents or hysteresis in the core. There is, of course, an I^2R loss in the winding itself. Reactors are placed in the leads of large generators to limit the current that will flow in case of a short circuit. The [illegible] winding of a large

generator, if short-circuited with full field on, are sufficient to tear the windings apart and wreck the machine. A reactor connected in series with a generator will limit the current, in case of a short circuit, to a safe value

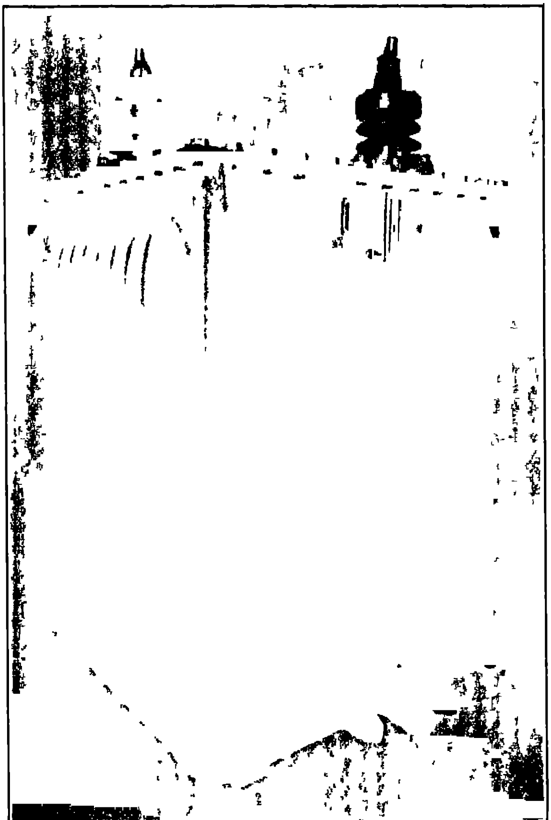


Fig. 303. — Exterior View of Oil Immersed Reactor. (Westinghouse Electric and Mfg. Co)

Reactors are sometimes placed between sections of busses to limit the current that will flow into a section in case of a short circuit. When thus placed, reactors prevent circuit breakers

from opening except in the section where there is the short circuit, thereby keeping the remainder of the system in service.

Figure 303 shows another kind of current-limiting reactor, known as the oil-immersed type. This has a coil which is wound and braced similar to the coil in a transformer. The core, how-

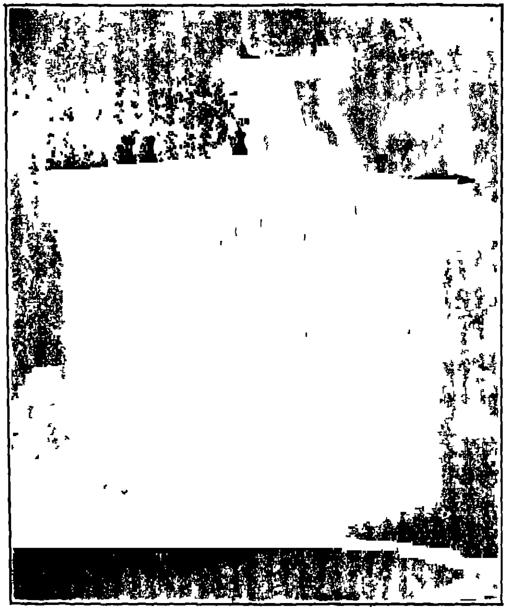


Fig 304. — Method of Shielding Reactor, Showing Coil with Shield Above, Below and on Two Sides of Coil. (Westinghouse Electric and Mfg. Co.)

ever, is of air instead of iron. In order to prevent the lines of force that extend outside the coil from cutting the case, a method of shielding by a laminated iron path of low reluctance outside the coil prevents leakage lines from cutting the case. Figure 304 shows the coil with shields above and below and on two sides of the coil.

Oil-immersed reactors have a high factor of safety against flash-over; they are compact and may be mounted anywhere a

transformer can be mounted, they do not have stray fields as leakage lines are practically shielded, and they are enclosed and protected against dust, water, pieces of metal, etc.

Induction Generator. An induction motor, with its stator excited by alternating current and its rotor turned above synchronism by an outside source of mechanical power, will act as an alternating-current generator.

Consider what happens when an ordinary squirrel-cage motor is running as a motor Polyphase currents are fed into the stator and produce a revolving field which cuts the rotor. This revolving field produces currents in the rotor that react on the field with the result that the rotor turns in the same direction that the field is turning.

The frequency in the rotor is the same as the frequency in the stator or the line frequency, when the rotor is stationary, but becomes less and less as the rotor speeds up. The rotor cannot turn as fast as the field on account of friction, windage, eddy currents, etc., acting as a drag or brake to hold it back

The number of revolutions a minute that it drops behind the speed of the field, expressed as a per cent of the field speed or synchronous speed, is called the slip of the motor.

Consider next, that when near synchronism, a direct-current adjustable-speed motor is coupled to the induction motor and the direct-current motor made to pull up the speed of the rotor to exactly synchronism. There will be no cutting of lines of force by the rotor, since its conductors move just as fast as the field. Suppose next, that the direct-current motor be made to turn the rotor faster than synchronous speed, then its conductors will cut the stator field in the opposite direction from what they cut it when running as a motor, so the rotor will have E. M. F.'s and currents induced in it in the opposite direction and in turn give power back to the stator.

No matter at what speed the rotor turned as a motor, below synchronism the primary or stator frequency remains constant and the same as that of the line and the motor receives power. Similarly, when the rotor is turned above synchronism, the stator

frequency remains the same but the rotor, in cutting the stator field, transfers power to the stator by means of its magnetic flux acting on the stator conductors.

The excitation of the machine must come from the A.C. line when operated either as a motor or generator. We can think of the induction generator as an alternating-current generator which receives its exciting current from the A.C. line in the same way that a transformer or induction motor receives its exciting current. The power component of current is supplied by the driving motor that turns the rotor against the magnetic pull from the flux set up by the current from the line.

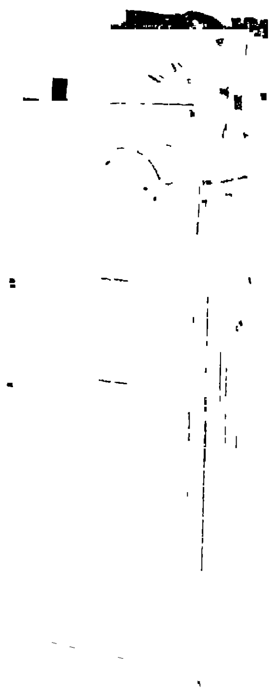


Fig. 305 — Single Phase Feeder Regulator with Automatic Auxiliaries.
(General Electric Company.)

Induction generators are rugged in construction and not subject to extremely large short-circuit currents. They require other alternators to operate with them to supply the excitation, just as synchronous generators require direct-current machines to supply the excitation. It is interesting that with induction generators, exciting and load currents both flow over the same lines.

Induction-Feeder Regulator. When feeders are run a considerable distance from the station or point of distribution, it is common to install apparatus for raising the voltage on individual feeders to take care of the line-drop due to load. One form of apparatus used for this purpose is known as an induction-feeder regulator. Figure 305 shows a General Electric single-phase automatic regulator of this type. The regulator is shown disassembled

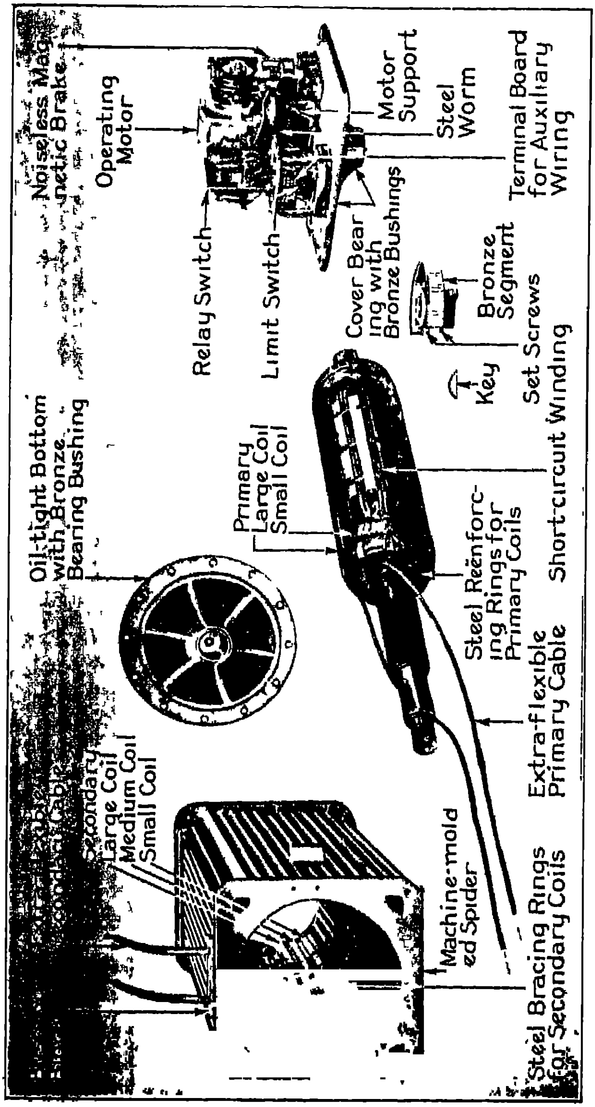


Fig 306. — Dissembled Feeder Regulator.
(General Electric Company)

by Fig. 306 In construction it resembles an induction motor with a wound rotor. Instead of the usual pulley, however, it has a sector of a worm gear keyed to the rotor shaft. This gear meshes with a worm keyed to the shaft of a small motor. The motor is provided with a suitable automatic control mechanism

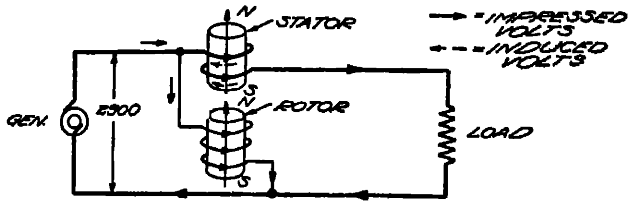


Fig 307 — Rotor in Position of Maximum Lower in Voltage. Poles of Rotor and Stator unlike.

so that it can turn the rotor in either direction, one way, if the feeder voltage is to be raised and the other way, if the voltage is to be lowered. Figures 307 and 308 show the regulator schematically in positions of maximum lower and boost of voltage. The stator consists of a winding suitable to be connected in series

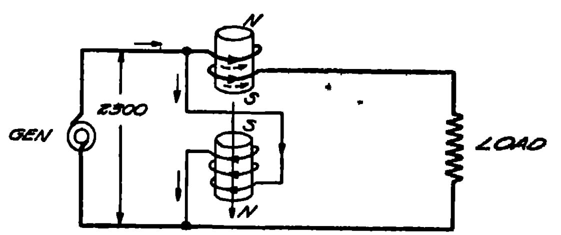


Fig 308 — Rotor in Position of Maximum Boost in Voltage Poles of Rotor and Stator Alike.

with the feeder. The rotor is connected across the two feeders. If the impressed voltages are denoted by full arrows and the induced voltages by dotted arrows, it is clear from Fig. 307 that the field set up by the rotor produces a voltage opposed to the feeder voltage, or lowers it. This happens when the poles of rotor and stator that are opposite each other are unlike. Figure 308 shows that when the rotor has been turned 180° and poles

are alike that the dotted arrows are in the same direction as full arrows or the feeder voltage is boosted. It follows that there will be a partial boost or lower for intermediate positions of rotor

Figure 309 shows the actual arrangement of rotor and stator and their windings The coil R_1 is the active rotor winding and R_2 is an auxiliary winding at 90° from R_1. R_2 is short-circuited. The purpose of R_2 is to decrease the reactance of the rotor as R_1 is turned toward the neutral position.

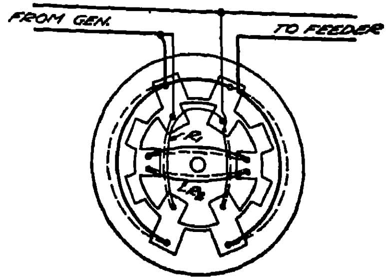


Fig 309 — Arrangement of Rotor and Stator Windings in a Feeder Regulator. (General Electric Company)

Regulators are built single-phase or polyphase. The designations are type LRS for single-phase, IRQ for quarter-phase, and IRT for three-phase.

Relays. A relay is a piece of apparatus that is designed to perform some operation, such as tripping a circuit breaker, locking a switch, assisting another relay to operate, or operating some form of signal.

Relays may be obtained for protection against practically any abnormal condition in a circuit. Among these are: relays for over- or under-current, over- or under-voltage, over- or under-power, overheating of apparatus, reversed polarity, wrong phase rotation, wrong frequency, wrong direction of flow of power, open-phase or unbalanced phases.

In some cases it is desirable that relays shall disconnect apparatus instantly, in others, a momentary disturbance will not injure the apparatus, so a relay is needed that will not shut down machinery at once in case of a sudden disturbance, but will operate after a time, if the disturbance lasts long enough to do harm to the apparatus.

Many relays operate on the principle of the magnetic trip used on the ordinary circuit breaker. In case it is desirable that the plunger of the electromagnet shall move slowly, thereby allowing

a little time to elapse before the circuit is opened, some form of dash pot is attached to the plunger. Other relays operate on the induction principle that has been explained in describing the induction watthour meter.

The Induction Relay. The type CO relay made by the Westinghouse Electric and Mfg Co. is typical of the induction relay and will therefore be described. Its construction is similar to that of the induction watthour meter. In fact, many of its parts are exactly the same as used in the Westinghouse watthour meter.

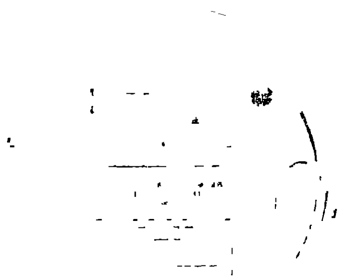


Fig 310 — Type CO Induction Over-Current Relay (Westinghouse Electric and Mfg Co)

Figure 310 shows one of these relays and Fig. 311 is a schematic diagram by which its operation will be explained.

When current flows in the line L, which may be either the line to be protected or the secondary circuit of a current transformer whose primary is connected in the line, the magnet F is energized. Magnets E_1 and E_2 are also energized by current from the transformer Tc, which is called a torque compensator. The disk D will turn and close the contacts C_1C_2 After D has started to turn, the time taken to close the contacts will depend on the speed of the disk, and on how far apart the contacts are set. The speed of the disk is controlled by connecting more or less turns in the coil of the magnet F by means of a screw in the terminal block TB. The settings on the block are usually for 4, 5, 6, 7, 8, 10, and 12 amperes. The setting of the contacts is made by means of a time lever with an index which may be moved along a scale S. This scale is numbered from 1 to 10, and settings on it are used with a graph etched on the name plate of the instrument. Such a curve is shown by Fig. 312. With a setting on the terminal block of 4, for instance, the disk will start to turn when the current in the line L reaches 4 amperes.

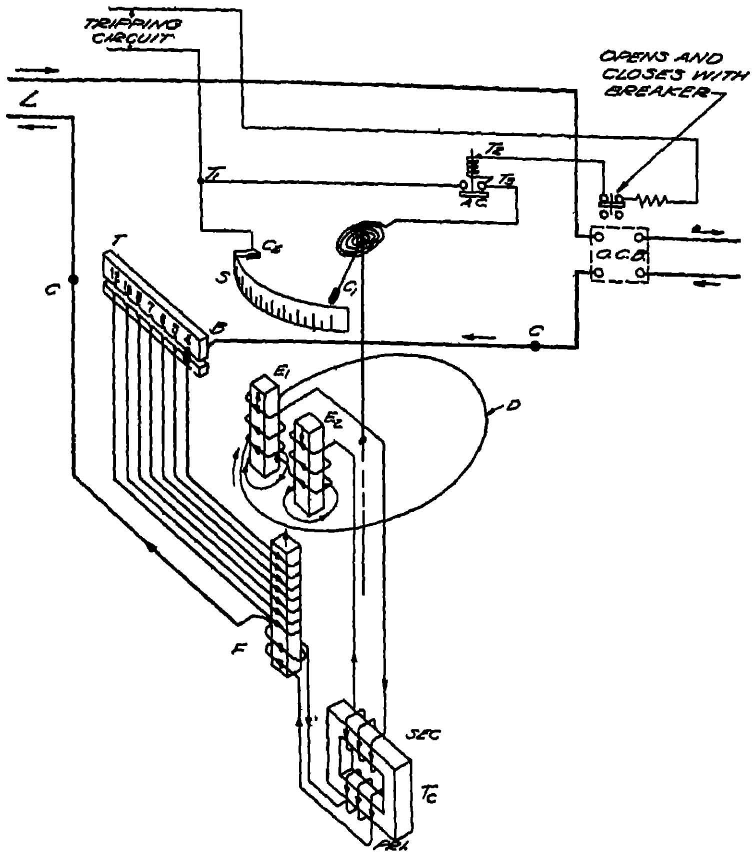


Fig. 311. — Circuits of CO Induction Over-Current Relay (Westinghouse Electric and Mfg Co.)

With a setting of the time-lever index of 10 on the scale S, we refer to the graph which is really a time-current curve plotted from test readings with the time lever set at 10, and find how long it will take to close the contacts for any desired per cent of the current value 4. Suppose that with the current setting of 4, we decide that 40 amperes or 1000% current is the value at which we wish the relay to trip. On the division marked 1000 on the horizontal scale we read upwards to the curve and then

horizontally to the left where we find the time to be 2 seconds. In order to determine a time setting for any other current and time, multiply the required time by 10 and divide by the time as read from the curve, or,

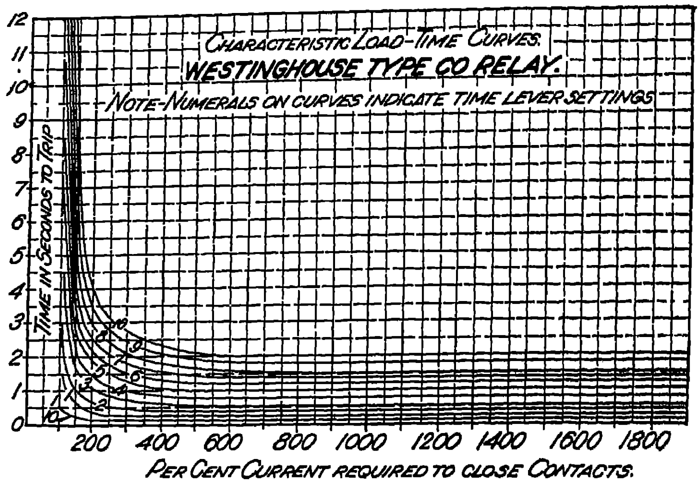


Fig. 312. — Characteristic Load-Time Curves for Westinghouse Type CO Relay

$$\text{Required time index setting} = \frac{\text{Required time} \times 10}{\text{Time read from curve}} \qquad (57)$$

For example, if the relay is to trip in .2 sec., the proper setting of the time lever index would be $\frac{.2 \times 10}{2} = 1$ or at point No. 1 on the scale S.

Thus far, only the mechanism for closing the contacts C_1C_2 has been considered. The trip circuit operates as follows: When C_1C_2 close, current in the trip circuit, which is usually direct current at 110 volts, energizes the auxiliary contactor AC. This closes and shunts the trip current through T_1T_2, thus relieving the main contacts CC_1 of all duty. The trip circuit will remain closed, even though C_1C_2 should open for any reason. The trip circuit should be opened by means of a pallet switch on the oil circuit breaker in the line protected. This switch should be

mechanically connected to the movable part of the breaker and will open when the breaker opens.

PROBLEMS

1. Explain the dynamometer principle used in the construction of several makes of instruments

2. Explain the Weston dynamometer-type wattmeter. Wherein does the ammeter differ from the wattmeter?

3. Explain the principle used in the electromagnetic type of instrument.

4 How is the electrodynamometer principle used in the inclined-coil instrument?

5 What is an electrostatic voltmeter? Mention a place where you would consider it especially desirable.

6. Explain the operation of the induction watthour meter.

7. What is an oscillograph? Mention several places where it can be used.

8. What is a synchroscope?

9. What principle is used in the Westinghouse power-factor meter?

10. Explain the mechanical rectifier

11. What is a kenotron? On what does its action depend?

12 Wherein does the Tungar rectifier differ from the kenotron?

13 Explain the action of the three-element vacuum tube

14. On what property of an electric circuit does the generation of high-frequency currents by means of a vacuum tube depend? Explain

15 Explain the mercury-arc rectifier. For what purpose is it used to a very large extent?

16. What is the function of the horn gap in a lightning arrester? What is the function of the choke coil? Why is a resistance used in series with the gap?

17. Upon what principle does the aluminum cell arrester depend?

18. Upon what principle does the Westinghouse autovalve arrester depend?

19. Upon what principle does the General Electric Oxide-film arrester depend?

20. What are current-limiting reactors used for? Wherein do they differ from transformers?

21. What is an induction regulator? Explain its construction and operation.

22. Give several places where relays can be used

23. Describe the Westinghouse CO relay and the method of setting it for a given current and time to trip.

CHAPTER XII

PRACTICAL TESTS AND MEASUREMENTS

General. The tests and measurements included in this chapter can be carried out in laboratories having a fair amount of equipment. All tests have been selected to have a direct bearing on the theory in the textbook.

In performing experiments and writing up reports, each laboratory will have its own methods. The following general suggestions are offered.

Study the experiment and try to decide what sized instruments will be suited to the work in hand Sketch out the connections you intend to use and connect your apparatus in convenient and systematic order.

Use a good note book for recording observations and rule off a frame work in which to place the actual readings. Letter the various headings of the tabulated work rather than write them.

Where observations have to be substituted in a formula, show the formula and then a sample calculation. In general, transpose the formula so that the quantity you wish to obtain stands at the left of the equality sign, then substitute.

Be systematic and accurate Do not, however, carry out calculations to a degree of accuracy that is not warranted by the instruments you use. For instance, the ordinary voltmeter can be read to 10ths and estimated to 100ths. The figure in the 100ths may or may not be correct. It will be misleading then, to divide such a quantity as 4.57 volts by 3 and give as an answer 1.5233. The correct reading may have been 4.56 or 4.58. Then $4.56 \div 3 = 1.52$ and $4.58 \div 3 = 1.52+$, and to call the answer 1.5233 would indicate an accuracy that is not justified by the original observations.

TEST NO. 1

Connecting Windings of Alternators. In order that the various generator armature-connections, such as star or delta, can be made, a revolving-field type generator, with the terminals of each coil brought to a circular terminal board, is desirable. Such a machine is shown by Fig. 217

If such a machine is not available, one end shield of a revolving-field type machine can be removed and the windings opened. It will not be necessary to break into the coil groups forming individual poles but open the windings between poles.

Run the machine at a constant speed and keep the field current constant Record the volts given by each pole. Then connect the poles of one phase together and read total volts per phase. Try a reading with one pole reversed.

Make the star connection and then try the delta connection, measuring in each case the phase and line voltage. Try reversing one phase and reading the voltages with this phase reversed.

Make a diagram showing each connection you try and draw vectors showing to scale and in proper phase relation, the voltages that you measure.

TEST NO. 2

Voltage Wave of an Alternator. If an alternator be driven at a constant speed and with constant field, a picture of its voltage wave may be obtained either by means of an oscillograph or by a point-by-point method of plotting.

The alternator is connected across a non-inductive resistance of a value sufficient to keep the current to the value desired and pressure leads taken off the resistance at such a distance apart that there will be sufficient drop between them to operate the vibrator of the oscillograph, or if the point-by-point method is used, at such distance apart as to give good readings on the meters used in this method. If the wave of the alternator is desired at no load, the non-inductive resistance should be very high so that the current will be negligible. If the wave is desired

for full load, the resistance should be such as to carry the full-load current of the alternator.

The connections for the point-by-point method of obtaining a voltage wave are shown by Fig. 313.

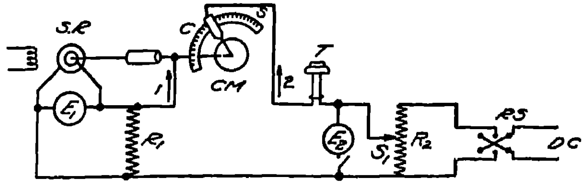


Fig 313 — Connections for Obtaining Voltage Wave of an Alternator Point-by-Point Method.

CM is a contact maker coupled directly to the armature shaft. The contact C may be set on the sector S at any convenient degrees at which readings are desired. R_1 is a non-inductive resistance connected across the slip rings SR of the alternator. E_1 is connected in to obtain the effective A. C. voltage of the machine. A source of D. C. is connected to the resistance R_2 through a reversing switch RS. S is a slider that is used to balance the drop through R_2 against the instantaneous A. C. voltage across the contact maker.

Assuming that at the instant of contact the A. C. voltage is in the direction shown by arrow 1, and the D. C. voltage is in the direction shown by arrow 2, then if arrow 1 balances arrow 2, no current will flow through the telephone receiver. Voltmeter E_2 may be switched in to read the D. C. voltage that balances the A. C. voltage. If the telephone receiver clicks, then S is moved along until no click is heard The reversing switch is necessary in order to obtain the second half of the wave. Values of E_2 are plotted as ordinates and degrees from the sector S are plotted as abscissas. A curve through the points obtained will be the wave of the machine for the load R_1.

TEST NO. 3

Current Wave of Alternator. If a shunt S_n be placed in series with the line from the alternator carrying load, and the apparatus

or obtaining the voltage wave be connected across the shunt instead of the resistance, the drop across the shunt will be proportional to the current so the current wave for the particular type of load can be plotted. By means of R_1 and S_h and a double-throw switch, both E M. F. and current waves may be obtained.

TEST NO. 4

No-Load Magnetization or Saturation Curve for an Alternator. In order to obtain a knowledge of the relation of terminal volts to field current at no load for an alternator, use is made of a no-load magnetization or saturation curve. Readings from which the curve is plotted are obtained by running the alternator at constant normal speed and reading field amperes and terminal volts. There should be no load on the alternator except the voltmeter which is of course negligible

Readings are taken from zero field current up to a value of field current that will give about 125% rated terminal volts It will be found that readings of terminal volts taken with ascending values of field current will not be the same as readings taken with descending values of field current. This difference in readings is due to hysteresis in the magnetic circuit. If, in taking readings with ascending values of field current, it should be necessary to go back to check a reading, reduce the field current to zero and then bring up to the value desired Similarly, with descending values of field current, raise the current to a high value and then reduce it, in case it is necessary to check a reading.

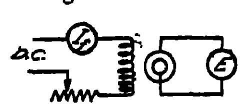


Fig. 314. — Connections for Obtaining Data for Magnetization Curve at No Load.

Connect as in Fig. 314. Take a complete set of readings of terminal volts with ascending values of field current and a similar set with descending values. Plot on the same sheet, curves for each set of readings using terminal volts as ordinates and field currents as abscissas.

TEST NO. 5

Full-Load Magnetization or Saturation Curve for an Alternator. The terminal volts with full load on an alternator will be less than those measured with no load on the machine, due to the impedance drop in the armature and to field distortion. With an inductive load on an alternator there is a further drop in voltage due to the demagnetizing action of the lagging armature current.

The purpose of this test is to show how the terminal voltage of an alternator drops off with load. In this test a non-inductive load such as a water rheostat or bank of lamps should be used. A similar test might be run with an inductive load to show the effect of the lagging current on the terminal voltage.

Connect as in Fig 315. Take readings of field current and terminal volts with constant full-load armature current. The resistance R, which forms the load, will have to be adjusted for each value of field current in order to keep the load constant. Take a set of readings with ascending values of field current and another set with descending values, observing the precautions mentioned in Experiment No. 4 to prevent errors due to hysteresis. Keep the speed constant. Plot the full-load magnetization curve on the same sheet as the no-load curve of Experiment No. 4.

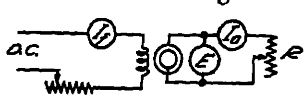


Fig 315. — Connections for Obtaining Data for Magnetization Curve at Full Load

TEST NO. 6

External Characteristic of an Alternator, Non-Inductive Load. The purpose of this test is to show how the terminal voltage of an alternator changes as the load is increased. The test is to be run with a non-inductive load on the machine. As explained on p. 38, the behavior of an alternator will be different when the nature of the load is such as to draw a leading or a lagging current.

Connect as Fig. 316. Adjust the field rheostat to give normal rated volts with the machine running on open circuit. Do not change the setting of field rheostat during the test. Cut in all the

resistance in the armature circuit and then cut out enough to bring the load current to 25% full load. Then take other readings at 50%, 75%, 100% and 125% load. Read load and terminal volts. Keep the speed constant. Record the value of field current that you use.

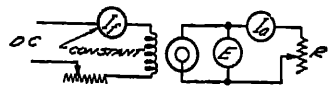


Fig 316 — Connections for Obtaining Data for External Characteristic Curve of an Alternator

Plot a curve with terminal volts as ordinates and loads as abscissas. Letter the abscissas both in amperes and per cent full load.

TEST NO. 7

Parallel Operation of Alternators. In order to gain practical experience in starting alternators and in synchronizing them, the connections of Fig. 317 may be used, and the two alternators synchronized by means of lamps.

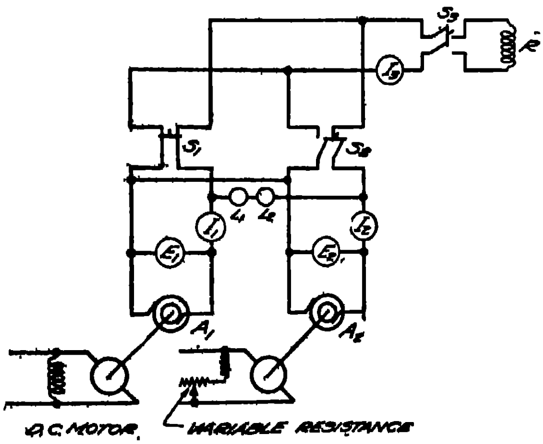


Fig. 317. — Connections for Parallel Operation of Generators

Start alternator A_1 and get it up to speed and on the busses. Then start alternator A_2. When the voltage of A_2 has been adjusted to that of A_1 and its speed about the same, the lamps

L_1L_2 will flicker. Adjust the speed of the driving motor of A_2 until the lamps light and go out slowly. When the period of darkness is about two seconds or more, close the machine switch S_2 The machines should run in parallel. Read the ammeters of the two machines at the time you close the switch and again after the machines are running in parallel. Try varying the field of A_2 and note the effect on the ammeters.

Try loading the machines by a load R and then vary the field rheostats and note the effect on the ammeters. Adjust the rheostats until you get what you consider the most economical condition of running. Explain.

TEST NO. 8

Measurement of Power Factor in Single-Phase Circuit. The power factor of a single-phase circuit may be measured by means of a voltmeter, ammeter and wattmeter. The connections should be as in Fig. 318. Read W, I, and E. The power factor is calculated from the formula,

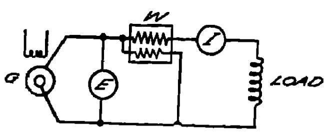


Fig 318 —Connections for Measuring Power Factor.

$$W = EI \times P.F.$$

$$P.F. = \frac{W}{IE}$$

TEST NO. 9

Measurement of Inductance by the Impedance Method. In order to measure inductance by means of the impedance method, use is made of the formula,

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}} \tag{33}$$

I, E and f are carefully measured, using the connection shown

by Fig. 319. R is measured by a separate test using either the D.O.P. or wheatstone bridge method.

The capacity of an ordinary coil is so low that its effect is negligible on ordinary lighting and power frequencies so it may be neglected.

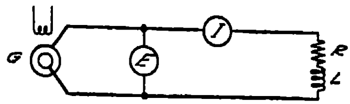


Fig 319 — Connections for Measuring Inductance.

Then $$I = \frac{E}{\sqrt{R^2 + (2\pi fL)^2}}$$

squaring $$I^2 = \frac{E^2}{R^2 + (2\pi fL)^2}$$

clearing of fractions,

$$I^2R^2 + I^2(2\pi fL)^2 = E^2$$

and $$I^2(2\pi fL)^2 = E^2 - I^2R^2$$

From which $$L^2 = \frac{E^2 - I^2R^2}{(2\pi f)^2I^2} = \frac{E^2}{(2\pi f)^2I^2} - \frac{I^2R^2}{(2\pi f)^2I^2}$$

$$L = \sqrt{\frac{E^2}{(2\pi f)^2I^2} - \frac{R^2}{(2\pi f)^2}}$$

$$= \frac{1}{2\pi f}\sqrt{\frac{E^2}{I^2} - R^2} \qquad (58)$$

If the coil contains an iron core, L will vary to some extent with the current and frequency.

TEST NO. 10

Measurements of Capacity. The condenser to be used in this test must be large enough to draw an appreciable current at the voltage and frequency used. The capacity is calculated from the formula,

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}} \qquad (33)$$

It is assumed that R and $2\pi fL$ are both zero, when the con-

denser is connected in the circuit with short wires of fairly large diameter, and the formula becomes,

$$I = \frac{E}{\sqrt{0^2 + \left(0 - \frac{1}{2\pi fC}\right)^2}}$$

$$= \frac{E}{\sqrt{\left(-\frac{1}{2\pi fC}\right)^2}} = \frac{E}{\frac{1}{2\pi fC}}$$

$$I = 2\pi fCE$$

from which
$$C = \frac{I}{2\pi fE} \tag{59}$$

Connect as shown by Fig 320 and measure carefully I, E, and f.

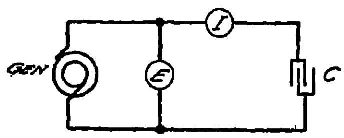


Fig 320 — Connections for Measuring Capacity.

TEST NO. 11

Reactance and Resistance in Series. If a highly-inductive coil such as the primary of a transformer (secondary open) and a non-inductive resistance such as a bank of lamps be connected in

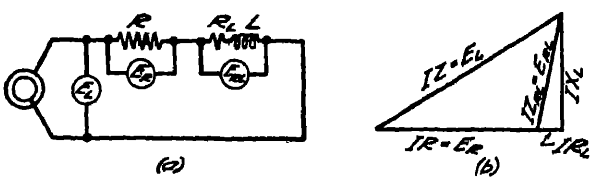


Fig. 321. — Connections and Diagram for Impedances in Series.

series and voltages across the coil, lamps, and line be measured, a triangle can be drawn illustrating the resistance, reactance, and impedance drops across impedances in series. The resistance of the transformer coil should be measured as the transformer is

not entirely reactance. The lamps may be considered to have negligible reactance.

The connections should be as in Fig. 321(a).

The diagram can be constructed as at (b).

TEST NO. 12

Impedances in Parallel. The principles explained under parallel circuits can be illustrated experimentally by connecting a non-inductive resistance in parallel with either an inductance or a capacity.

Connect as in Fig. 322. Read E, I, I_R and I_C. Draw vectors representing E, I_R and I_C in proper phase relation to each other. Combine I_R and I_C and compare with the actual reading of I. Find the power factor of the circuit.

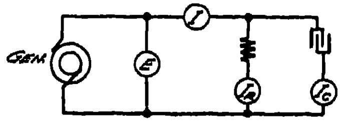


Fig. 322. — Connections for Studying a Circuit which Has Impedances in Parallel.

TEST NO. 13

Resonance in a Series Circuit by Varying Inductance. As explained under resonance in a series circuit, the current becomes maximum when $2\pi fL = \frac{1}{2\pi fC}$. At the point of resonance the current becomes in phase with the voltage. When $2\pi fL$ is greater than $\frac{1}{2\pi fC}$ the current lags and when $\frac{1}{2\pi fC}$ is greater than $2\pi fL$ the current leads. The tangent of the phase angle is

$$\operatorname{Tan} \phi = \frac{2\pi fL - \frac{1}{2\pi fC}}{R}.$$

All these facts are apparent from the formula,

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}} \qquad (33)$$

and the triangle of Fig. 323.

To obtain a condition of resonance in a series circuit, select a non-inductive resistance of known value that, when placed across the circuit to be used, will allow current to pass within range of the ammeter you have. Select a condenser and a variable inductive reactance. The two should be so proportioned that $2\pi fL$ may be made equal to $\frac{1}{2\pi fC}$ by varying L. An inductance coil with taps or one coil sliding within another may be used for L.

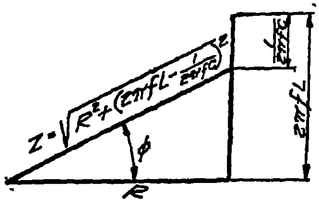


Fig. 323 — Triangle Showing Relation of Resistance, Inductive Reactance and Capacity Reactance.

Connect as in Fig 324.

Take readings of amperes I with several settings of L, starting with a few turns in circuit and ending with all turns in. Plot

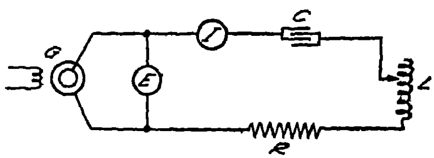


Fig 324 —Connections for Showing Resonance in a Series Circuit

curves with L as abscissas and current as ordinates. Figure the phase angle for each current value and plot a second curve on the same sheet with the first curve showing the phase angles. Use L as abscissas and phase angles as ordinates.

TEST NO. 14

Resonance in a Series Circuit by Varying Frequency. The general scheme for obtaining resonance in a series circuit is the same as that outlined for obtaining resonance by varying the inductance. Resonance may be obtained by varying f if at some frequency the equation $2\pi fL = \frac{1}{2\pi fC}$ is satisfied.

TEST NO. 15

Connecting Transformers. Using standard transformers, check ie connections given by Figs. 191 to 203 by applying voltage to ie side and reading voltages on the other side. Full voltage need ot be used, as at no load satisfactory results may be obtained y using voltages of any convenient proportion of the rated voltges. It is well to take several readings and average.

TEST NO. 16

Core Loss of a Transformer. The true core loss of a transrmer consists of the hysteresis and eddy-current losses in the on due to the rapid reversals of magnetism in the iron core.

If one side of a transformer be connected to a line of proper oltage and frequency for the winding used, and the other side left open-circuited, a wattmeter connected in the line side will ad the watts used up in the transformer at no load. These atts will practically all be used up in supplying the hysteresis d eddy-current losses in the iron but there will be a small numr of watts used in the windings themselves due to the small exing current flowing in the primary winding and eddy currents up in the primary and secondary windings These copper sses are usually so small as to be neglected, so that the watteter reading is called the core loss of the transformer.

An ammeter in the line side of the transformer will read the citing current. In order to compare one transformer with aner, core loss should be taken at normal voltage and frequency d preferably with an alternator giving a true sine wave.

The behavior of a transformer on other than normal frequency d voltage can be determined by testing on these frequencies d voltages and noting the effect on core loss and exciting curnt. Several different readings should be taken when such data to be obtained and the results plotted into curves. For innce, 25 %, 50 %, 75 %, 100 % and 125 % voltage at one frequency one curve and the same per cent voltages at another frequency another curve.

The connections for the core-loss test should be as in Fig. 325. Measure volts, amps, watts and frequency. Either side of the

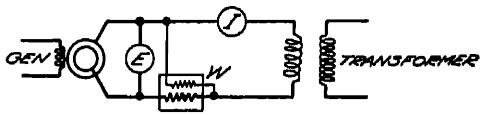


Fig. 325 — Connections for Measuring the Core Loss of a Transformer

transformer may be used for making the core-loss test, but usually the low-voltage side is better adapted to use with the instruments at hand.

TEST NO. 17

Copper Loss of a Transformer — Impedance. The copper loss in a transformer can be calculated by measuring the resistance of the primary and secondary windings and then computing the I^2R loss in each, using for I the normal primary and secondary currents for the respective windings.

The copper loss can be measured directly by means of a wattmeter. To make this measurement, one side of the transformer (usually the low side) is short-circuited and enough voltage impressed across the other side to send full-load current through the line side or primary side. The full-load primary current will cause full-load secondary current to circulate in the secondary. These currents will heat the windings and a wattmeter connected in the primary circuit will read the watts used in heating the copper. In reality there is a very small core loss included in this reading but since the voltage necessary to send full-load current through the short-circuited transformer is only a small per cent of the normal primary voltage, this core loss is negligible.

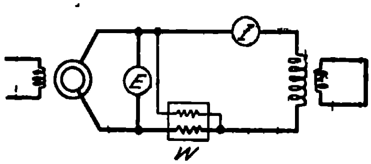


Fig 326 — Connections for Measuring the Copper Loss and Impedance of a Transformer

A voltmeter across the primary side of the transformer will read the voltage necessary to overcome the impedance of the winding or the "impedance volts"

Connect as in Fig. 326. Adjust current to normal full-load value, using normal frequency Read volts, amps, watts and frequency. Calculate the impedance Z

TEST NO. 18

Efficiency of a Transformer. The efficiency of a small transformer can be measured by actually loading it and reading the watts input and the watts output. Then the efficiency is,

$$E\% = \frac{\text{output}}{\text{input}} \times 100 \tag{60}$$

This method is not practical with large transformers or where many small transformers are to be tested, since the method requires loading the transformers and therefore using a large amount of power The same results can be obtained far more economically by measuring the losses, which are only a small per cent of the output, and adding the losses to the rated output to get the input.

Since the appreciable losses are only those in the iron and copper, the efficiency can be obtained from the formula,

$$E\% = \frac{\text{rated output}}{\text{rated output} + \text{core loss} + \text{copper loss}} \times 100 \tag{61}$$

Measure the core loss and copper loss of a transformer by the methods described under core loss and copper loss and calculate [illegible] efficiency of a transformer Take readings suitable for ob-[illegible] efficiency at 75%, 100%, 125% full load.

TEST NO. 19

[illegible] a **Transformer.** It would be possible to load a tra[illegible]ead its primary and secondary voltages and then thro[illegible]nd read the secondary voltage again and from

the full-load and no-load voltage readings calculate the per cent rise in voltage from full load to no load. This would be the regulation. This method is expensive and impractical. The rise in voltage is a small per cent of the secondary voltage and the small difference cannot be read accurately on the voltmeter connected to the secondary.

Regulation can be calculated from the readings taken during the impedance test by the method described on pp. 156 158.

In addition to the name plate data, the following readings are required.

Primary resistance
Secondary resistance
Impedance volts
Impedance watts
Impedance current

TEST NO. 20

Heat Run of Transformers. In addition to the tests for core loss and copper loss, heating tests are run on transformers to determine whether they will actually carry their rated loads without undue heating. It would be too expensive to actually load the transformers with their rated loads, using for instance, motors or water rheostats, so methods have been developed that serve the same purpose, so far as load conditions are concerned, but require far less energy. These methods require only enough energy to supply the losses in the transformers. Several transformers may be run at once.

The method is known as the opposition or "bucking" method and requires two sources of alternating current, one for supplying the iron losses or "exciting" the transformers and the other for supplying the copper losses or "loading" the transformers. The generator for exciting should give normal transformer voltage and frequency. The generator for loading should be capable of supplying full-load current but need not give normal frequency. A frequency lower than normal may be used. With a low frequency, lower voltage will send the load current through the transformers.

When an even number of transformers of the same rating are to be run they are bucked as shown by Fig. 327. Generator

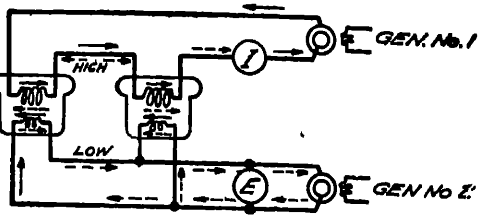


Fig. 327 — Opposition Method of Loading Two Transformers for Heat Run

No. 1 has its voltage adjusted to give twice the impedance volts of one transformer, when two are run. This voltage will send current through the transformers at an instant in the direction shown by the full arrows, and induce currents in the secondary windings as shown also by full arrows The currents will heat the windings just as much as the regular load currents will heat them.

Generator No. 2 has its voltage adjusted to the normal low-voltage rating of the transformers. This machine must give rated frequency of the transformers. It sends current through the low-voltage windings as shown by the dotted arrows, and induces voltage in the high-voltage windings, also shown by dotted arrows. It is seen from the drawing that these voltages in the high-voltage windings oppose each other and that the transformers receive current from generator No 2 equal to twice the exciting current of one transformer, and that this current divides, half going to each. The transformers will be heated by this exciting current and the current supplied and induced by generator No. 1 just as much as if the low-voltage winding were carrying [illegible] normal voltage.

[illegible] 328 shows the method of connecting four transformers [illegible] run. When the number of transformers is such that [illegible] grouped in threes, the open-delta method may be used [illegible] similar transformers. It was stated under Genera[illegible] that when three windings giving equal

voltages and spaced 120° apart on an armature were connected in delta, the instantaneous voltages of two would always balance

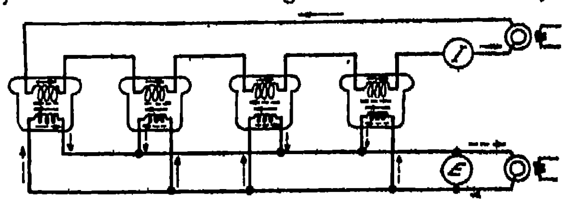


Fig 328. — Opposition Method of Loading Four Transformers for Heat Run

the third. From this it follows that if we open the corner of a delta, there will be no voltage across the opening. This principle is made use of in the open-delta method of loading transformers. The method is, in general, similar to the bucking method just described except that a three-phase machine must be used for exciting.

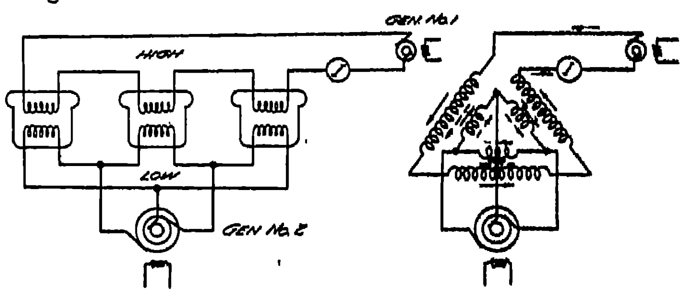


Fig. 329. — Open-Delta Method of Loading Three Transformers for Heat Run.

Figure 329 shows the connections for making a heat run on three similar transformers connected open delta.

TEST NO. 21

Brake Test of a Motor. A motor can be readily tested by loading it by means of a Prony brake and its input and output measured; the input electrically by means of a wattmeter and its output mechanically by means of the brake. If it is desired to

measure the power factor, a voltmeter and ammeter or a power factor meter are needed in addition to the wattmeter. The method of testing by means of a brake has the disadvantage that it requires power equal to the input of the motor and is therefore expensive where a large motor or several small motors are to be tested.

Figure 330 shows the connections to use for testing a single-

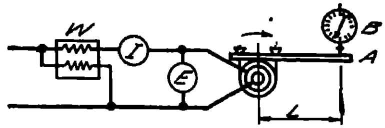


Fig. 330 — Connections for Making a Brake Test of a Single-Phase Motor.

phase motor. The wattmeter, ammeter and voltmeter will give the watts input and the volt-amperes input. From these the power factor can be calculated.

The brake is, in effect, a constant load being pulled at a distance L from the center of the shaft The pull on the balance B may be thought of as the pull on a rope by which the motor is lifting a weight by means of a drum 2L feet in diameter. Hence, if free to move, the end of the arm A would move in one revolution, $2\pi L$ feet (L should be in feet)

In N revolutions per minute A would move $2\pi LN$ feet. If the effective pull on the balance, which is the difference between the pull from the motor and the weight of the arm at A, is P, the foot pounds developed by the motor per minute will be $2\pi LNP$. The horse power will be,

$$\text{H.P.} = \frac{2\pi \text{LNP}}{33000} \tag{62}$$

Measure L in feet. Get the weight of the arm at A. This may be done with sufficient accuracy by taking off the brake and supporting it on a knife edge and reading the weight on the balance.

Read the revolutions per minute and the pull due to the motor and arm. Call the pull due to the motor and arm p_1, and the pull due to the arm alone p_2. The net of effective pull is then $P = p_1 - p_2$.

Obtain readings for plotting curves at 25%, 50%, 75%, 100% and 125% load. Plot the following curves.

(*a*) Horse power output abscissas, efficiency ordinates.

(*b*) Horse power output abscissas, power factor ordinates.

(*c*) Horse power output abscissas, current ordinates.

TEST NO. 22

Test of a Synchronous Motor. The purpose of this test is to obtain practical experience in synchronizing and to show how the motor behaves with weak and strong fields.

Connect the motor the same as for synchronizing a generator and synchronize. Arrange the connections of the driving motor so that they can be changed over to make the driving motor act as a generator after you have synchronized, and use this generator with, say, a water rheostat to load the synchronous motor.

With a light load on the synchronous motor, take several readings of field current, starting with a low value, that is, with the motor under-excited, and ending with a high value of current, or with the motor over-excited. For each value of field current, read the armature volts and amperes for the synchronous motor.

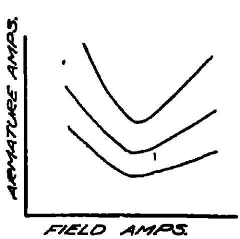


Fig. 331. — V-Curves for Synchronous Motor, Light, Medium and Heavy Loads

Adjust the load on the generator used for load, so that the synchronous motor carries a medium load and take a second set of readings. Take a third set with a heavy load on the synchronous motor.

Plot curves with armature amperes as ordinates and field amperes as abscissas. The curves will have the general shape of those of Fig. 331 and are known as "V" curves for the motor.

The test may be extended to include the efficiency of the synchronous motor. If the efficiency of the D. C. generator has previously been determined and an efficiency curve for various loads has been plotted, the input of the generator, which is the output of the

synchronous motor, may be determined by dividing the generator output by its efficiency for any particular load. The machines should be coupled together rather than belted for this test to avoid losses in the belt.

The input of the synchronous motor may be measured by putting a wattmeter in its armature circuit and adding to the reading of the wattmeter the watts taken by the field of the synchronous motor to obtain the total input of the synchronous motor.

The output of the synchronous motor divided by its input when expressed as a per cent will be the efficiency of the synchronous motor.

TEST NO. 23

Circle Diagram for a Three-Phase Induction Motor. The theory of the circle diagram is outlined on pp. 194–198 and on p. 198 the necessary readings to be taken are listed.

Connect the motor so that it can be run with no load and the input measured. The readings necessary for this part of the test are: volts, amperes and watts per phase.

The rotor is next blocked and reduced voltage applied to give about full-load current. Volts, amperes and watts are read again.

The resistance of the motor is next measured. In getting effective resistance of rotor and stator it is best to take several readings with rotor in different positions and average them.

To construct the diagram, draw OX and lay off OV 90° from OX. OV is drawn to scale to equal volts per phase.

Since $P = \sqrt{3}EI \cos \phi$ in a three-phase circuit, (38a) $\cos \phi = \frac{P}{\sqrt{3}EI}$, so OI_0 may be laid off at an angle with OV such that $\cos \phi_0 = \frac{P_0}{\sqrt{3}\,EI}$ where $P_0 =$ the power running light per phase, E the volts and I_0 the current per phase.

With the rotor blocked, the current OI_B that would flow if full voltage were impressed upon the motor would be $I_B = I_R \times \frac{OV}{E_R}$

where OV is full volts per phase, E_R reduced volts per phase and I_R the current that flows when the voltage E_R is impressed.

When P is the total power input to the motor with the rotor blocked, the power per phase is $\frac{P}{3}$ and $\angle VOI_B = \cos\phi_B = \frac{P_R}{3 I_B E_R}$.

Having located OI_0 and OI_B, draw I_0I_B and I_0X_0. Erect a perpendicular at the center of I_0I_B. This will cut I_0X_0 at a point which will be the center for the semicircle of the diagram. Draw the semicircle passing through I_0 and I_B.

From the diagram find,

(*a*) core loss friction and windage
(*b*) maximum input
(*c*) maximum output
(*d*) maximum power factor
(*e*) efficiency
(*f*) slip
(*g*) primary and secondary copper loss full load.

CHAPTER XIII

TRIGONOMETRY USEFUL IN SOLVING VECTOR PROBLEMS

Functions of an Angle. When CAB, Fig. 332, is a right-angle triangle,

$$\text{Sin A} = \frac{a}{c} \quad (63) \qquad \text{Csc A} = \frac{c}{a} \quad (66)$$

$$\text{Cos A} = \frac{b}{c} \quad (64) \qquad \text{Sec A} = \frac{c}{b} \quad (67)$$

$$\text{Tan A} = \frac{a}{b} \quad (65) \qquad \text{Cot A} = \frac{b}{a} \quad (68)$$

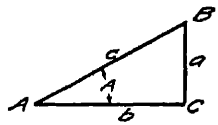


Fig. 332 —Right-Angle Triangle.

Oblique Triangles. In any triangle as Fig. 333.

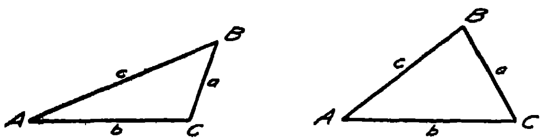


Fig. 333 —Oblique Triangles.

$$\frac{a}{b} = \frac{\text{Sin A}}{\text{Sin B}} \quad (69) \qquad a = \sqrt{b^2 + c^2 - 2bc \cos A} \quad (72)$$

$$\frac{a}{c} = \frac{\text{Sin A}}{\text{Sin C}} \quad (70) \qquad b = \sqrt{a^2 + c^2 - 2ac \cos B} \quad (73)$$

$$\frac{b}{c} = \frac{\text{Sin B}}{\text{Sin C}} \quad (71) \qquad c = \sqrt{a^2 + b^2 - 2ab \cos C} \quad (74)$$

Algebraic Sign of Functions. The algebraic sign of the functions Sin, Cos, Tan, Csc, Sec, and Cot will depend upon the quadrant in which the angle is located. Let the angle be formed

by a radius AB turning about a center A, Fig. 334. Read X plus if at right of A and minus if at left of A. Read Y plus above A and minus below A. Read radius AB plus in any position. The algebraic signs of Sin, Cos, Tan, Csc, Sec and Cot will then be as below:

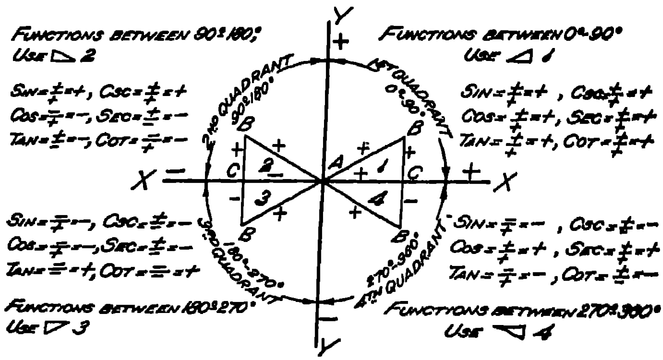


Fig. 334 — Diagram Showing Algebraic Signs of Functions.

From the above diagram the algebraic sign of a function in the first quadrant (0 to 90°), such as Tan, is +. The sign of Tan in the second quadrant (90° to 180°) is −, etc.

Use of Functions. Given the hypothenuse c and the angle A, (Fig. 335), to find side a.

From (63) $\frac{a}{c} = \text{Sin A}$

Multiplying by c

$a = c \text{ Sin A}$

We have given $c = 100$

$A = 30°$

From Table D p. 312

$\text{Sin A} = .5$

Substituting in $a = c \text{ Sin A}$

$a = 100 \times .5$

$= 50$ Ans.

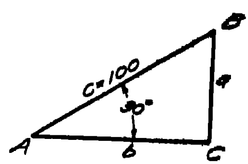


Fig. 335 — Right Angle Triangle with Hypothenuse C and Angle A given.

Given the sides a and b of a right-angle triangle Fig. 336, to find angle A and side c.

To find angle A.

From (65) $\frac{a}{b} = \text{Tan } A$

We have given $a = 36.4$
$b = 100$

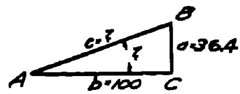


Fig. 336 — Right-Angle Triangle with Sides a and b Given.

Substituting $\frac{a}{b} = \text{Tan } A$

$$\frac{36.4}{100} = .364$$

From Table D p. 312

Angle $A = 20°$ Ans.

To find side c

From (66) $\text{Csc } A = \frac{c}{a}$

Multiplying by a

$$a \text{ Csc } A = c$$

We have given $a = 36.4$
$A = 20°$

From Table D p. 312

$\text{Csc } 20° = 2.9238$

Substituting in $a \text{ Csc } A = c$

$$36.4 \times 2.9238 = 106.4 \text{ Ans.}$$

Given the sides a and b of an oblique triangle, to find the side c and the angle CAB.

From (74) $c = \sqrt{a^2 + b^2 - 2ab \text{ Cos } C}$

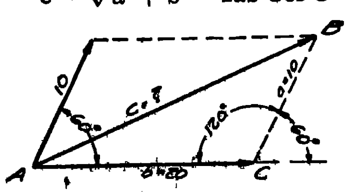


Fig. 337. — Oblique Triangle with Sides a and b Given.

We have given

$$a = 10$$
$$b = 20$$
$$C = 120°$$

From Fig. 334 it will be seen that

$$-\text{Cos } 120° = +\text{Sin } 30°$$

or

$$\text{Cos } 120° = -\text{Sin } 30°$$

Substituting in

$$c = \sqrt{a^2 + b^2 - 2ab \text{ Cos } C}$$
$$c = \sqrt{10^2 + 20^2 - 2 \times 10 \times 20 \times -.5}$$
$$= \sqrt{100 + 400 + 200}$$
$$c = 26\ 5 \text{ Ans}$$

To find the angle CAB

From (70) $$\frac{a}{c} = \frac{\text{Sin } A}{\text{Sin } C}$$

Multiply by Sin C $$\frac{a}{c} \text{Sin } C = \text{Sin } A$$

or

$$\text{Sin } A = \frac{a}{c} \text{Sin } C$$

We have

$$a = 10$$
$$c = 26\ 5$$
$$\text{Sin } C = .5$$

Substituting in

$$\text{Sin } A = \frac{a}{c} \text{Sin } C$$
$$\text{Sin } A = \frac{10}{26.5} \times .5$$
$$= .1887$$

Then

$$.1736 = \text{Sin } 10° \text{ from Table of Sines}$$ p. 312
$$.1887 = \text{Sin } X°$$
$$.1908 = \text{Sin } 11° \text{ from Table of Sines}$$

Difference for 1° = .1908 − .1736 = .0172
Difference for X° = .1887 − .1736 = .0151

$$X^\circ = 10^\circ + \frac{.0151}{.0172} \times 60' = 10^\circ\, 53' \quad \text{Ans.}$$

Find E_{AO} and angle α, Fig. 338.

$$E_{AB} = 12$$
$$E_{BO} = 20$$
$$\angle OE_{AB}\, E_{AO} = 110^\circ$$
$$\text{Cos } 110^\circ = -\text{ Cos } 70^\circ$$
$$= -\ .342$$

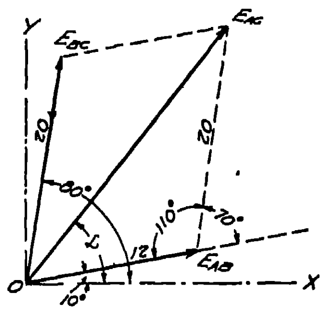


Fig. 338 — Vector Diagram for Solution by Trigonometry.

From (74)

$$OE_{AO} = \sqrt{12^2 + 20^2 - 2 \times 12 \times 20 \times -.342}$$
$$= 26.6$$

From (70)
$$\frac{\text{Sin } E_{AB}OE_{AO}}{\text{Sin } 110^\circ} = \frac{20}{26.6}$$

$$\text{Sin } E_{AB}OE_{AO} = \text{Sin } 110^\circ \times \frac{20}{26.6}$$
$$\text{Sin } 110^\circ = \text{Sin } 70^\circ$$
$$\text{Sin } 70^\circ = .9397$$
$$= .9397 \times \frac{20}{26.6}$$
$$= .7065$$

From the table of sines (Table D) the angle whose natural sine is .7065 is 44° 57′.

Hence $\alpha = 44^\circ\, 57' + 10^\circ = 54^\circ\, 57'$ Ans.

Table D. Table of Natural Functions

Angle	sin	cos	tan	cot	sec	csc	Angle
0°	0000	1 0000	0000		1 0000	. . .	90°
1	0175	9998	0175	57 2900	1 0002	57 2987	89
2	0349	9994	0349	28 6363	1 0006	28 6537	88
3	0523	9986	0524	19 0811	1 0014	19.1073	87
4	0698	9976	0699	14 3007	1 0024	14 3356	86
5	0872	.9962	0875	11 4301	1 0038	11 4737	85
6	1045	9945	1051	9 5144	1 0055	9 5668	84
7	1219	.9925	1228	8 1443	1 0075	8.2055	83
8	1392	9903	1405	7 1154	1 0098	7.1853	82
9	1564	9877	1584	6 3138	1 0125	6.3925	81
10	1736	9848	1763	5 6713	1 0154	5.7588	80
11	1908	9816	1944	5 1446	1 0187	5 2408	79
12	2079	9781	2126	4 7046	1 0223	4 8097	78
13	2250	9744	2309	4 3315	1 0263	4 4454	77
14	2419	9703	2493	4 0108	1 0306	4 1336	76
15	2588	9659	2679	3 7321	1 0353	3 8637	75
16	2756	9613	2867	3 4874	1 0403	3.6280	74
17	2924	9563	3057	3 2709	1 0457	3 4203	73
18	3090	.9511	3249	3 0777	1 0515	3 2361	72
19	3256	9455	3443	2 9042	1 0576	3 0716	71
20	3420	9397	3640	2 7475	1 0642	2 9238	70
21	3584	9336	3839	2 6051	1.0711	2.7904	69
22	3746	9272	4040	2 4751	1 0785	2.6695	68
23	3907	9205	4245	2 3559	1 0864	2.5593	67
24	4067	9135	4452	2 2460	1 0946	2.4586	66
25	4226	9063	4663	2 1445	1 1034	2 3662	65
26	4384	8988	4877	2.0503	1.1126	2 2812	64
27	4540	8910	5095	1 9626	1 1223	2 2027	63
28	4695	8829	5317	1 8807	1 1326	2 1301	62
29	4848	.8746	5543	1.8040	1 1434	2 0627	61
30	5000	8660	5774	1 7321	1 1547	2.0000	60
31	5150	.8572	6009	1 6643	1.1666	1 9416	59
32	5299	8480	6429	1.6003	1 1792	1.8871	58
33	.5446	.8387	6449	1 5399	1 1924	1.8361	57
34	5592	8290	6745	1 4826	1 2062	1 7883	56
35	.5736	.8192	.7002	1 4281	1 2208	1.7434	55
36	.5878	.8090	.7265	1 3764	1 2361	[illegible]	[illegible]
37	.6018	.7986	.7536	1 3270	1 2521	[illegible]	[illegible]
38	.6157	.7880	7813	1 2799	1.2690	[illegible]	52
39	.6293	.7771	8098	1 2349	1.2868	[illegible]	51
40	.6428	.7660	8391	1.1918	[illegible]	1 5557	50
41	6561	.7547	8693	1.1504	1.3250	1 5243	49
42	6691	7431	9004	1 1106	[illegible]	1 4945	48
43	.6820	.7314	9325	1 0724	[illegible]	1 4663	47
44	.6947	.7193	.9657	1.0355	1 3902	1 4396	46
45	.7071	.7071	1 0000	1 0000	1 4142	1.4142	45
Angle	cos	sin	cot	tan	csc	sec	Angle

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